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Radiation and Thermal Diffusion Effects on MHD Unsteady Maxwell Fluid Past a Porous Flat Plate Through Porous Medium

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Radiation and thermal diffusion effects of magnetohydrodynamic flow for non Newtonian fluid through a porous medium past an infinite porous flat plate arc presented. The flow under consideration obeys Maxwell rheological model. Solutions for velocity, temperature and concentration distributions arc obtained with the help of finite difference method. The effects of various parameters such as relaxation parameter λ of the Maxwell fluid, permeability of the fluid K, magnetic parameter M, Dufour number D_f, Soret number S_r, Prandtl number P_r, radiation parameter N and Schmidt number S_c on the velocity, temperature and concentration profiles are studied and illustrated graphically. We obtained also the rate of heat transfer and concentration gradient during the course of discussion.

Keywords: radiation effects, thermal diffusion, Maxwell fluid.

1. Introduction

The analysis of heat and mass transfer plays an important role in ice formation and damage of an organ, preserve cells and tissues through freezing also studying non-Newtonian fluids in porous medium has many applications in industries such as ground-water hydrology, petroleum reservoir, nuclear waste disposal, geothermal energy production, transpiration cooling, design of solid matrix heat exchange and packed-bed chemical catalytic reactors.

There are many authors who interested in studying this field. Smith *et al.* [1] investigated the problem of mass transfer during freezing in rat prostate tumor tissue. Mass transfer effects on the non-Newtonian fluids past a vertical plate embedded in a porous medium with non-uniform surface heat flux is carried out by El-Hakiem *et* *al.* [2]. El-dabe *et al.* [3] investigated the problem of MHD flow of viscoelastic fluid through a porous medium on an inclined porous plane in the presence of surface stress. Lal *et al.* [4] discussed the problem of magnetohydrodynamics unsteady flow of Maxwell fluid past a flat plate. Verma [5] investigated the flow of viscoelastic Maxwell fluid between torsionally oscillatory discs. Two-dimensional steady suction now of the upper-convected Maxwell fluid in a porous channel has been studied by Choi *ct al.* [6]. Michael [7] studied the initial-value problems with inflow boundaries for Maxwell fluids. Rainieri *et al.* [8] discussed the problem of convective heat transfer to temperature dependent property fluids in the entry region of corrugated tubes. The problem of forced convection in channels partially filled with porous substrates is studied by Alkam *et al.* [9]. Numerical study of simultaneous natural convection heat transfer from both surfaces of a uniformly heated thin plate with arbitrary inclination is investigated by Wei *et al.* [10]. Takhar *et al.* [11] investigated the radiation effects on MHD free convection flow of a gas past semi-infinite vertical plate. The problem of natural convection from an inclined plate embedded in a variable porosity porous medium due to solar radiation is studied by Chamkha [12].

The main aim of our paper is to study the effect of radiation and thermal diffusion on MHD Maxwell fluid flow past a vertical porous flat plate through porous medium in the presence of magnetic field in order to investigate the relation between velocity, temperature, concentration of the fluid and different parameters of the problem as Dufour number D_r, Soret number S_r, Prandtl number P_r, Schmidt number S_c, radiation parameter N and time relaxation parameter λ where this problem plays an important role in biomedical process.

2. Basic equations

The basic equations of MHD motion neglecting displacement current and free charges are Maxwell's equations

$$\frac{\partial H_i}{\partial x_i} = 0\,, \tag{1}$$

$$J_i = \epsilon_{ijk}\frac{\partial H_k}{\partial x_j}\,, \tag{2}$$

$$\frac{\partial H_i}{\partial t} = -\mu_e \epsilon_{ijk}\frac{\partial E_k}{\partial x_j}\,, \tag{3}$$

Ohm's equation

$$J_i = \sigma(E_i + \mu_e \epsilon_{ijk}\nu_j H_k)\,. \tag{4}$$

The momentum equations

$$\rho\frac{D\nu_i}{Dt} = \frac{\partial S_{ij}}{\partial x_j} + \mu_e \epsilon_{ijk} J_j H_k\,. \tag{5}$$

The continuity equation

$$\frac{D\rho}{Dt} = \rho e_{ii} = 0\,. \tag{6}$$

The energy equation

$$\rho c\left[\frac{\partial T}{\partial t}+\nu_j\frac{\partial T}{\partial x_j}\right]=k\frac{\partial^2 T}{\partial x_i\partial x_i}-\frac{\partial q}{\partial x_j}+Dk_T\frac{\partial^2 C}{\partial x_i\partial x_i}\,. \tag{7}$$

The concentration equation

$$\frac{\partial C}{\partial t}+\nu_j\frac{\partial C}{\partial x_j}=D\frac{\partial^2 C}{\partial x_i\partial x_i}+\frac{Dk_T}{T_m}\frac{\partial^2 C}{\partial x_i\partial x_i}\,. \tag{8}$$

The stress tensor for a linear, isotropic, Maxwell fluid is given by

$$S_{ij}=-p\delta_{ij}+\tau_{ij}\,. \tag{9}$$

τ_{ij} is a tensor usually related to the strain tensor in the constitutive equations of Maxwell fluid as [5]

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\tau_{ij}=2\mu e_{ij}\,, \tag{10}$$

where

$$e_{ij}=\frac{1}{2}\left(\nu_{i,j}+\nu_{j,i}\right)\,. \tag{11}$$

3. Mathematical Analysis

We consider the unsteady flow between two infinite parallel planes of Maxwell fluid through porous medium in the presence of radiation effect, thermal diffusion and diffusion thermo-effects. The x-axis is taken along the wall and y-axis perpendicular to it. A uniform magnetic field H_0 is assumed to be applied in y direction, since the plate is infinite in extent, all quantities are functions of the space coordinate y and time t [3], $\bar{\nu}=(u,v,0)$, the momentum, energy, Maxwell fluid and concentration equations can be written as

$$\frac{\partial v}{\partial y}=0\,, \tag{12}$$

$$\rho\left[\frac{\partial u}{\partial t}+\frac{\partial u}{\partial y}\right]=\frac{\partial \tau_{xy}}{\partial y}-\left(\sigma\mu_e^2H_0^2+\frac{\mu}{K^*}\right)u+\rho g\beta\left(T^*-T_\infty\right)+\rho g\beta_c\left(C^*-C_\infty\right)\,, \tag{13}$$

$$\left(1+\lambda\frac{\partial}{\partial t}\right)\tau_{xy}=\mu\frac{\partial u}{\partial y}\,, \tag{14}$$

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{\partial q}{\partial y}+\frac{D}{c_p}\frac{k_T}{c_s}\frac{\partial^2 C}{\partial y^2}\,, \tag{15}$$

$$\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+\frac{Dk_T}{T_m}\frac{\partial^2 T}{\partial y^2}\,, \tag{16}$$

the term $\sigma\mu_e^2H_0^2$ represents the effect of magnetic field, the terms $\rho g\beta(T^*-T_\infty)$ $\rho g\beta_c(C^*-C_\infty)$ are the forced heat convection and forced mass convection $\frac{\mu}{K^*}$, is the porosity of medium, the last two terms in eqs. (15) and (16) refer to the thermal diffusion and diffusion thermo-effect where these terms play an important role in most chemical reaction. The radiation effects is represented in the term $\frac{\partial q}{\partial y}$.

By using Rosselant approximations [13]

$$q = -\frac{4\sigma^*}{3k^*}\nabla T^4 \,. \tag{17}$$

where σ^* is the Stefan-Boltzman and k^* mean absorption coefficient. We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T and neglecting higher order terms, thus we have

$$T^4 = 4T_w^3 T - 3T_w^4 \,. \tag{18}$$

The boundary conditions for the problem are

$$\left.\begin{array}{llll} u = 0\,, & T = T_w\,, & C = C_w\,, & \text{at} \quad y = 0\,,\ t > 0\,, \\ u = 0\,, & T = T_\infty\,, & C = C_\infty\,, & \text{at} \quad y \to \infty\,,\ t \leq 0\,, \end{array}\right\} \tag{19}$$

where T_∞ is the temperature at infinity, C_∞ is the mass concentration of species at infinity, T_w and C_∞ are the temperature and mass concentration at the plate.

Let us introduce the following non-dimensional variables as:

$$\left.\begin{array}{llll} y^* = \frac{U}{v}y\,, & t^* = \frac{U^2}{\nu}t\,, & u^* = \frac{u}{U}\,, & \tau^*_{xy} = \frac{\tau_{xy}}{\rho U^2}\,, \\ v^* = \frac{v}{U}\,, & \lambda^* = \frac{U^2}{v}\lambda\,, & S_c = \frac{\nu}{D}\,, & P_r = \frac{\mu c_p}{k}\,, \\ T = \frac{T^* - T_\infty}{T_w - T_\infty}\,, & C^* = \frac{C - C_\infty}{C_w - C_\infty}\,, & & D_f = \frac{Dk_T(C_w - C_\infty)}{c_p c_s (T_w - T_\infty)}\,, \\ G = \frac{\nu g\beta(T_w - T_\infty)}{v_0^3}\,, & G_0 = \frac{\nu g\beta(C_w - C_\infty)}{v_0^3}\,, & & S_r = \frac{Dk_T(T_w - T_\infty)}{T_m \nu (C_w - C_\infty)}\,, \end{array}\right\} \tag{20}$$

where U is the mean fluid velocity. Substituting from (20) in equations (12)–(16) and after dropping star mark, we obtain the following equations

$$\frac{\partial v}{\partial y} = 0\,, \tag{21}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{\partial \tau_{xy}}{\partial y} - \left(M + \frac{1}{K}\right)u + GT + G_0 C\,, \tag{22}$$

$$\left(1 + \lambda\frac{\partial}{\partial t}\right)\tau_{xy} = \frac{\partial u}{\partial y}\,, \tag{23}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \left(\frac{1}{P_r} + N\right)\frac{\partial^2 T}{\partial y^2} + D_f\frac{\partial^2 C}{\partial y^2}\,, \tag{24}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} + S_r\frac{\partial^2 T}{\partial y^2}\,, \tag{25}$$

where

$$M = \frac{\sigma H_0^2 \nu}{\rho U^2}\,, \quad N = \frac{16\sigma^* T_1^3}{3\rho c_p k^*} \text{ and } K = \left(\frac{U}{\nu}\right)^2 K^*\,.$$

The initial and boundary conditions subjected to the dimensionless quantities can be written in the following form:

$$\left.\begin{array}{llll} u = 0\,, & T = 1\,, & C = 1\,, & \text{for} \quad y = 0\,, \text{and } t > 0\,, \\ u \to 0\,, & T \to 0\,, & C \to 0\,, & \text{for} \quad y \to \infty\,, \text{and } t \leq 0\,, \end{array}\right\} \tag{26}$$

4. Numerical technique

To solve the system of equations which describe the radiation and thermal diffusion effects on MHD unsteady Maxwell fluid past a porous flat plate through porous medium under the boundary and initial conditions (26), a finite difference scheme of Crank-Nicolson type can be applied on equations (22)–(25). The region of integration is considered as a rectangle with sides $y_{max} = 18$, where y_{max} corresponds to $y = \infty$ and $t_{max} = 20$, the appropriate mesh sides $\Delta y = 0.01$ and time steps $\Delta t = 0.004$ arc considered for calculations. From eq.(21) it is observed that $v = \chi(t)$, therefore the finite difference equations corresponding to equations (22)–(25) can be written in the following form:

$$u_{i,j+1} = u_{i,j} + \Delta t \left[D_0 \tau_{i,j} + G T_{i,j} + G_0 C_{i,j} - \left(M + \frac{1}{K} \right) u_{i,j} - \chi_j D_0 u_{i,j} \right], \quad (27)$$

$$\tau_{i,j+1} = \left(\frac{1}{\lambda} - \Delta t \right) \tau_{i,j} + \Delta t D_0 u_{i,j}\,, \quad (28)$$

$$T_{i,j+1} = T_{i,j} + \Delta t \left[\frac{1}{P_r} D_+ D_- T_{i,j} + D_f D_+ D_- C_{i,j} - \chi_j D_0 T_{i,j} \right], \quad (29)$$

$$C_{i,j+1} = C_{i,j} + \Delta t \left[\frac{1}{S_c} D_+ D_- C_{i,j} + S_r D_+ D_- T_{i,j} - \Psi_j D_0 C_{i,j} \right], \quad (30)$$

where $u_{i,j} = u(i\Delta y, j\Delta t)$, $T_{i,j} = T(i\Delta y, j\Delta t)$, $C_{i,j} = C(i\Delta y, j\Delta t)$, i is an integer, j is non-negative integer denote the grid functions which approximate the exact solutions of $u(y,t)$, $T(y,t)$ and $C(y,t)$. D_+, D_- and D_0 are numerical operators defined in [14].

According to the stability conditions which computed by Von-Neuman technique we calculated numerical values the velocity, temperature and concentration distributions for different parameters of our problem. We also obtained the non-dimensional forms of rate of heat transfer and concentration gradient as following

1. The dimensionless rate of heat transfer can be expressed

$$Q = -\left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (31)$$

2. The concentration gradient in dimensionless can be written as

$$S = -\left(\frac{\partial C}{\partial y} \right)_{y=0} \quad (32)$$

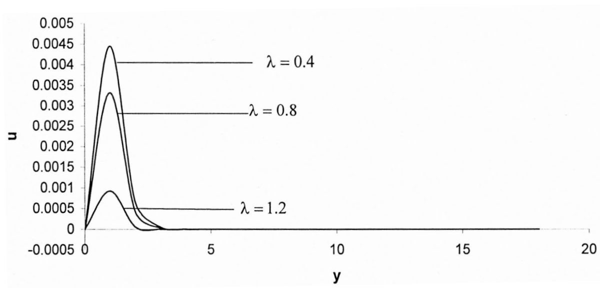

Figure 1 Velocity distribution plotted against y for different time relaxation parameter λ, magnetic parameter M, permeability K, radiation parameter N, local mass Grashof number G_0 and Grashof number G, respectively

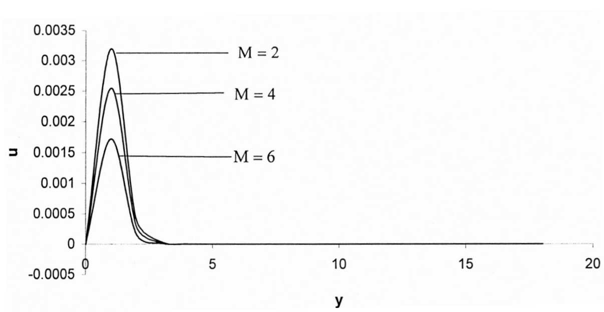

Figure 2 Velocity distribution plotted against y for different time relaxation parameter λ, magnetic parameter M, permeability K, radiation parameter N, local mass Grashof number G_0 and Grashof number G, respectively

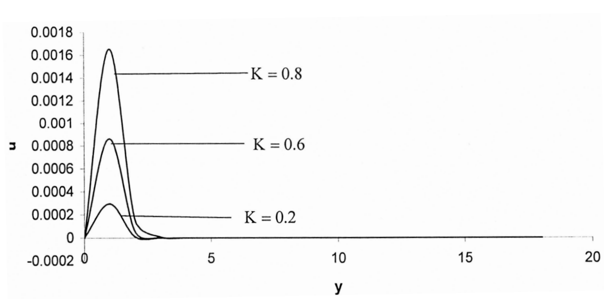

Figure 3 Velocity distribution plotted against y for different time relaxation parameter λ, magnetic parameter M, permeability K, radiation parameter N, local mass Grashof number G_0 and Grashof number G, respectively

5. Results and discussion

The radiation and thermal diffusion effects on unsteady MHD flow in porous medium over an infinite vertical flat plate are studied. The momentum, energy and concentration equations are solved numerically by using finite difference methods. In the course of our discussion we computed the numerical values of velocity, temperature and concentration according to stability conditions which obtained by Von-Neuman technique. In computing the functions of the problem we put

$$\chi(t) = \cos(t)\,.$$

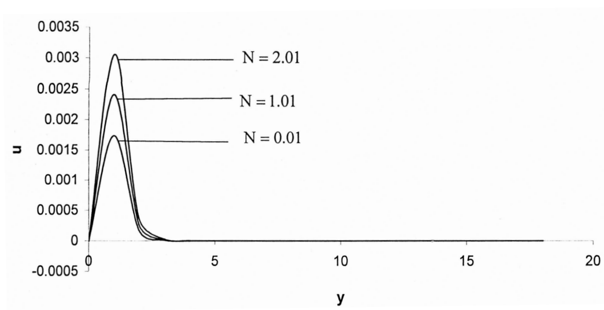

Figure 4 Velocity distribution plotted against y for different time relaxation parameter λ, magnetic parameter M, permeability K, radiation parameter N, local mass Grashof number G_0 and Grashof number G, respectively

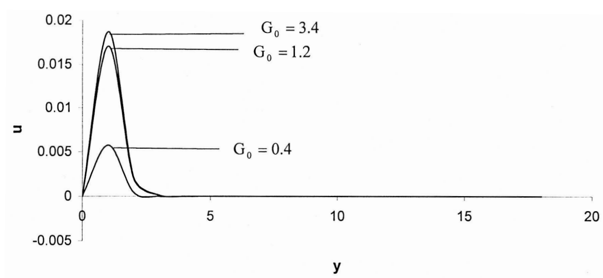

Figure 5 Velocity distribution plotted against y for different time relaxation parameter λ, magnetic parameter M, permeability K, radiation parameter N, local mass Grashof number G_0 and Grashof number G, respectively

The effect of time relaxation parameter λ on velocity distribution is showed in Figure 1 where it is clear that the velocity distribution decreases as time relaxation parameter λ increases. Figure 2 shows the effect of magnetic parameter M on the velocity distribution. It is observed that the velocity decreases as the magnetic parameter M increases. Figures 3 and 4 indicate that the velocity increases as permeability parameter K and radiation parameter N increase. The effects of local mass Grashof number G_0 and local temperature Grashof number G on the velocity can be shown from Figures 5 and 6. It is clear that the velocity distribution increases as both local mass Grashof number G_0 and local temperature Grashof number G increase.

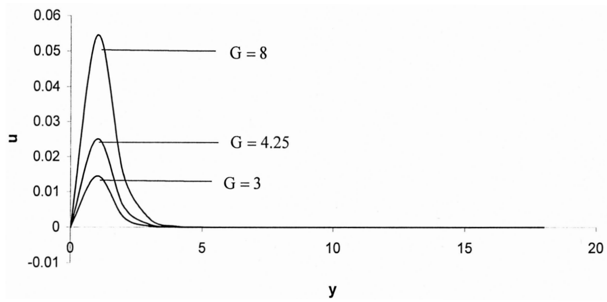

Figure 6 Velocity distribution plotted against y for different time relaxation parameter λ, magnetic parameter M, permeability K, radiation parameter N, local mass Grashof number G_0 and Grashof number G, respectively

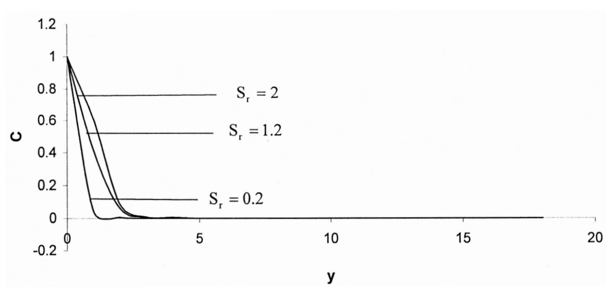

Figure 7 Concentration distribution plotted against y for different thermal diffusion parameter S_r, and Schmidt number S_c

Figure 7 indicates that the thermal diffusion parameter S_r, increases as concentration distribution increases while it decreases as Schmidt number S_c increases as shown in Figure 8. Figure 9 shows that the skin friction coefficient increases as magnetic parameter increases.

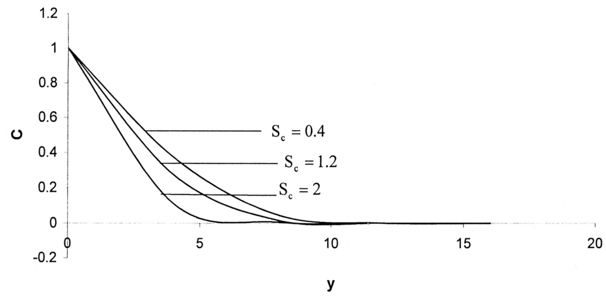

Figure 8 Concentration distribution plotted against y for different thermal diffusion parameter S_r, and Schmidt number S_c

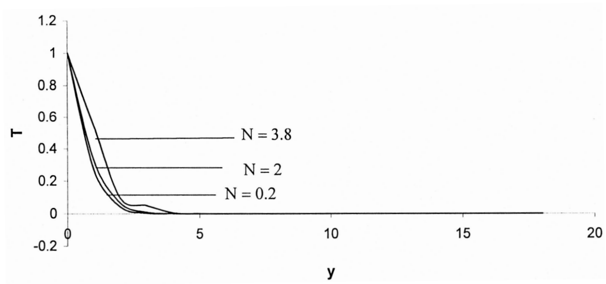

Figure 9 Temperature distribution plotted against y for different radiation parameter N and Prandtl number P_r

The effect of radiation parameter N on the temperature distribution can be shown from Figure 9. It is clear that temperature distribution decreases as radiation parameter N increases, while it is observed from Figure 10 that the temperature distribution increases with the increasing of the Prandtl number P_r. Figures 11 and 12 show that the rate of heat transfer and concentration gradient increase with the increasing of both radiation parameter N and thermal diffusion parameter S_r.

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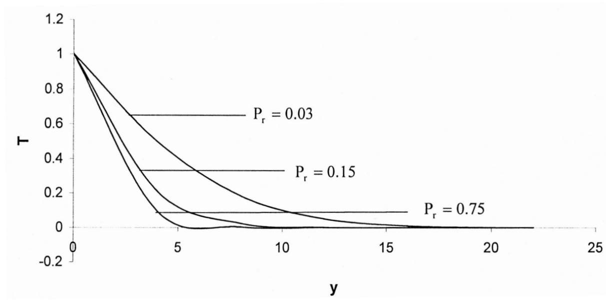

Figure 10 Temperature distribution plotted against y for different radiation parameter N and Prandtl number P_r

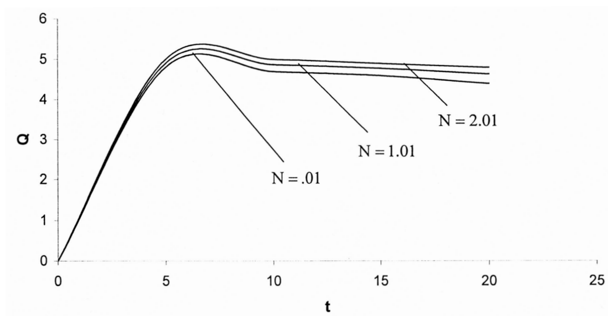

Figure 11 Rate of heat transfer Q plotted against time t

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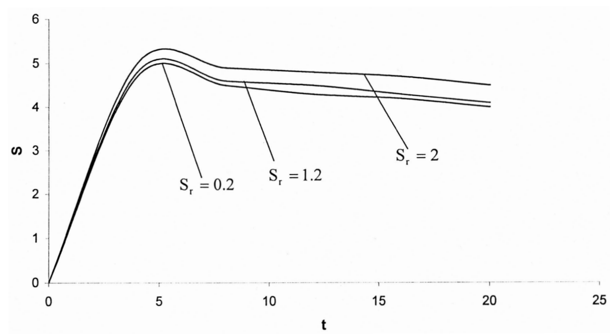

Figure 12 Rate of concentration gradient S plotted against time t

Nomenclature

σ	electrical conductivity
ρ	density
K	permeability
S_{ij}	stress tensor
p	pressure
H_0	the strength of magnetic induction applied parallel to y-axis
Q	rate of heat transfer
T	temperature
P_r	Prandtl number
C	concentration
S_C	Schmidt number
c_p	specific heat
k	thermal conductivity of the fluid
μ_e	the magnetic permeability
λ	relaxation time
e_{ij}	strain rate tensor
E_c	Eckert number
$\bar{H}$	magnetic field
$\bar{E}$	electric field
ϵ_{ijk}	third order tensor
G	local temperature Grashof number
G_0	local mass Grashof number
β	coefficient of the thermal expansion
β_c	coefficient of the thermal expansion with concentration
ν	kinematic viscosity
D	molecular diffusivity
D_f	Dufour number
S_r	Soret number
k_T	thermal diffusion ratio
T_m	mean fluid temperature
c_s	concentration susceptibility
N	radiation parameter
q	radiative heat flux