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Effect of the Magnetic Field on Forced Convection Flow Along a Wedge with Variable Viscosity

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This paper presents a study of the flow of a viscous incompressible fluid along a heated wedge, taking into account the variation of the viscosity with temperature. The flow is under the influence of a magnetic field $B(x)$ along y direction applied perpendicular to the surface of the boundary layer along x direction and an electric field $E(x)$ along z direction. The boundary layer equations are transformed to nonlinear ordinary differential equations and are solved numerically. The effects of the magnetic field on the velocity and the temperature and the shear stress on the surface (τ_w) are studied. It is found that the velocity of the fluid increases with increasing the magnetic field parameter M, with the other parameters kept constant. It is also established that the temperature of the wedge decreases with increasing M value. The value of the skin friction increases whereas the rate of heat transfer decreases owing to increasing the magnetic field parameter and also according to decreasing the viscosity.

Keywords: convection, saturated medium, buoyancy.

1. Introduction

Laminar forced convection and heat transfer of incompressible Falkner-Skan flows for an isothermal wedge has been studied by many investigators. The viscosity of gases generally increases with temperature, whereas liquid viscosity decreases with temperature. Therefore, for heating a fluid, the effect of temperature on viscosity is to decrease transport in gases and to increase transport in liquid.

The different relations between the physical properties of fluid and temperature are given by [1–4]. Elbashbeshy and Ibrahim [5] studied the flow of viscous incompressible fluid along a heated vertical plate, taking into account the variation of the viscosity and thermal diffusivity with temperature. Elbashbeshy and Dimian [6] studied the effect of radiation on the flow and heat transfer over an isothermal wedge with variable viscosity.

The action of the magnetic field on the fluid has many practical applications for example, plasma welding, nuclear industry and many other, so the effect of an applied magnetic field on the fluid past a semi-infinite plate was studied by many researchers [7,8,9,10]. Ibrahim and Terbeche [7] studied, the boundary layer flow of a power law non-Newtonian fluid in the presence of magnetic field $B(x)$ applied perpendicularly to the surface and an electric field $E(x)$ perpendicular to the $B(x)$.

They have also and the direction of the longitudinal velocity in the boundary layer with constant viscosity. The unsteady laminar boundary layer flow of an electrically conducting fluid past a semi-infinite flat plate with an aligned magnetic field has been studied by Takhar *et. al.* [8]. They assumed that when time the plate is impulsively moved with a constant velocity which is in the same or opposite direction to that of free stream velocity. They took into account the effect of the induced magnetic field and the viscosity was constant. Ibrahim [9] presented an analytical and numerical solution for the momentum and thermal boundary layer equations of a non-Newtonian power law fluid over a wedge. Recently, Elbashbeshy [10] studied the steady free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of a magnetic field.

The present work has the advantage that it improves the results obtained earlier in [6] and [7] as follows:

(i) In [6] the effect of the magnetic field can be inserted,

(ii) In [7] the variation of the viscosity with temperature can be taken into account.

Moreover similar solution of the boundary layer equations is obtained. The variations of the dimensionless velocity, temperature profiles with the parameters of the flow are obtained. The skin friction coefficient and the Nusselt number are also calculated for considered case.

2. Formulation of the problem

Consider a steady forced convection along a wedge imposed in a magnetic field. We assume that the origin is taken to be the front point of the wedge, x is the direction of the surface of the wedge and y is normal to it upward as shown in Figure 1. Let $\bar{V}$ be the velocity of the fluid and u and v are the velocity components in x and y directions, respectively. We also assume that the magnetic field $\bar{B}(x)$ is perpendicular to the surface of the boundary layer, i.e., it is function of x and parallel to y-axis $\bar{B}(x) = B(x)\bar{j}$.

Here we assume that the induced magnetic field which is produced by the motion of the electrically conducting fluid is negligible because it is small compared with the magnetic field. This assumption is valid for small magnetic Reynolds number. Let the electric field $\bar{E}(x)$ be perpendicular to both $\bar{B}(x)$ and the direction of the longitudinal velocity in the boundary layer, i.e., $\bar{E}(x) = E(x)\bar{k}$. Also let the temperature of the wedge surface be T_w and T_∞ be the temperature of the free stream having the velocity U_∞.

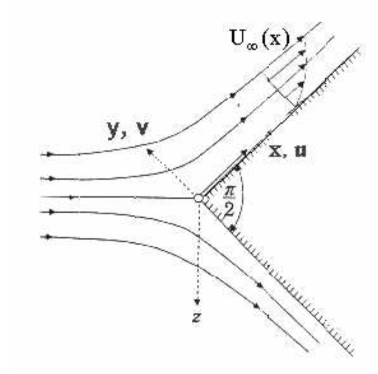

Figure 1 Coordinates (x, y, z) and the components of the velocity over a wedge

Thus we can write the boundary layer equations which are the conservation of mass, momentum and heat as [7,11,12]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\frac{1}{\rho}\left(\bar{J}\wedge\bar{B}\right)_x\,, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}\,, \tag{3}$$

$$\bar{J} = \sigma\left(\bar{E}+\bar{V}\wedge\bar{B}\right)\,. \tag{4}$$

Subject to boundary conditions:

$$u=v=0\,,\quad T=T_w\quad \text{at } y=0\,,$$
$$u\to U_\infty(x)=Cx^{\frac{1}{3}}\,,\quad T\to T_\infty\quad \text{as } y\to\infty\,, \tag{5}$$

where ρ is the density of the fluid, P is the pressure of the fluid, μ is the variable viscosity coefficient, $\bar{J}$ is the electric current vector, $(\bar{J}\wedge\bar{B})_x$ is the x-component of $(\bar{J}\wedge\bar{B})$, α is the thermal diffusivity, σ is the electrical conductivity, T is the temperature of the fluid, $U_\infty(x)$ is the main stream velocity at the edge of the boundary layer and C is constant. The velocity component u will in general increase from zero value at the wedge surface to the value $U_\infty(x)$ at the edge of the boundary layer. If $\bar{i}$ and $\bar{k}$ are unit vectors in the x and z directions respectively, then

$$\bar{J}=\sigma(E+uB)\bar{k}\,, \tag{6}$$

$$\left(\bar{J}\wedge\bar{B}\right)_x=-\sigma B(E+uB)\,. \tag{7}$$

On the edge of the boundary layer, the velocity u equals the outer flow velocity $U_\infty(x)$ while the pressure P does not change, thus equations (2) and (7) will give

$$U_\infty\frac{dU_\infty}{dx}=-\frac{1}{\rho}\frac{\partial P}{\partial x}-\frac{\sigma B}{\rho}(E+U_\infty B)\,. \tag{8}$$

Eliminating $\frac{\partial P}{\partial x}$ from equations (2) and (8) and then using (7), we get

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\frac{\sigma B^2}{\rho}\left(U_\infty-u\right). \tag{9}$$

Thus our governing equations are (1), (9) and (3) subject to the boundary conditions (5). The continuity equation (1) is satisfied by introducing the stream function $\psi(x,y)$ such that

$$u=\frac{\partial\psi}{\partial y}\,,\quad v=-\frac{\partial\psi}{\partial x}\,. \tag{10}$$

To transform the partial differential equations (9) and (3) into a set of nonlinear ordinary differential equations, the following dimensionless variables are introduced [6]:

$$\begin{array}{ll}\eta=y\sqrt{\frac{2C}{3\nu}}x^{\frac{1}{3}}\,, & \psi=\sqrt{\frac{3C\nu}{2}}x^{\frac{2}{3}}f(\eta)\,,\\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\,, & \nu=\frac{\mu_0}{\rho}\,,\end{array} \tag{11}$$

where $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature and $\nu=\frac{\mu_0}{\rho}$ is the kinematic viscosity and μ_0 is the viscosity of the fluid outside the boundary layer (as $y\to\infty$).

It has been proved that, if the outer flow velocity $U_\infty(x)$ (potential flow) is proportional to the arc length x raised to a power m, then the similarity solution for equations (9) and (3) is possible only when $m=\frac{1}{3}$, which represents flow past a wedge of included angle $\frac{\pi}{2}$. Also for the magnetic field $B(x)\bar{j}$, it is found that the power of x must equal to $-\frac{1}{3}$. Thus,

$$U_\infty(x)=Cx^{\frac{1}{3}}\,,\quad B^2(x)=B_0^2x^{-\frac{2}{3}}\,, \tag{12}$$

where C and B_0 are constants.

The variation of the viscosity with temperature θ, is taken in the form [5,9,16]:

$$\mu=\mu_0\mathrm{e}^{-\gamma\theta}\,, \tag{13}$$

where γ is the viscosity parameter depending on the nature of the fluid. By using the transformations (11)–(13), equations (9) and (3) are transformed to

$$f'''-\gamma f''\theta'+\mathrm{e}^{\gamma\theta}ff''+\frac{1}{2}\mathrm{e}^{\gamma\theta}(1-f'^2)+M\mathrm{e}^{\gamma\theta}(1-f')=0\,, \tag{14}$$

$$\theta''+\mathrm{Pr}f\theta'=0\,, \tag{15}$$

with the boundary conditions:

$$\begin{array}{lll}f=f'=0\,, & \theta=1 & \text{at } \eta=0\,,\\ f'\to1\,, & \theta\to0 & \text{as } \eta\to\infty\,,\end{array} \tag{16}$$

where $M=\frac{3\sigma b^2}{2\rho C}$ is the magnetic field parameter and Pr$=\frac{\nu}{\alpha}$ is the a Prandtl number. The prime denotes differentiation with respect to η.

3. Results and discussion

The nonlinear ordinary differential equations (13) and (14) with the boundary conditions (15) have been solved by the fourth-order Runge Kutta integration scheme along with the Nachtshem-Swigert shooting technique [12] with error of order 10^{-6}. The procedure is to estimate the unknown values of $f''(0)$ and $\theta'(0)$. If we neglect the electric and magnetic fields and take the viscosity constant ($M = 0, \gamma = 0$), then equations (13) and (14) reduce to those of heat transfer for the Falkner-Skan flows [13,14]

$$f''' + ff'' + \beta(1 - f'^2) = 0\,, \tag{17}$$

$$\theta'' + \mathrm{Pr} f\theta' = 0\,, \tag{18}$$

subject to the same boundary conditions

$$\begin{aligned} f = f' = 0\,, \quad \theta = 1 \quad \text{at } \eta = 0\,, \\ f' \to 1\,, \quad \theta \to 0 \quad \text{as } \eta \to \infty\,, \end{aligned} \tag{19}$$

(for the special case if the angle of the wedge $= \frac{\pi}{2}$, so $\beta = \frac{1}{2}$). In order to verify the accuracy of our present method, we have compared our results with those of [14] at Pr$= 0.733$, $\beta = 0.5$ we got $f'' = 0.927685$, but in [14] $f'' = 0.9277$, which is in good agreement.

If the viscosity is constant ($\gamma = 0$), our equations will reduce to those of Ibrahim [9] in his special case for Newtonian fluid ($n = 1$) and if there is no dissipation of energy due to the viscosity and the electric field.

The physical quantities of interest are the skin friction coefficient and the Nusselt number. The shearing stress on the surface is defined as

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \mathrm{e}^{-\gamma}\mu_0\sqrt{\frac{2C^3}{3\nu}}f''(0)\,, \tag{20}$$

where $f''(0)$ is the skin friction coefficient.

The local Nusselt number for heat transfer in the present case is defined by

$$\mathrm{Nu} = -x\frac{\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty} = -x^{-\frac{2}{3}}\sqrt{\frac{2C}{3\nu}\theta'(0)}\,, \tag{21}$$

where $\theta'(0)$ is the heat rate transfer.

The range of variation of the parameters of the flow y and the Prandtl number, Pr, can be taken as follows [15]:

(i) for air: $-0.7 \leq \gamma \leq< 0$, Pr=0.733,

(ii) for water: $0 \leq \gamma \leq 0.6$, $2 \leq \mathrm{Pr} \leq 6$.

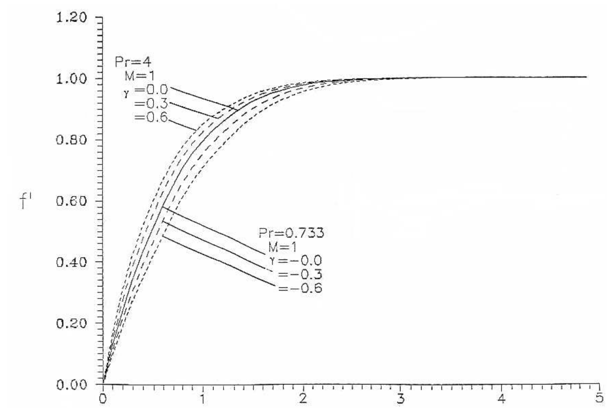

Figure 2 Representation of the velocity f' with different values of the viscosity parameter γ at $M = 1$, Pr=0.733 for air and Pr=4 for liquid

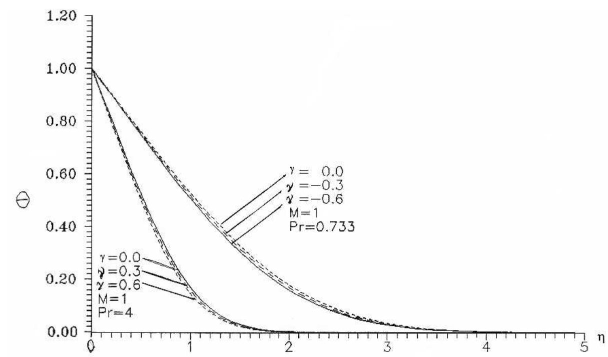

Figure 3 Representation of the temperature θ with different values of the viscosity parameter γ at $M = 1$, Pr=0.733 for air and Pr=4 for liquid

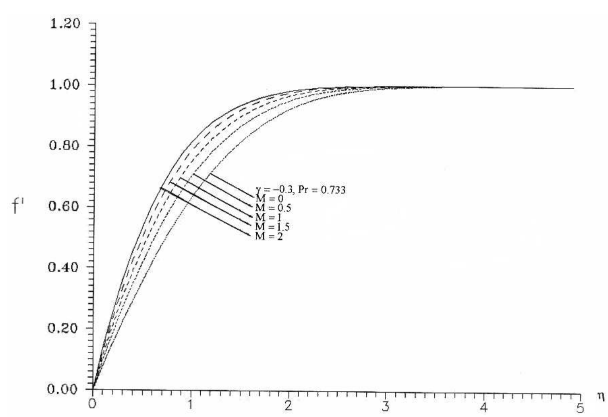

Figure 4 Representation of the velocity f' with different values of the magnetic field parameter M ($M = 0.0$, 0.5, 1.0, 1.5, 2.0), $\gamma = -0.3$, Pr=0.733

It is clear from Figure 2 that the velocity f' increases with increasing the viscosity parameter γ (decreasing the viscosity μ) for both air and liquid, at constant value of magnetic field $M = 1$ and at Pr= 0.733 for air and Pr= 4 for liquid. If we assume that there are neither electric nor magnetic fields, this result agrees with that obtained by Elbashbeshy and Dimian [6] for their special case when the radiation effect is negligible. From Figure 3 we can see that the temperature θ decreases with increasing γ at $M = 1$. From Figure 4 we can see that the velocity f' of air increases with increasing the magnetic field parameter M for $\gamma = 0$ and Pr= 0.733. We also obtained like these curves for liquid. This result agrees with that obtained by Ibrahim and Terbeche [7] for their special case of Newtonian fluid ($n = 1$). From Figure 5 we can notice that the effect of M on the temperature resembles that of γ.

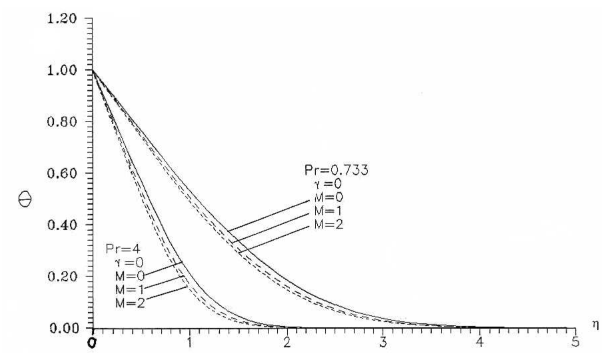

Figure 5 Representation of the temperature θ with different values of the magnetic field parameter M ($M = 0.0$, 0.5, 1.0, 1.5, 2.0), $\gamma = -0.3$, Pr=0.733

That is the temperature θ decreases as the magnetic field M increases, for both air and liquid. Figure 6 (for gases) represents the effect of both the viscosity parameter γ and the magnetic field M together on the velocity f'. It is seen that the effect of the magnetic field M only is greater than the effect of the viscosity parameter γ only for the given values in the figure. But the effect of the two parameters M and γ together is greater than the effect of any one of them alone.

For example, at ($\eta = 1$) any vertical plane, we can notice that the increase in the value of f' corresponding to an increase in γ only is 0.06. The increase in the value of corresponding to an increase in M only is 0.14. The increase in the

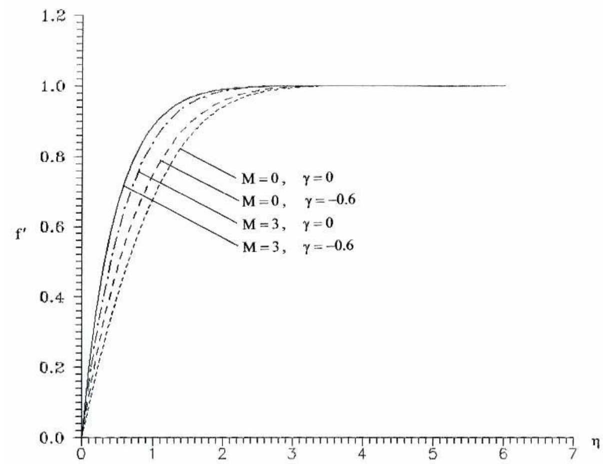

Figure 6 Representation of the effect of the magnetic field parameter M and viscosity parameter γ together on the velocity f' when Pr=0.733 (for air)

value of corresponding to an increase in M and together is 0.20. So, the magnetic field parameter M together with the viscosity parameter γ have strong effect on the velocity. They together increase more than any one alone. From Figure 7, we can see an inverse effect of M and together on the temperature θ. It is shown that the decrease in θ corresponding to increase in M is greater than the decrease in θ corresponding to increase in γ. If we use the two parameters M and γ, so we will get lowest temperature. Figure 8 (for liquids) represents the effect of both M and γ on the velocity f'. We notice that the effect of the magnetic field M only on the velocity f' is greater than the effect of both M and γ together. Finally Figure 9 represents the effect of M and γ together on the temperature θ for gases. It is shown that the decrease in θ corresponding to increase in M only is greater than the decrease in θ corresponding to M and γ together.

From Table 1 we notice that:

1. The skin friction coefficient $f''(0)$ increases as γ increases (the viscosity of air or water decreases) for different values of magnetic parameter M.

2. The rate of heat transfer $\theta'(0)$ at the wedge decreases very small as γ increases for different values of the magnetic field parameter M.

3. The skin friction coefficient $f''(0)$ increases as increasing the magnetic field parameter M for different values of γ.

4. The rate of heat transfer $\theta'(0)$ at the wedge decreases with increasing of M.

5. It is clear that the the skin friction coefficient $f''(0)$ has the largest value when we use M and γ together.

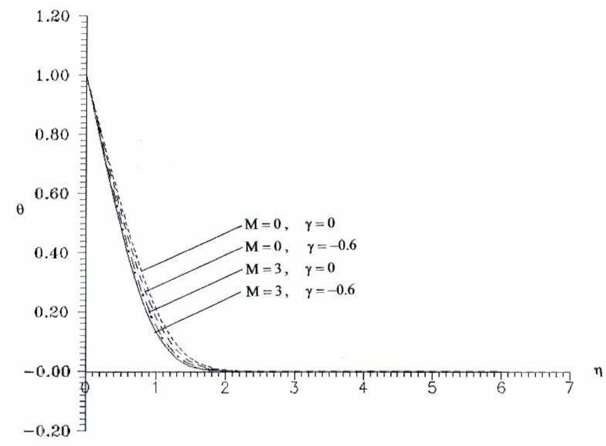

Figure 7 Representation of the temperature θ with different values of the magnetic field parameter M ($M = 0.0$, 0.5, 1.0, 1.5, 2.0), $\gamma = -0.3$, Pr=0.733

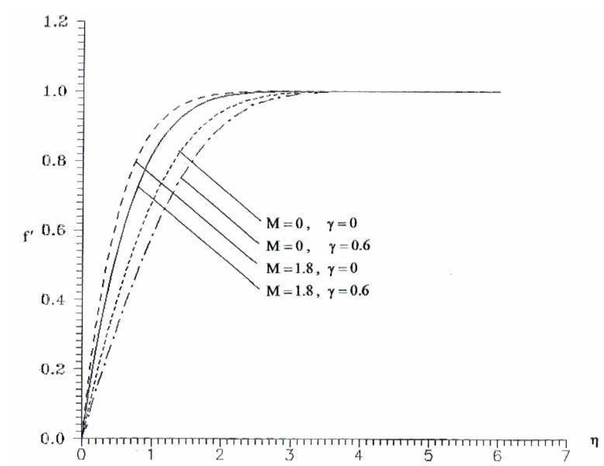

Figure 8 Representation of the effect of the magnetic field parameter M and viscosity parameter γ together on the velocity f' when Pr=0.733 (for air)

		$M = 0.0$		$M = 0.1$		$M = 1.0$	
	γ	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
Pr=0.733	0.0	0.92768	0.47889	0.92768	0.47889	1.35991	0.51061
	-0.3	0.80109	0.46619	0.84525	0.47058	1.14018	0.49585
	-0.6	0.69194	0.45363	0.72977	0.45803	1.01014	0.48584
Pr=4.0	0.0	0.92768	0.89609	0.97919	0.90725	1.35991	0.97827
	0.3	1.07387	0.92420	1.13396	0.93554	1.58535	1.00843
	0.6	1.24314	0.95302	1.31325	0.96453	1.83010	1.03716

Table 1 Represents the value of the skin friction $f''(0)$ at the surface and the temperature gradient $\theta'(0)$ with different values of γ and M at Prandtl number Pr=0.733 for air and Pr=4 for liquid

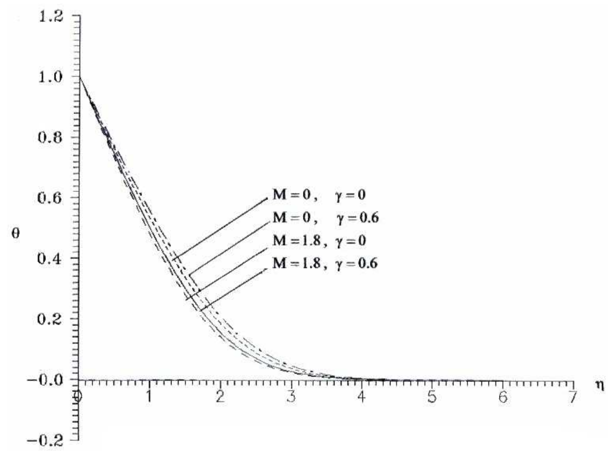

Figure 9 Representation of the effect of the magnetic field parameter M and viscosity parameter γ together on the velocity f' when Pr=0.733 (for air)

4. Conclusions

1. Increasing the magnetic field with the presence of electric field causes increase of the velocity of the flow but decrease the temperature of the flow. So the magnetic field can therefore be used to control the flow characteristics.

2. The value of the skin friction $f''(0)$ increases whereas the rate of heat transfer $\theta'(0)$ decreases owing to increase of magnetic field in presence of an electric field.

3. The value of the skin friction $f''(0)$ increases whereas the rate of heat transfer $\theta'(0)$ decreases with decreasing the viscosity (γ increases).

4. The magnetic field parameter M together with the viscosity parameter γ have strong effects on the velocity f' and the temperature θ. They together increase f' and decrease θ more than any one alone (for liquids).

5. The magnetic field parameter M (alone) has strong effects on the velocity f' and the temperature θ (for gases). It increases f' and decreases θ more than M and γ together.

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