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On the possibility of overstable motion of a rotating viscoelastic fluid layer heated from below under the effect of magnetic field with one relaxation time

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The convective stability of a horizontal layer of viscoelastic conducting liquid (Walters' liquid B') heated from below and rotating about a vertical axis in the presence of a magnetic field and thermal relaxation has been investigated. Linear stability theory and normal mode analysis are used to derive an eigenvalue system of eighth order, and an exact eigenvalue equation for a neutral instability is obtained. Critical Rayleigh numbers and wave numbers for the onset of instability are presented graphically as functions of Taylor number for various values of the Chandrasekhar number and the relaxation time at a Prandtl number $P_r = 1$.

Keywords: viscoelsastic liquid, critical Rayleigh number, overstable motion

1. Introduction

An important stability problem is the thermal convection in a horizontal thin layer of fluid heated from below. A detailed account of thermal convection in a horizontal thin layer of Newtonian fluid heated from below, under varying assumptions of hydrodynamics, has been given by Chandrasekhar [1]; however, very little is known about the thermal instability in a viscoelastic fluid layer. The problem of the onset of thermal instability in a horizontal layer of a viscoelastic fluid heated from below is both of theoretical and practical interest.

To the author's knowledge, the first work which deals directly with this problem appears in a brief report by Green [2]. His analysis, which is restricted to the case when both bounding surfaces are free, was carried out in terms of a two-time-constant mode due to Oldroyd [3,4]. The same problem was also attacked in some detail by Vest and Arpaci [5] who employed a one-time-constant model due to Maxwell fluid [6,7]. This latter work has recently been extended by Takashima [8,9] to the case when the fluid layer is rotating about a vertical axis at a constant rate. All these investigations show that the presence of elasticity in a viscoelastic fluid destabilized the fluid layer heated from below.

In technological fields there exists an important class of fluids, called non-Newtonian fluids, are also being studied extensively because of their practical applications, such as fluid film lubrication, analysis of polymers in chemical engineering etc. One such fluid is called viscoelastic fluid and Walters [10], and Beard and Walters [11] deduced the governing equations for the boundary layer flow for a prototype viscoelastic fluid, which they have designated as liquid B', when this liquid has a very short memory. The problem of two-dimensional magnetohydro-dynamic flow and heat transfer through a non-Newtonian viscoelastic incompressible porous fluid obeying the rheological equations of state due to Walters is studied by El-Dabe and Sallam [12]. Singh and Singh [13] have studied the magnetohydro-dynamic flow of viscoelastic fluid past an accelerated plate.

The method of the matrix exponential, proposed by Bohar [14], and applied by Ezzat [15] and [14], which constitutes the basis of the state space approach of modern control theory is applied to the non-dimensional equations of a viscoelastic fluid flow of hydromagnetic free convection flows. Ezzat and Abd-Elaal [16] have been studied the effects of free convection currents with one relaxation time on the flow of a viscoelastic conduction fluid through a porous medium, which is bounded by a vertical plane surface. In these works, more general model of magnetohydrodynamic free convection flow which also includes the relaxation time of heat convection and the electric permeability of the electromagnetic field are used. The inclusion of the relaxation time and electric permeability modify the governing thermal and electro-magnetic equations, changing them from parabolic to hyperbolic type, and there by eliminating the unrealistic result that thermal disturbance is realized instantaneously everywhere within a fluid.

Ezzat and Othman [17] studied the influence of a transverse a.c. electric field on the thermal instability of a rotating micropolar fluid layer. Othman and Ezzat [18] have studied the stability of viscoelastic conducting liquid (Walters' liquid B') heated from below in the presence of a magnetic field. Othman [19] have studied the problem of the onset of stability in a horizontal layer of viscoelastic dielectric liquid (Walters' liquid B') under the simultaneous action of a vertical ac electric field and a vertical temperature gradient.

In Section 2 the basic equations after perturbation and boundary conditions are written for viscoelastic conducting liquid B'. The solution of the problem was presented in Section 3. In Section 4 the influence of the magnetic field on the overstability of a rotating fluid layer heated from below with one relaxation time has studied and illustrated graphically. In order to simplify the mathematics somewhat, artificial boundary conditions are adopted.

2. Formulation of the problem

Consider an infinite horizontal layer of an electrically conducting, viscoelastic fluid layer (Walters' liquid B'), occupying the space between two horizontal rigid boundaries, which are at distance L apart. We choose the origin on the lower boundary, let us introduce the Cartesian-coordinate system x, y, z in which z is measured at right angles to the boundaries. Let the system be rotating (round the z-axis) with uniform angular velocity $\Omega = (0, 0, \Omega)$ and is permeated by a uniform external magnetic field $h = (0, 0, H_0)$ of intensity H_0 aligned in the vertical direction. The lower surface at $z = 0$ and the upper surface at $z = L$ are maintained at constant temperatures T_0 and T_1, respectively, and the fluid in the quiescent state is heated from below such that $\beta = \frac{T_0 - T_1}{L}$ is the adverse temperature gradient.

The basic equations are as follows

$$\frac{\partial v_i}{\partial x_i} = 0 \tag{1}$$

$$\begin{aligned} \rho\left[\frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k}\right] &= \rho g_i - \frac{\partial P}{\partial x_i} + \eta_0 \frac{\partial^2 v_i}{\partial x_k \partial x_k} - \frac{\mu_0 h_j}{4\pi}\frac{\partial h_i}{\partial x_j} - \\ & K_0\left[\frac{\partial}{\partial t}\frac{\partial^2 v_i}{\partial x_k \partial x_k} + v_m \frac{\partial^3 v_i}{\partial x_m \partial x_k \partial x_k} - \right. \\ & \left. \frac{\partial v_i}{\partial x_m}\frac{\partial^2 v_m}{\partial x_k \partial x_k} - 2\frac{\partial v_m}{\partial x_k}\frac{\partial^2 v_i}{\partial x_m \partial x_k}\right] + 2\mathrm{e}_{ijk}\rho v_j \Omega_k\,, \end{aligned} \tag{2}$$

$$\rho C_v\left[\frac{\partial}{\partial t} + v_k \frac{\partial T}{\partial x_k}\right] = k_c \frac{\partial^2 T}{\partial x_k \partial x_k} - \tau \rho C_v \frac{\partial}{\partial t}\left[\frac{\partial T}{\partial t} + v_k \frac{\partial T}{\partial x_k}\right], \tag{3}$$

$$\frac{\partial h_i}{\partial t} + \frac{\partial}{\partial x_j}(v_j h_i - v_i h_j) = \eta \frac{\partial^2 h_i}{\partial x_k \partial x_k}\,, \tag{4}$$

and

$$\frac{\partial h_i}{\partial x_i} = 0\,. \tag{5}$$

The equation of state is given by

$$\rho = \rho_0\left[1 - \alpha(T - T_0)\right] \tag{6}$$

where ρ is the mass density, ρ_0 is the reference density at the lower boundary, α is the coefficient of volume expansion, $v_i = (u, v, w)$ is the velocity of the fluid, P is the pressure, $g_i = (0, 0, -g)$ is the gravitational acceleration, K_0 is the elastic constant of Walters' liquid B', k_c is the thermal diffusivity, c_v is the specific heat at constant volume, T is the temperature of the liquid, τ is the relaxation time, μ_0 is the magnetic permiability and η_0 is the magnetic diffusivity.

We first obtained the following steady solutions (denoted by an over bar)

$$\bar{u} = \bar{v} = \bar{w} = 0\,, \tag{7}$$

$$\bar{T} = T_0 - \beta \mathrm{z}\,, \tag{8}$$

$$\bar{\rho} = \rho_0(1 + \alpha\beta z)\,, \tag{9}$$

$$\bar{h}_x = 0\,, \quad \bar{h}_y = 0\,, \quad \bar{h}_z = H_0\,. \tag{10}$$

Under Boussinesq approximation, the equations governing the disturbances can be written as (Chandrasekhar [1]):

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0\,, \tag{11}$$

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\nabla^2 w' = \alpha g \nabla_1^2 T' - \frac{K_0}{\rho_0}\frac{\partial}{\partial t}\nabla^4 w' + \frac{\mu_0 H_0}{4\pi\rho_0}\frac{\partial}{\partial z}\nabla^2 h' - 2\Omega\frac{\partial \zeta'}{\partial z}\,, \quad (12)$$

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\zeta' = 2\Omega\frac{\partial w'}{\partial z} - \frac{K_0}{\rho_0}\frac{\partial}{\partial t}\nabla^2\zeta'\,, \quad (13)$$

$$\left[\frac{\partial T'}{\partial t} - \beta w'\right] = k_c\nabla^2 T' - \tau\frac{\partial}{\partial t}\left[\frac{\partial T'}{\partial t} - \beta w'\right], \quad (14)$$

$$\left(\frac{\partial}{\partial t} - \eta\nabla^2\right)h' = H_0\frac{\partial w'}{\partial z}\,. \quad (15)$$

where ν is the kinematic viscosity. The dependent variables w', T', ζ' and h' represent respectively the z-component of the perturbation in the velocity, the temperature, the vorticity and the z-component of the perturbation in the magnetic field. There ∇^2 and ∇_1^2 represent respectively $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, the time is represented by t and $\zeta' = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the vorticity.

Now, introducing the nondimensional variables, given by L, $\frac{L^2}{k_c}$, $\frac{k_c}{L}$, βL, $\frac{k_c}{L^2}$ and $\frac{k_c H_0}{\eta}$ as the units of length, time, velocity, temperature, vorticity and magnetic field, respectively, we obtain the equations governing the disturbances as:

$$\left[P_r^{-1}\frac{\partial}{\partial t} - \nabla^2\right]\nabla^2 w' = R\nabla_1^2 T' - K_0^* P_r^{-1}\frac{\partial}{\partial t}\nabla^4 w' + Q\frac{\partial}{\partial z}\nabla^2 h' - S\frac{\partial \zeta'}{\partial z}\,, \quad (16)$$

$$\left[P_r^{-1}\frac{\partial}{\partial t} - \nabla^2\right]\zeta' = S\frac{\partial w'}{\partial z} - K_0^* P_r^{-1}\frac{\partial}{\partial t}\nabla^2\zeta'\,, \quad (17)$$

$$\left[\frac{\partial}{\partial t}(1 + \tau\frac{\partial}{\partial t}) - \nabla^2\right]T' = \left(1 + \tau\frac{\partial}{\partial t}\right)w'\,, \quad (18)$$

$$\left[P_m P_r^{-1}\frac{\partial}{\partial t} - \nabla^2\right]h' = \frac{\partial w'}{\partial z}\,. \quad (19)$$

There, $P_r = \frac{\nu}{k_c}$ is the Prandtl number, $P_m = \frac{\nu}{\eta}$ is the magnetic Prandtl number, $R = \frac{\alpha\beta g L^4}{\nu k_c}$ is the Rayleigh number,$Q = \frac{\mu_0 H_0^2 L^2}{4\pi\sigma\nu\eta}$ is the Chandrasekhar number, $\sigma = \mu_0\rho_0$, $K_0^* = \frac{K_0}{\rho_0 L^2}$ is an elastic parameter, $S = \frac{2\Omega L^2}{\nu}$ is the Taylor number, and $k_c = \frac{k}{\rho_0 c_v}$.

Following the normal mode analysis we assume that the solutions of Eqs. (16)-(19) are given by

$$[w', \zeta', T' h',] = [W(z), Y(z), \Theta(z), H(z)]\exp\left[ct + \mathrm{i}(ax + by)\right]\,. \quad (20)$$

There, $\lambda = \sqrt{a^2 + b^2}$ is the horizontal wavenumber and c is the stability parameter which is, in general, a complex constant. For solutions having the dependence of the form (20), Eqs. (16)–(19) yield:

$$\begin{aligned}&\left[P_r^{-1}c - (D^2 - \lambda^2)\right](D^2 - \lambda^2)W + R\lambda^2\Theta + \\ P_r^{-1}K_0^* c(D^2 - \lambda^2)^2 W - QD(D^2 - \lambda^2)H + SDY &= 0\,,\end{aligned} \quad (21)$$

$$\left[P_r^{-1}c + (P_r^{-1}K_0^* c - 1)(D^2 - \lambda^2)\right]Y = SDW\,, \quad (22)$$

$$\left[c(1+\tau c)-(D^2-\lambda^2)\right]\Theta=(1+\tau c)W\,, \tag{23}$$

$$\left[P_mP_r^{-1}c-(D^2-\lambda^2)\right]H=DW\,. \tag{24}$$

Since $P_mP_r^{-1}$ is exceedingly small under most terrestrial conditions, the first term on the left-hand side of Eq.(24) may be ignored. Consequently, we can eliminate H from Eqs. (21) and (24) without any differentiation; thus,

$$\begin{aligned}\left[P_r^{-1}c-(D^2-\lambda^2)\right](D^2-\lambda^2)W+R\lambda^2\Theta+\\ P_r^{-1}K_0^*c(D^2-\lambda^2)^2W+QD^2W+SDY\quad=\quad 0\,.\end{aligned} \tag{25}$$

This means that under the above application the solution for the underlying problem can be carried out independently of the boundary conditions on the magnetic field.

In seeking solutions of these equations we must impose certain boundary conditions at the lower surface $z=0$ and the upper surface $z=1$. In this paper we shall restrict ourselves to the case when both boundary surfaces are stress-free, non-deformable and isothermal.

The boundary conditions for W, Y and Θ are given by

$$W=D^2W=DY=\Theta=0\text{ at }z=0,1\,. \tag{26}$$

This case, although admittedly an artificial one to consider, is of importance since its exact solution is readily obtained. Furthermore, from pas experience with problems of this kind (see, for example, Chandrasekhar [1] and Takashima [20]), one may feel fairly confident that the general features of the physical situation will be disclosed by discussion of this case equally as well as by a discussion of solutions satisfying less artificial boundary conditions.

Equations (22), (23) and (25) subject to the boundary conditions (26) constitute an eigenvalue system of eighth order.

3. Solution

The eigenvalue system defined by Eqs. (22), (23) and (25) can readily be combined to yield

$$\begin{aligned}&[c(1+\tau c)-(D^2-\lambda^2)]\left\{[P_r^{-1}c+(P_r^{-1}K_0^*c-1)(D^2-\lambda^2)][(P_r^{-1}c-\right.\\ &\quad\left.D^2+\lambda^2)(D^2-\lambda^2)+P_r^{-1}K_0^*c(D^2-\lambda^2)^2+Q]+S^2D^2\right\}W+\\ &\quad R\lambda^2(1+\tau c)[P_r^{-1}c+(P_r^{-1}K_0^*c-1)(D^2-\lambda^2)]W\quad=\quad 0\,,\end{aligned} \tag{27}$$

together with

$$W=D^{(2m)}W=0\text{ at }z=0,1,\quad(m=1,2,3,\ldots)\,. \tag{28}$$

Examination of Eq. (28) and (28) indicates that the relevant solution for W (characterizing the lowest mode) (see, for example, Takashima [20], Rama Rao [21], Sharma and Kumar [21] and Othman and Ezzat [18]) is given by

$$W=W_0\sin\pi z\,, \tag{29}$$

where W_0 is a constant. Substitution of this solution for W in Eq. (28) leads to required eigenvalue equation

$$\begin{aligned} \left\{\left[c(1+\tau c)+(\pi^2+\lambda^2)\right]\left[P_r^{-1}c-(P_r^{-1}K_0^*c-1)(\pi^2+\lambda^2)\right]\right. \\ \left.\left[-(\pi^2+\lambda^2)(P_r^{-1}c+\pi^2+\lambda^2)+P_r^{-1}K_0^{-1}c(\pi^2+\lambda^2)^2-\pi^2 Q\right]-\pi^2 S^2\right\} \\ \left[c(1+\tau c)+(\pi^2+\lambda^2)\right]+R\lambda^2(1+\tau c)\left[P_r^{-1}c-(P_r^{-1}K_0^*c-1)(\pi^2+\lambda^2)\right] &= 0\,, \end{aligned} \tag{30}$$

where it must be remembered that c can be complex. Letting

$$B=\pi^2+\lambda^2\,, \tag{31}$$

we can rewrite Eq. (30) in the form

$$\begin{aligned} R = {} & \frac{\pi^2[c(1+\tau c)+B]}{\lambda^2(1+\tau c)}\,Q+ \\ & \frac{B\,[c-B(K_0^*c-P_r)]\,[c(1+\tau c)+B]}{P_r\lambda^2(1+\tau c)}+\frac{\pi^2 P_r S^2}{\lambda^2(1+\tau c)\,[c-(K_0^*c-P_r)]}\,. \end{aligned} \tag{32}$$

4. Overstability motions and conclusions

Let us now separate the right-hand side of Eq. (32) into the real and imaginary parts after setting $c=i\omega$ with ω being real. Then, we have

$$R=X+i\omega Y\,. \tag{33}$$

There, X and Y are real-value functions of P_r, Q, K_0^*, τ_0, λ, S and ω, and the explicit expressions for these functions are as follows:

$$\begin{aligned} X = {} & \frac{\pi^2 BQ}{\lambda^2\left(1+\omega^2\tau^2\right)}+ \\ & \frac{B}{\lambda^2 P_r\left(1+\omega^2\tau^2\right)}\left[B^2P_r-\omega^2\left(1-BK_0^*\right)\left(1-B\tau+\omega^2\tau^2\right)\right]+ \\ & \frac{\pi^2P_rS^2\left[BP_r-\omega^2\tau\left(1-BK_0^*\right)\right]}{\lambda^2\left\{\left[BP_r-\omega^2\tau\left(1-BP_r\right)\right]^2+\omega^2\left[BP_r\tau+1-BK_0^*\right]^2\right\}}\,, \end{aligned} \tag{34}$$

$$\begin{aligned} Y = {} & \frac{\pi^2 Q\left(1-\tau B+\omega^2\tau^2\right)}{\lambda^2\left(1+\omega^2\tau^2\right)}+ \\ & \frac{B^2}{\lambda^2 P_r\left(1+\omega^2\tau^2\right)}\left[P_r\left(1-\tau B+\omega^2\tau^2\right)+\left(1-BK_0^*\right)\right]+ \\ & \frac{\pi^2P_rS^2\left[BP_r\tau+1-BK_0^*\right]}{\lambda^2\left\{\left[BP_r-\omega^2\tau\left(1-BP_r\right)\right]^2+\omega^2\left[BP_r\tau+1-BK_0^*\right]^2\right\}}\,. \end{aligned} \tag{35}$$

It is apparent from Eq. (36) that for arbitrarily assigned values of P_r, Q, λ, K_0^*, τ, S and ω, R will be complex, but the physical meaning of R requires it to be real.

Consequently, from the condition that R must be real we have either

$$R = X \text{ and } \omega = 0\,, \tag{36}$$

or

$$R = X \text{ and } Y = 0\,. \tag{37}$$

From Eq. (36) we obtain the eigenvalue equation for a neutral stationary instability,

$$R = \frac{1}{\lambda^2}\left[\pi^2 BQ + B^3 + \pi^2 P_r S^2\right]\,. \tag{38}$$

For a Newtonian viscous fluid, when the magnetic field is absent i.e. $Q = 0$ and without rotation i.e. $S = 0$, Eq. (34) reduces to

$$R = \frac{B^3}{\lambda^2}\,, \tag{39}$$

which agrees with the classical result (Chandrasekhar [1]). Equation (38) will give the critical Rayleigh number R_c for the onset of stationary instability.

On the other hand, Eq. (37) leads, after some rearrangements, to

$$\begin{aligned} R \;=\; & \frac{\pi^2 BQ}{\lambda^2\left(1+\omega^2\tau^2\right)} + B\frac{B^2 P_r - \omega^2\left(1-BK_0^*\right)\left(1-B\tau+\omega^2\tau^2\right)}{\lambda^2 P_r\left(1+\omega^2\tau^2\right)} + \\ & \frac{\pi^2 P_r S^2\left[B^2 P_r - \omega^2\left(1-BK_0^*\right)\right]}{\lambda^2\Big\{\left[BP_r - \omega^2\tau\left(1-BP_r\right)\right]^2 + \omega^2\left[BP_r\tau + \left(1-BK_0^*\right)\right]^2\Big\}}\,, \end{aligned} \tag{40}$$

and

$$A_0\sigma^3 + A_1\sigma^2 + A_2\sigma + A_3 = 0\,. \tag{41}$$

where,

$$\sigma = \omega^2\,, \tag{42}$$

$$A_0 = P_r\tau^4(1-BP_r)^2(\pi^2 Q + B^2)\,, \tag{43}$$

$$\begin{aligned} A_1 \;=\; & P_r\pi^2 Q\Big\{\tau\left(1-B\tau\right)\left(1-BP_r\right)^2 + \\ & \tau\Big[\left(BP_r+\tau+1-BK_0^*\right)^2 - 2BP_r\tau\left(1-BP_r\right)\Big]\Big\} + \\ & B^2\tau^2\Big\{\left(1-BP_r\right)^2\left[P_r\left(1-B\tau\right)+\left(1-BK_0^*\right)\Big] + \\ & P_r\left[B^2P_r^2\tau\left(2+\tau\right)+1-2BK_0^*\left(1+BP_r\tau\right)+B^2K_0^*\right]\,, \end{aligned} \tag{44}$$

$$\begin{aligned} A_2 \;=\; & P_r\pi^2 Q\Big\{B^2P_r^2\tau^2 + \left(1-B\tau\right)\Big[BP_r\tau+1-BK_0^2\Big)^2 - \\ & 2BP_r\tau\left(1-BP_r\right)]\}\, B^2\left\{B^2P_r^2\tau^2 + [P_r(1-B\tau)+(1-BK_0^*)]\right. \\ & \Big[B^2P_r^2\tau(2+\tau)+1-2BK_0^*(1+BP_r\tau)\Big] + B^2K_0^2\Big]\Big\} + \\ & \tau^2(BP_r\tau+1-BK_0^*)\,, \end{aligned} \tag{45}$$

$$A_3 = B^2\pi^2 Q^2 P_r^3 (1 - B\tau) + B^4 P_r^2 [P_r (1 - B\tau) + (1 - BK_0^*)] + \pi^2 P_r^2 S^2 (BP_r\tau + 1 - BK_0^*) \ . \quad (46)$$

For assigned values of P_r, τ, K_0^*, Q, and S, Eqs. (36) and (37) define R as a function of B; the minimum of this function determines the critical Rayleigh number R_c for the onset of oscillatory instability. The critical Rayleigh number for the onset of oscillatory instability (i.e., overstability) should be compared with that for the onset of stationary instability (i.e., ordinary convection). The type of instability, which takes place in practice, will be that corresponds to the lower value of the critical Rayleigh number.

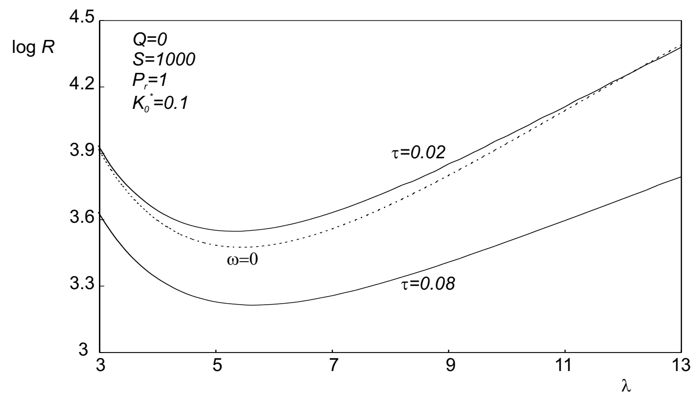

Figure 1 The variation of R for the onset of stability as a function of λ for various values of τ at: $P_r = 1$, $K_0^* = 0.1$, $Q = 0$ and $S = 0$. A broken line represents the onset of stationary convection

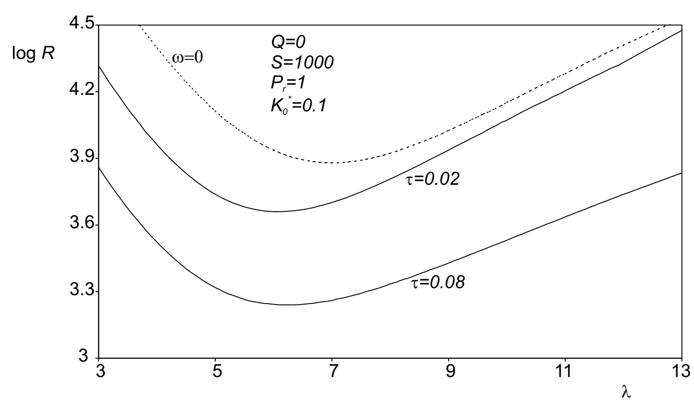

Figure 2 The variation of R for the onset of stability as a function of λ for various values of τ at: $P_r = 1$, $K_0^* = 0.1$, $Q = 0$ and $S = 1000$. A broken line represents the onset of stationary convection

In order to evaluate the conditions under which instability sets in a overstability P_r, Q, τ, K_0^*, S and B were first assigned fixed values. Then, the positive root of the cubic Eq. (41) was sought numerically and substituted in Eq. (40). When

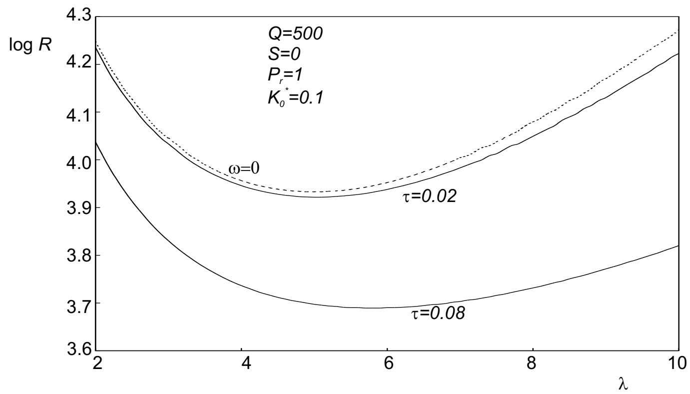

Figure 3 The variation of R for the onset of stability as a function of λ for various values of τ at: $P_r = 1$, $K_0^* = 0.1$, $Q = 500$ and $S = 0$. A broken line represents the onset of stationary convection

more than one such root was found, the one yielding the lowest value of R was, of course, taken. When no such root was found, the neutral state was considered to be stationary. This procedure was then repeated for several values of B in order to locate the minimum of R.

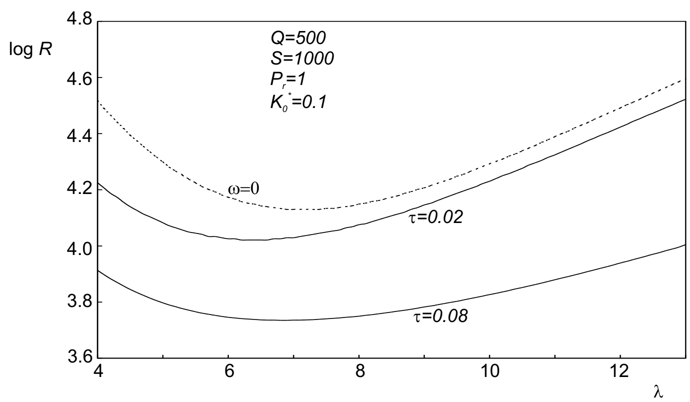

Figure 4 The variation of R for the onset of stability as a function of λ for various values of τ at: $P_r = 1$, $K_0^* = 0.1$, $Q = 500$ and $S = 1000$. A broken line represents the onset of stationary convection

We have plotted the variation of the Rayleigh number R with the wave number λ using Eq. (40) satisfying Eq. (41) in the stationary and the overstable case for values of the dimensionless parameters $P_r = 1$, $K_0^* = 0.1$, $\tau = 0.02$, 0.08, and $S = 0,\ 1000$. Figures 1–4 correspond to two values $Q = 0$ and $Q = 500$, respectively, of the magnetic field. Figures 1-4 show that the Rayleigh number R increases with an increase in the magnetic field and the Taylor number S and decreases as the relaxation time τ increase i.e. the onset of instability is delayed as Q and S increase while it is hastened as τ increases.

The critical Rayleigh number R_c and the critical wave number λ_c obtained in

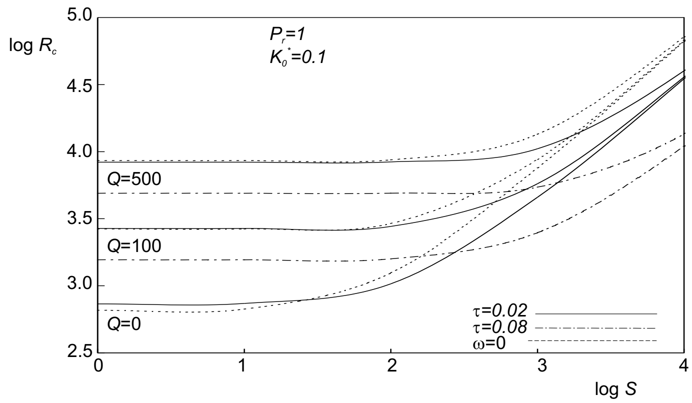

Figure 5 The variation of R_c for the onset of stability as a function of S for various values of τ and Q at: $P_r = 1$, $K_0^* = 0.1$. A broken line represents the onset of stationary convection

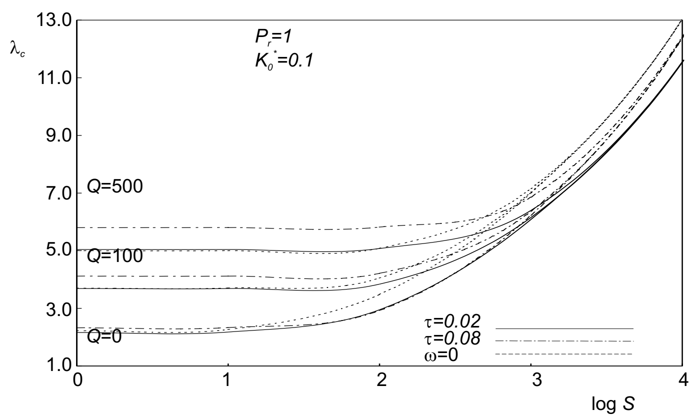

Figure 6 The variation of λ_c for the onset of stability as a function of S for various values of τ and Q at: $P_r = 1$, $K_0^* = 0.1$. A broken line represents the onset of stationary convection

that manner for both stationary instability and overstability are shown in Figs. 5 and 6, respectively, as a function of S for values of the dimensionless parameters $P_r = 1$, $K_0^* = 0.1$ and $\tau = 0.02$, 0.08 for various assigned values of Q. It is seen from Fig. 5 that the critical Rayleigh number R_c decrease as the relaxation time τ increases and increase as the rotation and the magnetic field increase. From Fig. 6 we see that the critical wave number λ_c increases as the relaxation time, the magnetic field and the rotation increase. The critical Rayleigh number obtained by Chandrasekhar [1] for the onset of stationary convection is also superimposed in Figs. 1–6 by broken line. We can directly read from Figs. 1–6 the type of instability, which takes place in practice, for the various values of parameters for which the calculations have been made.

From the above findings we conclude that the destabilizing effect of the relaxation time and the rotating are controlled by the presence of the magnetic field.

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