Mechanics and Mechanical Engineering
Vol. 7, No. 2 (2004) 89–98

Unsteady Free Convection Flow in the Stagnation-Point Region of a Rotating Sphere Embedded in a Porous Medium

I.A. HASSANIEN, F.S. IBRAHIM and Gh.M. OMER
Department of Mathematics, Faculty of Science, Assiut University
Assiut, Egypt

Received (22 September 2003)
Revised (20 October 2003)
Accepted (12 May 2004)

The problem of the unsteady free convection flow in a fluid saturated porous medium of a rotating sphere in the presence of a solid matrix exerting first and second resistance is studied. Numerical solutions are obtained for the cases of constant wall temperature (CWT) and constant heat flux (CHF). The non-linear coupled partial differential equations governing the flow have been solved numerically using finite difference. The effects of the first resistance parameter ξ, the buoyancy parameter λ, the Prandtl number Pr, the variation of the angular velocity with time ($\epsilon > 0$) on the skin friction and heat transfer rate ue discussed. is shown the figures. It is found that the buoyancy force enhanced both the skin fraction and the heat transfer rate. The effect of the presence of the first resistance decreases the skin friction and the heat transfer rate.

Keywords: convection, saturated medium, buoyancy.

1. Introduction

Free convection heat transfer in a porous medium has been studied widely in literature. Cheng [1] provides an extensive review of literature on ee convection in fluid saturated porous media with regard to application in geothermal systems. Nield and Bejan [2] given an excellent summary of this subject. Excellent reviews of the literature dealing with the free convection flows are presented by Ede[3J, Gebhart [4] and Gebhart et al. [5]. The problem of free convection arises in a fluid when the temperature changes cause density variations leading to buoyancy forces. Some of the recent studies on the free convection flows include those of Merkin [6,7], Watanabe [8] and Takhar *et al.* [9]. Self-similar solution of the unsteady flow in the stagnation point region of a rotating sphere with a magnetic flied have been studied by Takhar *et al.* [10,11]. The heat transfer mechanism involving flows in contact with the walls, which undergo a transient change, is important in industry such as in power transformers and nuclear reactor. Although steady free convection problems have been studied extensively, the analogous unsteady problems have received comparatively less attention.

The present analysis deals the unsteady free convection flow in the forward stagnation-point region of a rotating sphere in the presence of a solid matrix, exerting first and second-order resistance. The unsteadiness is caused by the rotation of the sphere, which varies arbitrary with time. Both constant wall temperature and constant heat flux conditions have been considered.

2. Governing eguations

Let us consider an unsteady free convection boundary layer flow in the forward stagnation-point region of a heated sphere, which is rotating with time dependent angular velocity Ω in a fluid saturated porous medium, under a gravitational field which is parallel to the axis of rotation. The temperature of the ambient fluid is taken as a constant. Also the temperature at the wall or the heat flux at the wall is taken as a constant. Assuming that:

(a) the fluid has constant properties except the density changes which produce buoyancy forces,

(b) the effect of the buoyancy-induced streamwise pressure gradient terms on the flow and temperature fields is negligible,

(c) the fluid and the porous medium are the local thermal equilibrium,

(d) The viscous dissipation terms are negligible.

Under the above-mentioned assumptions, the boundary layer equations governing the flow can be expressed as

$$\frac{\partial u}{\partial x} + \frac{u}{x} + + \frac{\partial u}{\partial y} = 0\,, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{w^2}{x} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{gx}{l}\beta(T - T_\infty) - \frac{\nu\epsilon}{K}u - \frac{\Gamma\epsilon^2}{K^{\frac{1}{2}}}u^2\,, \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + \nu\frac{\partial w}{\partial y} + \frac{uw}{x} = \nu\frac{\partial^2 w}{\partial y^2}\,, \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + \nu\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}\,. \tag{4}$$

The initial and boundary conditions are given by

$$\begin{aligned} u(x,y,0) &= u_i(x,y)\,, \\ v(x,y,0) &= v_i(x,y)\,, \\ w(x,y,0) &= w_i(x,y)\,, \\ T(x,y,0) &= T_i(x,y)\,, \\ u(x,0,t) &= \nu(x,0,t) = 0\,, \\ w(x,0,t) &= \Omega(t)x\,, \end{aligned} \tag{5}$$

$$
\begin{aligned}
T(x,0,t) &= T_w \text{ or } -k\frac{\partial T(x,0,t)}{\partial y} = q_w\,, \\
u(x,\infty,t) &= w(x,\infty,t) = 0\,, \\
T(x,\infty,t) &= T_\infty\,.
\end{aligned} \tag{6}
$$

where x, y and z are coordinates measured from the forward stagnation point along the surface, normal to the surface and in the rotating direction, respectively; u, v and w are the velocity components along the x, y and z directions, respectively; t is the time; g is the acceleration due to gravity; L is the characteristic length, α and ν are thermal diffusivity and kinematic viscosity, respectively; k is the thermal conductivity, ϵ is the porosity; K is the permeability; Γ empirical constant in the second-order resistance; T is the temperature in the boundary layer; β is the volumetric coefficient of thermal expansion; q_w is the heat flux at the wall; T_w and T_∞ are the wall temperature and the ambient temperature, respectively; Ω is the angular velocity of the body and Ω_0 is its value at $t = 0$; the subscripts i, w and ∞ denote initial condition, condition at the wall and ambient condition, respectively.

We apply the following transformations for the constant wall temperature case (CWT),

$$
\begin{aligned}
\eta &= 2^{\frac{1}{2}}\left(\frac{\Omega_0}{\nu}\right)^{\frac{1}{2}} y\,, \\
\Omega &= \Omega_0\phi(t^*), \quad (t^*) = \Omega_0 t\,, \\
u &= \Omega_0 x\phi(t^*)f'(\eta,t^*)\,, \\
w &= \Omega_0 x\phi(t^*)s(\eta,t^*)\,, \\
\nu &= -2^{\frac{1}{2}}\left(\Omega_0\nu\right)^{\frac{1}{2}}\phi(t^*)f(\eta,t^*)\,, \\
\frac{T-T_w}{T_w-T_\infty} &= G(\eta,t^*)\,, \\
Gr_L &= g\beta(T_w-T_\infty)\frac{L^3}{\nu^2}\,, \\
\lambda &= \frac{Gr_L}{\mathrm{Re}_L^2}\,, \\
\mathrm{Pr} &= \frac{\nu}{\alpha}\,, \\
\mathrm{Re}_L &= \Omega_0\frac{L^2}{\nu}\,, \\
\xi &= \frac{\nu\epsilon}{2K\Omega_0}\,, \\
\psi_x &= \frac{\Gamma\epsilon^2 x}{K^{\frac{1}{2}}}\,.
\end{aligned} \tag{7}
$$

Substituting with this non-dimension transformation in Eqs. (1–4),the governing momentum and energy equations for the CTW case can be written as:

$$
f''' + \frac{\phi}{2}\left[2ff'' - f'^2 + s^2\right] - \frac{\phi}{2}\psi_x f'^2 = \xi f' - (2\phi)^{-1}\frac{d\phi}{dt^*}f' - 2^{-1}\frac{\partial f'}{\partial t^*} + (2\phi)^{-1}\lambda G = 0\,, \tag{8}
$$

$$s'' + \phi(fs' - f's) - (2\phi)^{-1}\frac{d\phi}{dt^*}s - 2^{-1}\frac{\partial s}{\partial t^*} = 0\,, \tag{9}$$

$$\mathrm{Pr}^{-1}G'' + \phi f G' - 2^{-1}\frac{\partial G}{\partial t^*} = 0\,, \tag{10}$$

and the boundary conditions (5) and (6) can be expressed as:

$$\begin{gathered} f(0,t^*) = f'(0,t^*) = 0, \quad s(0,t^*) = 1, \\ G(0,t^*) = 1, f'(\infty,t^*) = s(\infty,t^*) = G(\infty,t^*) = 0\,. \end{gathered} \tag{11}$$

On the other hand for the constant heat flux case (CHF) we apply the following transformations,

$$\begin{aligned} u &= \Omega_0 x\phi(t^*)F'(\eta,t^*)\,, \\ w &= \Omega_0 x\phi(t^*)S(\eta,t^*)\,, \\ \nu &= -2^{\frac{1}{2}}\left(\Omega_0\nu\right)^{\frac{1}{2}}\phi(t^*)F(\eta,t^*)\,, \\ T - T_\infty &= \frac{q_w L}{K}\mathrm{Re}_L^{-\frac{1}{2}}\theta(\eta,t^*)\,, \\ Gr_L^* &= \frac{g\beta q_w L^4}{k\nu^2}\,, \\ \lambda_1 &= \frac{Gr_L^*}{(\mathrm{Re}_L)^{\frac{5}{2}}} \end{aligned} \tag{12}$$

For the constant heat flux case (CHF) the equations corresponding to equations (8)–(10) are given by

$$\begin{aligned} F''' + \frac{\phi}{2}\left(2FF'' - F'^2 + S^2\right) - \frac{\phi}{2}\psi_x F'^2 - \xi F' - & \\ (2\phi)^{-1}\frac{d\phi}{dt^*}F' - 2^{-1}\frac{\partial F'}{\partial t^*} + 2^{-\frac{3}{2}}\phi^{-1}\lambda_1\theta &= 0\,, \end{aligned} \tag{13}$$

$$S'' + \phi(FS' - F'S) - (2\phi)^{-1}\frac{\partial\phi}{\partial t^*}S - 2^{-1}\frac{\partial S}{\partial t^*} = 0\,, \tag{14}$$

$$\mathrm{Pr}^{-1}\theta'' + \phi F\theta' - 2^{-1}\frac{\partial\theta}{\partial t^*} = 0\,. \tag{15}$$

The boundary conditions for the CHF case are given by

$$\begin{gathered} F(0,t^*) = F'(0,t^*) = 0, \quad S(0,t^*) = 1, \\ \theta(0,t^*) = -1, F'(\infty,t^*) = S(\infty,t^*) = \theta(\infty,t^*) = 0\,. \end{gathered} \tag{16}$$

The initial conditions are given by the steady-state equations obtained by putting

$$t^* = 0\,, \quad \phi(t^*) = 1\,, \quad \frac{d\phi}{dt^*} = f_{t^*} = s_{t^*} = G_{t^*} = F_{t^*} = S_{t^*} = \theta_{t^*} = 0\,,$$

in equations (9)–(14). The steady state equations for the CWT case are given by

$$f''' + ff'' - \frac{1}{2}\left[f'^2 - s^2\right] - \frac{1}{2}\psi_x f'^2 - \xi f' + 2^{-1}\lambda G = 0\,, \tag{17}$$

$$s'' + (fs' - f's) = 0\,, \tag{18}$$

$$\mathrm{Pr}^{-1}G'' + fG' = 0\,. \tag{19}$$

The boundary conditions for the CWT case can be expressed as

$$f(0) = f'(0) = 0, \quad s(0) = 1, G(0) = 1, \quad f'(\infty) = s(\infty) = G(\infty) = 0\,. \tag{20}$$

The steady state equations for the CHF case are expressed as

$$F''' + FF'' - 2^{-1}\left[F'^2 + \psi_x F'^2 - S^2\right] - \xi F' + 2^{-\frac{3}{2}}\lambda_1\theta = 0\,, \tag{21}$$

$$S'' + (FS' - F'S) = 0\,, \tag{22}$$

$$\mathrm{Pr}^{-1}\theta'' + F\theta' = 0\,. \tag{23}$$

The boundary conditions for the CHF case are given by

$$F(0) = F'(0) = 0, \quad S(0) = 1, \quad \theta(0) = -1, \quad F'(\infty) = S(\infty) = \theta(\infty) = 0\,. \tag{24}$$

here η is the transformed dimensionless variable; t^* is the dimensionless time; f and F are the dimensionless stream functions for the CWT and CHF cases, respectively; f' and s are, respectively, the dimensionless velocities along x and z directions for the constant wall temperature case; F' and S are the corresponding velocities for the CHF case, G and θ are the dimensionless temperatures for the CWT and CHF cases, respectively; ξ is the first-order resistance; ψ_x is the second order resistance; Gr_L and Gr_L^* are the Grashof numbers for the CWT and CHF cases, respectively; Re_L is the Reynolds number; λ is the ratio of the Grashof number and the square of the Reynolds number for the CWT case; λ_1 is the ratio of Grashof number and the Reynolds number raised to the power 2.5 for the CHF case; Pr is the Prandtl number; prime denotes derivative with respect to η; and the subscript t^* denotes derivative with respect to t^*, ϕ is an arbitrary function of t^* having first-order continuous derivative.

The skin friction coefficients in x and z directions for the CWT case are respectively, given by

$$C_{fx} = \frac{\tau_1}{\rho\Omega_0^2 x^2} = 2^{\frac{1}{2}}(\mathrm{Re}_x)^{-\frac{1}{2}}\phi f''(0, t^*)\,, \tag{25}$$

$$C_{fz} = -\frac{\tau_2}{\rho\Omega_0^2 x^2} = 2^{\frac{1}{2}}(\mathrm{Re}_x)^{-\frac{1}{2}}\phi s'(0, t^*)\,, \tag{26}$$

where

$$\tau_1 = \mu\frac{\partial u}{\partial y}_{y=0}, \quad \tau_2 = -\mu\frac{\partial w}{\partial y}_{y=0}, \quad \mathrm{Re}_x = \frac{\Omega_0 x^2}{\nu}\,. \tag{27}$$

Similarly, the skin friction coefficients in the x and z directions for the CHF case are

$$\bar{C}_{fx} = \frac{\tau_1}{\rho\Omega_0^2 x^2} = 2^{\frac{1}{2}}(\mathrm{Re}_x)^{-\frac{1}{2}}\phi F''(0, t^*)\,, \tag{28}$$

$$\bar{C}_{fz} = \frac{\tau_2}{\rho\Omega_0^2 x^2} = 2^{\frac{1}{2}}(\mathrm{Re}_x)^{-\frac{1}{2}}\phi S'(0, t^*)\,, \tag{29}$$

The heat transfer coefficient in terms of Nusselt number for the CWT case can be expressed as

$$\mathrm{Nu} = -L\frac{\frac{\partial T}{\partial y}}{T_w - T_\infty} = -2^{\frac{1}{2}}\,(Re_L)^{\frac{1}{2}}\,G'(0,t^*)\,. \qquad (30)$$

The heat transfer coefficient for the CHF case is given in the form

$$\bar{\mathrm{Nu}} = 2^{\frac{1}{2}}\frac{(\mathrm{Re}_L)^{\frac{1}{2}}}{\theta(0,t^*)}\,. \qquad (31)$$

Here C_{fx} and C_{fz} are the skin friction coefficients in the x and z directions, respectively, for the CWT case and $\bar{C}_{fx}$ and $\bar{C}_{fz}$,are the corresponding expressions for the CHF case; Nu and $\bar{\mathrm{Nu}}$ are the Nusselt numbers for the CWT and CHF cases, respectively; τ_1 and τ_2 are the shear stress in the x and z directions, respectively; ρ and μ are the density and coefficient viscosity, respectively; and Re_x and Re_L are the Reynolds numbers defined with respect to x and L, respectively.

The systems of equations (8)–(11) and (13)–(16) have been solved by using the finite difference methods explained by Pereyra and Sparrow [12,13]. To conserve space the details of the solution procedure are omitted here.

3. Results and discussion

The non-linear coupled partial differential equations governing the flow have been transformed to a system of ordinary equation by using the non-similar technique as explained by Pereyra and Sparrow [12,13], ln order to validate our numerical solution, we have compared our steady state skin friction and heat transfer parameters for the constant wall temperature case $[f''(0), -s'(0), -G'(0)]$ with those by Takhar *et al.* [14], when $\lambda = 0$ (in the absence of the buoyancy force). The results are found to be in very good agreement. The comparison is shown in Fig. 1.

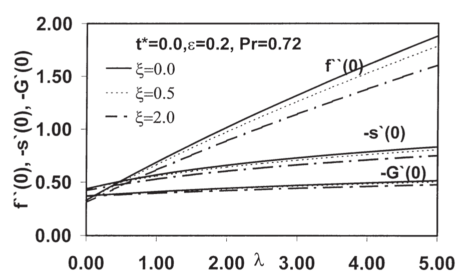

Figure 1 The variation of the skin parameter in x and z directions $[f''(0,t^*), -s'(0)]$ and the heat transfer parameter $[-G'(0)]$ for the CWT case when $t^* = 0$

For the constant wall temperature (CWT) case and for $\phi(t^*) = 1 + \epsilon t^{*2}$, $\epsilon > 0$ (i.e. when the angular velocity Ω increases with time) the variation of the skin friction

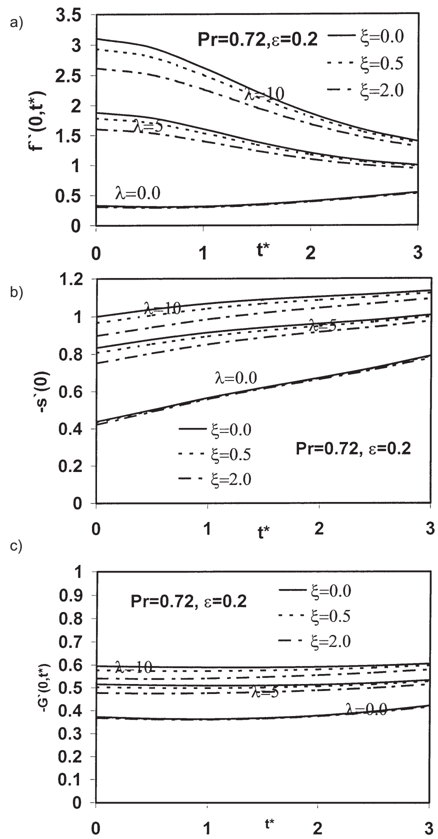

Figure 2 The variation of the skin parameter in a) x direction $[f''(0,t^*)]$ with time, b) z direction $[-s'(0,t^*)]$, and c) the variation of the heat transfer parameter $[-G'(0,t^*)]$, all for the CWT case when $\phi(t^*) = 1 + \epsilon t^{*2}$

parameters in x and z directions $[f''(0,t^*), -s'(0,t^*)]$ and heat transfer parameter $-G'(0,t^*)$ with time t^* for several values of the buoyancy parameter λ, and for several values of the first-order resistance parameter ξ is shown in Fig. 2(a–c). The skin friction and heat transfer $[f''(0), -s'(0), -G'(0)]$ are found to decrease with the first-resistance parameter ξ increases, and its found that the first resistance have not any effect in the absence of the buoyancy force ($\lambda = 0$). On contrary as the buoyancy parameter λ increases the skin friction and heat transfer rate decrease.

For the CWT case and for $\phi(t^*) = 1 + \epsilon t^{*2}$, $\epsilon < 0$ (i.e. when the angular velocity Ω decreases as t^* increases) the variation of the skin friction parameters in x and z directions $[f''(0,t^*), -S'(0,t^*)]$ with time t^* is presented in Fig. 3. It is seen that $f''(0,t^*)$ decreases with the first resistance and increases with time t^*. On the other hand $[-s'(0,t^*)]$ decreases with both the first resistance increase and time t^*.

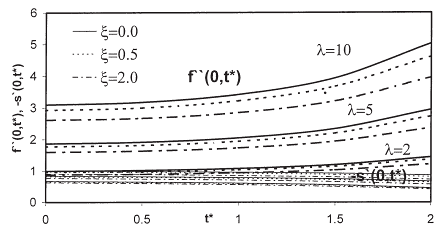

Figure 3 The variation of the skin parameter in x and z directions $[f''(0,t^*), -s'(0,t^*)]$ with time t^* for the CWT case

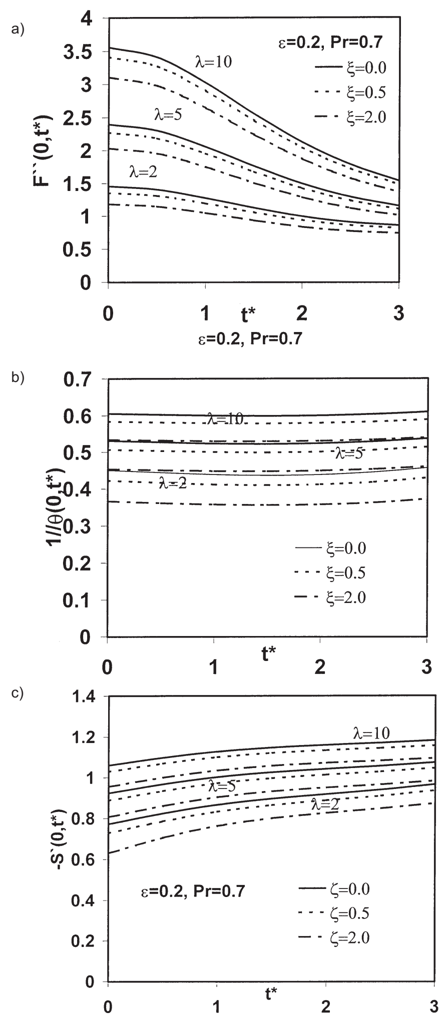

Figure 4 The variation of the skin parameter in a) x direction $[F''(0,t^*)]$ with time t^*, b) the variation of the heat transfer parameter $\frac{1}{\theta(0,t^*)}$, and c) in z direction $[-S'(0,t^*)]$, all for the CHF case when $\phi(t^*) = 1 + \epsilon t^{*2}$

For CHF case and for $\phi(t^*) = 1 + \epsilon t^{*2}$, $\epsilon > 0$ the variation of the skin friction parameters in x and z directions $[F''(0,t^*), -S'(0,t^*)]$ and heat transfer parameter $\frac{1}{\theta(0,t^*)}$ with time t^* for several values of the buoyancy parameter λ and for several values of the first-order resistance parameter ξ is shown in Fig. 4(a–c). The trend of the skin friction parameter and heat transfer $[F''(0,t^*), -S'(0,t^*), \frac{1}{\theta(0,t^*)}]$ is qualitatively similar to that of the CWT case. However, the skin friction and heat transfer are found to be more than those of the CWT case.

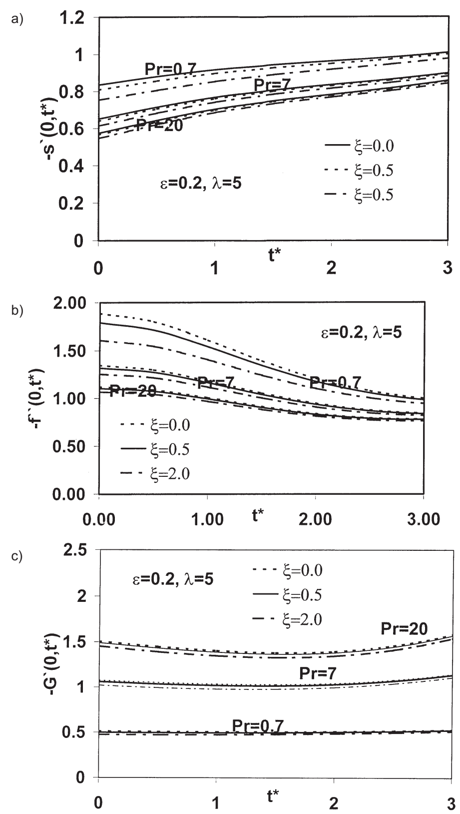

Figure 5 The variation of the skin friction parameter in a) x direction $[f''(0,t^*)]$ with time t^*, b) in z direction $[-s'(0,t^*)]$, and c) the variation of the heat transfer parameter $[-G'(0,t^*)]$ with time t^*, all for the CWT case when $\phi(t^*) = 1 + \epsilon t^{*2}$

For the CWT case, the variation of the skin friction and the heat transfer parameters $[f''(0), -s'(0), -G'(0)]$ with t^* for several values of the first-order resistance parameter ξ); and for several values of the Prandtl numbers Pr when $\phi(t^*) = 1 + \epsilon t^{*2}$, $\epsilon > 0$ is shown in Fig. 5(a–c). It is observed that the heat transfer decrease with the first order resistance parameter and increase with the Prandtl number Pr, but the skin friction parameters in the x and z directions $[f''(0,t^*), -s'(0,t^*)]$ decreases with the first order resistance parameter increases and decrease with increase the Prandtl number Pr. This results in a reduction of the thermal boundary layer thickness and an increase in the gradient of the temperature and hence the increase in the heat transfer rate. Since higher Pr implies more viscous fluid having a comparatively thicker boundary, layer these results in reduction in the friction parameters.

References

[1] **Cheng, P**: Heat transfer in geothermal systems, *Advances in Heat Transfer*, **4**, (1978,), 1-105.

[2] **Nield, DA** and **Bejan, A**: *Convection in porous media*, Springer, New York, (1992).

[3] **Ede, AJ**: *Advances in heat transfer*, **4**, Academic Press, New York (1967), 1.

[4] **Gebhart, B**: *Advances in Heat Transfer*, **9**, Academic Press, New York, (1973), 273.

[5] **Gebhart, B**, **Jaluria, Y**, **Mahajan, RL** and **Sammaria, R**: *Buoyancy Induced Flows and Transport Hemisphere*, New York, (1988).

[6] **Merkin, JH**: *J. Engny Math.*, **33**, (1989), 273.

[7] **Merkin, JH** and **Mahmood, T**: *J. Engny Math.*, **24**, 95, (1990).

[8] **Watanabe, T**: *Acta Mech.* **87**, 1, (1991).

[9] **Takhar, HS**, **Kumariad, M**, **Nath, G**: *Arch. App. Mech.*, **63**, 313, (1993).

[10] **Takhar, HS** and **Nath, G**: *Heat and Mass Transfer*, **36**, (2000), 89-96.

[11] **Takhar, HS**, **Chamkha, AJ** and **Nath, G**: *Heat mass transfer*, **37**, (2001), 397-402.

[12] **Sparrow, EM**, **Quack, H** and **Boerner, J**: *AIAA*, **8**, (1970), 1936-1942.

[13] **Pereyra, V**: *Lecture Note in Computer Science*, **76**, Springer, Berlin, (1978).

[14] **Takhar, HS**, **Slaouta, A**, **Kumari, M** and **Nath, G**: *Int. J. Non-linear Mechanics*, **33**, (1998), 857-865.