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Unsteady Couette-Poiseulle Flow with Variable Physical Properties in the Presence of a Uniform Magnetic Field

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The unsteady hydromagnetic Couette-Poiseulle flow and heat transfer of an electrically conducting fluid is studied in the presence of a transverse uniform magnetic field with variable physical properties. The viscosity and thermal conductivity of the fluid are assumed to vary with temperature. The fluid is subjected to a constant pressure gradient and an external uniform magnetic field perpendicular to the plates which are kept at different but constant temperatures. The effect of the magnetic field, the temperature dependent viscosity and thermal conductivity on both the velocity and temperature fields is reported.

Keywords: Fluid dynamics, magnetohydrodynamics, parallel channel flow, magnetic field, Couette flow, Couette- Poiseulle, heat transfer, steady flow, unsteady flow.

1. Introduction

The flow with heat transfer of a viscous incompressible electrically conducting fluid between two parallel plates is a classical problem that has important applications in magnetohydrodynamic (MHD) power generators and pumps, accelerators, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry. This problem has been considered by many researchers under different physical effects [1–5]. Most of these studies are based on constant physical properties, although some physical properties are varying with temperature and assuming constant properties is a good approximation as long as small differences in temperature are involved [6]. More accurate prediction for the flow and heat transfer can be achieved by considering the variation of these physical properties with temperature. Klemp et al. [7] studied the effect of temperature dependent viscosity on the entrance flow in a channel in the hydrodynamic case. The MHD fully developed flow and heat transfer of an electrically conducting fluid between two parallel plates with temperature dependent viscosity is studied in [8,9] without taking the Hall effect into consideration.

In the present work, the unsteady Couette-Poiseulle flow of a viscous incompressible electrically conducting fluid with heat transfer between two electrically insulating plates is studied in the presence of uniform magnetic field. The upper plate is moving with a constant speed and the lower plate is kept stationary while the fluid is acted upon by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected [1,5]. The two plates are kept at two constant but different temperatures while the viscosity and thermal conductivity of the fluid are assumed to vary with temperature. Thus, the coupled set of the nonlinear equations of motion and the energy equation including the viscous and Joule dissipations terms is solved numerically using finite differences to obtain the velocity and temperature distributions at any instant of time.

2. Formulation of the Problem

The fluid is assumed to be flowing between two infinite horizontal plates located at the $y = \pm h$ planes. The upper plate moves with a uniform velocity U_0 while the lower plate is stationary. The two plates are assumed to be electrically insulating and kept at two constant temperatures T_1 for the lower plate and T_2 for the upper plate with $T_2 > T_1$. A constant pressure gradient $\partial P/\partial x$ is applied in the x-direction. A uniform magnetic field B_0 is applied in the positive y-direction which is the only magnetic field in the problem as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number [1,5]. The viscosity of the fluid is assumed to vary exponentially with temperature while the thermal conductivity is assumed to depend linearly on temperature. The viscous and Joule dissipations are taken into consideration. The flow of the fluid is governed by the Navier-Stokes equation which has the form [1,5],

$$\rho \frac{\partial u}{\partial t} = -\frac{dP}{dx} + \mu \frac{\partial^2 u}{\partial y^2} + \frac{\partial \mu}{\partial y}\frac{\partial u}{\partial y} - \sigma B_0^2 u \tag{1}$$

where ρ is the density of the fluid, μ is the viscosity of the fluid, σ is the electric conductivity of the fluid, and $u = u(y,t)$ is the velocity component of the fluid in the x-direction. It is assumed that the pressure gradient is applied at $t = 0$ and the fluid starts its motion from rest. Thus

$$t = 0: \ u = 0 \tag{2a}$$

For $t > 0$, the no-slip condition at the plates that

$$y = -h: \ u = 0, \ y = h: \ u = U_0 \tag{2b}$$

The energy equation describing the temperature distribution for the fluid is given by [1,10]

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \mu \left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 u^2 \tag{3}$$

where T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The last two terms in the left-hand side of Eq. (3) represent, respectively, the viscous and Joule dissipations. The temperature of the fluid must satisfy the boundary conditions,

$$t = 0: \; T = T_1 \tag{4a}$$

$$t > 0: \; T = T_1, \; y = -h, \; T = T_2, \; y = h\,. \tag{4b}$$

The viscosity of the fluid is assumed to vary with temperature and is defined as, $\mu = \mu_0 f_1(T)$. By assuming the viscosity to vary exponentially with temperature, the function $f_1(T)$ takes the form [7], $f_1(T) = \exp(-a_1(T - T_1))$. In some cases a_1 may be negative, i.e. the coefficient of viscosity increases with temperature [8,9]. Also, the thermal conductivity of the fluid is assumed to vary with temperature as $k = k_0 f_2(T)$. We assume linear dependence for the thermal conductivity upon temperature in the form $k = k_0[1 + b_1(T - T_1)]$ [10], where the parameter b_1 may be positive or negative [10].

The problem is simplified by writing the equations in the non-dimensional form. To achieve this, we define the following non-dimensional quantities,

$$(\hat{x}, \hat{y}, \hat{z}) = \frac{(x, y, z)}{h}, \quad \hat{t} = \frac{tU_0}{h},$$

$$\hat{P} = \frac{P}{\rho U_0^2}, \quad (\hat{u}, \hat{v}, \hat{w}) = \frac{(u, v, w)}{U_0},$$

$$\theta = \frac{T - T_1}{T_2 - T_1}, \quad G = -\frac{d\hat{P}}{d\hat{x}}\,.$$

$\hat{f}_1(\theta) = \exp(-a_1(T_2 - T_1)) = \exp(-a\theta)$, a is the viscosity exponent,

$\hat{f}_2(\theta) = 1 + b_1(T_2 - T_1) \quad \theta = 1 + b\theta$, b is the thermal conductivity parameter,

$\mathrm{Re} = \rho U_0 h/\mu_0$, is the Reynolds number,

$\mathrm{Ha}^2 = \sigma B_0^2 h^2/\mu_0$, is the Hartmann number,

$\mathrm{Pr} = \mu_0 c_p/k_0$, is the Prandtl number,

$\mathrm{Ec} = U_0^2/c_p\,(T_2 - T_1)$, is the Eckert number.

$\tau_L = (\partial\hat{u}/\partial\hat{y})_{\hat{y}=-1}/\mathrm{Re}$ is the axial skin friction coefficient at the lower plate,

$\tau_U = (\partial\hat{u}/\partial\hat{y})_{\hat{y}=1}/\mathrm{Re}$ is the axial skin friction coefficient at the upper plate,

$\mathrm{Nu}_L = (\partial\theta/\partial\hat{y})_{\hat{y}=-1}$ is the Nusselt number at the lower plate,

$\mathrm{Nu}_U = (\partial\theta/\partial\hat{y})_{\hat{y}=1}$ is the Nusselt number at the upper plate.

In terms of the above non-dimensional quantities Eqs. (1) to (4) read (the hats are dropped for convenience)

$$\frac{\partial u}{\partial t} = G + \frac{1}{\mathrm{Re}} f_1(T) \frac{\partial^2 u}{\partial y^2} + \frac{1}{\mathrm{Re}} \frac{\partial f_1(T)}{\partial y} \frac{\partial u}{\partial y} - \frac{\mathrm{Ha}^2}{\mathrm{Re}} u \tag{5}$$

$$t = 0: \; u = 0 \tag{6a}$$

$$t > 0: \; u = 0, \; y = -1, \; u = 0, \; y = 1 \tag{6b}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}} f_2(T) \frac{\partial^2 T}{\partial y^2} + \frac{1}{\mathrm{Re}\,\mathrm{Pr}} \frac{\partial f_2(T)}{\partial y} \frac{\partial T}{\partial y} + \frac{\mathrm{Ec}}{\mathrm{Re}} f_1(T) (\frac{\partial u}{\partial y})^2 + \frac{\mathrm{Ec}}{\mathrm{Re}} \mathrm{Ha}^2 u^2 \tag{7}$$

$$t = 0: \; \theta = 0 \tag{8a}$$

$$t > 0: \; \theta = 0, \; y = -1, \; \theta = 1, \; y = 1\,. \tag{8b}$$

Equations (5) and (7) represent a system of coupled non-linear partial differential equations which can be solved numerically under the initial and boundary conditions (6) and (8) using the finite difference approximations. The Crank-Nicolson implicit method is used [11]. Finite difference equations relating the variables are obtained by writing the equations at the mid point of the computational cell and then replacing the different terms by their second order central difference approximations in the y-direction. The diffusion terms are replaced by the average of the central differences at two successive time levels. The non-linear terms are first linearized and then an iterative scheme is used at every time step to solve the linearized system of difference equations. All calculations have been carried out for $G = 5$, $R = 1$, $\mathrm{Pr} = 1$, and $\mathrm{Ec} = 0.2$.

3. Results and Discussion

Figures 1(a) and (b) present the velocity and temperature distributions as functions of y for various values of time t starting from $t = 0$ up to the steady-state. The figures are evaluated for $\mathrm{Ha} = 1$, $a = 0.5$, and $b = 0.5$. The velocity component u reaches the steady state faster than θ. This is expected as u is the source of T. Figure 1(b) shows that the temperature T inside the fluid may exceed the value 1, which is the temperature of the hot plate, especially at large times. This is due to the Joule and viscous dissipations.

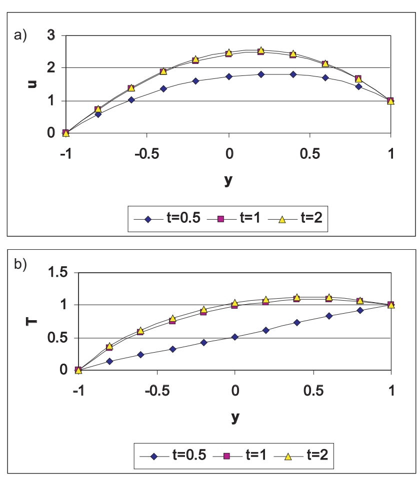

Figure 1 Time development of the profile of: (a) u; (b) T; $\mathrm{Ha} = 1$, $a = 0.5$, $b = 0.5$

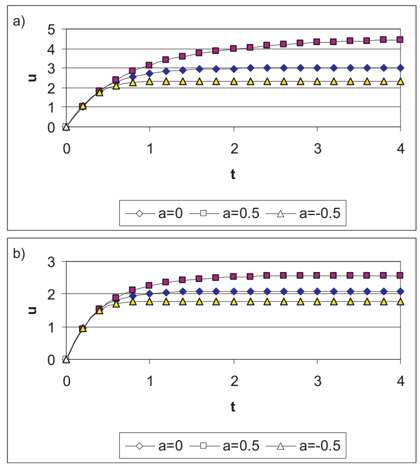

Figure 2 Time development of u at $y = 0$ for various values of a: (a) Ha $= 0$; (b) Ha $= 1$

Figures 2(a) and (b) present the time development of the velocity component u at the center of the channel ($y = 0$), for various values of the parameters a and Ha and for $b = 0$. Figures 2(a) and (b) show that increasing the parameter a decreases u for all values of Ha. It is also shown that the steady state time of u is not greatly affected by changing a. Comparing Figs. 1(a) and (b) indicates the damping effect of the magnetic field which decreases u for all values of a. Figures 3(a) and (b) present the time development of the temperature T at the center of the channel ($y = 0$), for various values of the parameters a and Ha and for $b = 0$. The figures show that increasing a decreases T for all values of H_a as a result of decreasing the velocity u and its gradient the function f_1 which decreases the viscous and Joule dissipations. It is also shown that the steady state value of T is not greatly affected by changing a. The comparison between Figs. 2(a) and (b) shows that increasing Ha increases T, for all values of a, due to the increase in the Joule dissipations.

Figures 4(a) and (b) present the time development of the temperature T at the center of the channel ($y = 0$), for various values of the parameters b and Ha and for $a = 0$. The figures show that the variation of the temperature θ with the parameter b depends on t where a crossover in $T - t$ charts occurs. The effect of b on T depends on t and increasing b increases T at small times, but decreases T when t is large. This occurs because, at low times, the center of the channel acquires heat by conduction from the hot plate, but after large time, when u is large, the Joule dissipation is large at the center and center looses heat by conduction. It is noticed that the parameter b has no significant effect on u inspite of the coupling between the momentum and energy equations. It is also shown in the figures that increasing the parameter b decreases the steady state time of T. Figure 3(b) indicates that increasing Ha increases T as the Joule dissipation increases and decreases the time at which the crossover in $T - t$ charts occurs.

Table 1 and 2 presents the variation of the steady state axial skin friction coefficients at both walls for various values of a and for Ha=0 and 3, respectively. It is clear that increasing a increases the magnitude of τ_L and τ_U for all values of Ha.

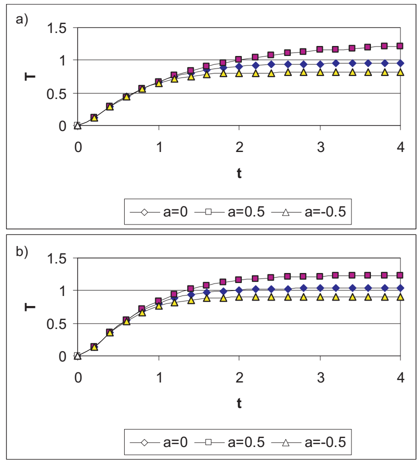

Figure 3 Time development of θ at $y = 0$ for various values of a: (a) Ha = 0; (b) Ha = 1

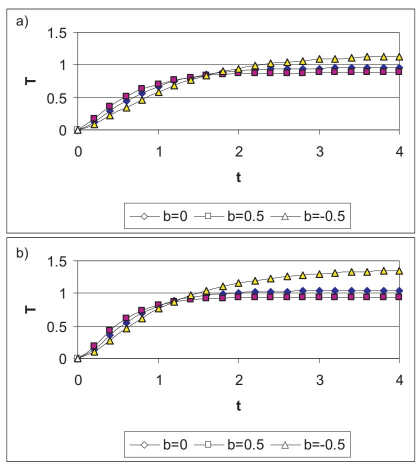

Figure 4 Time development of θ at $y = 0$ for various values of b: (a) Ha = 0; (b) Ha = 1

	$a = -0.5$	$a = -0.1$	$a = 0.0$	$a = 0.1$	$a = 0.5$
τ_L	5.2131	5.4494	5.4998	5.5458	5.6355
τ_U	-2.9642	-4.1236	-4.4999	-0.4918	-7.0608

Table 1 Variation of the steady state skin friction coefficients τ_L and τ_U for various values of a (Ha=0)

	$a = -0.5$	$a = -0.1$	$a = 0.0$	$a = 0.1$	$a = 0.5$
τ_L	1.6089	1.6535	1.6617	1.6690	1.6897
τ_U	0.9174	1.2451	1.3299	1.4168	1.7888

Table 2 Variation of the steady state skin friction coefficients τ_L and τ_U for various values of a (Ha=3)

T	$a=-0.5$	$a=-0.1$	$a=0.0$	$a=0.1$	$a=0.5$
b=-0.5	0.8652	1.0583	1.1322	1.2235	1.7873
b=-0.1	0.8181	0.9324	0.9706	1.0148	1.2737
b=0.0	0.8109	0.9151	0.9495	0.9888	1.2148
b=0.1	0.8045	0.9005	0.9317	0.9673	1.1682
b=0.5	0.7832	0.8569	0.8801	0.9061	1.0468

Table 3 Variation of the steady state temperature T at $y=0$ for various values of a and b (Ha=0)

T	$a=-0.5$	$a=-0.1$	$a=0.0$	$a=0.1$	$a=0.5$
$b=-0.5$	0.8948	0.8563	0.8482	0.8407	0.8177
$b=-0.1$	0.8254	0.8046	0.7999	0.7955	0.7811
$b=0.0$	0.8161	0.7973	0.7930	0.7890	0.7758
$b=0.1$	0.8079	0.7909	0.7869	0.7833	0.7711
$b=0.5$	0.7827	0.7700	0.7671	0.7642	0.7549

Table 4 Variation of the steady state temperature T at $y=0$ for various values of a and b (Ha=3)

Nu_L	$a=-0.5$	$a=-0.1$	$a=0.0$	$a=0.1$	$a=0.5$
$b=-0.5$	2.0662	2.4035	2.5176	2.6479	3.2568
$b=-0.1$	2.0626	2.4015	2.5110	2.6346	3.3095
$b=0.0$	2.0677	2.4066	2.5152	2.6374	3.3046
$b=0.1$	2.0739	2.4129	2.5210	2.6421	3.3013
$b=0.5$	2.1048	2.4453	2.5515	2.6697	3.3031

Table 5 Variation of the steady state Nusselt number Nu_L for various values of a and b (Ha=0)

Nu_L	$a=-0.5$	$a=-0.1$	$a=0.0$	$a=0.1$	$a=0.5$
$b=-0.5$	0.9978	0.9631	0.9559	0.9495	0.9309
$b=-0.1$	0.9712	0.9420	0.9356	0.9299	0.9114
$b=0.0$	0.9730	0.9445	0.9382	0.9325	0.9141
$b=0.1$	0.9763	0.9485	0.9424	0.9367	0.9184
$b=0.5$	0.9958	0.9698	0.9639	0.9584	0.9405

Table 6 Variation of the steady state Nusselt number Nu_L for various values of a and b (Ha=3)

Nu_U	$a=-0.5$	$a=-0.1$	$a=0.0$	$a=0.1$	$a=0.5$
$b=-0.5$	-0.9800	-1.6554	-1.8797	-2.1323	-3.0552
$b=-0.1$	-0.5221	-0.9089	-1.0353	-1.1795	-2.0039
$b=0.0$	-0.4715	-0.8200	-0.9331	-1.0615	-1.7918
$b=0.1$	-0.4318	-0.7488	-0.8508	-0.9665	-1.6194
$b=0.5$	-0.3358	-0.5682	-0.6414	-0.7234	-1.1741

Table 7 Variation of the steady state Nusselt number Nu_U for various values of a and b (Ha=0)

Nu_U	$a=-0.5$	$a=-0.1$	$a=0.0$	$a=0.1$	$a=0.5$
$b=-0.5$	-1.0377	-0.9168	-0.8872	-0.8586	-0.7536
$b=-0.1$	-0.5353	-0.4727	-0.4570	-0.4415	-0.3839
$b=0.0$	-0.4807	-0.4249	-0.4109	-0.3971	-0.3453
$b=0.1$	-0.4381	-0.3878	-0.3751	-0.3625	-0.3155
$b=0.5$	-0.3354	-0.2992	-0.2900	-0.2809	-0.2465

Table 8 Variation of the steady state Nusselt number Nu_U for various values of a and b (Ha=3)

Increasing Ha decreases the magnitude of τ_L and τ_U while reverses the direction of τ_U for all values of a. Tables 3 and 4, 5 and 6, 7 and 8 present the variation of the steady state temperature T at $y=0$, the Nusselt number at the lower and upper plates for various values of the parameters a and b and, respectively, for Ha=0 and Ha=3. It is clear from Tables 3, 5, and 7 that increasing a increases T, Nu_L, and the magnitude of Nu_U for all values of b. On the other hand, for large values of Ha, as shown in Tables 4, 6, and 8, increasing a decreases T, Nu_L, and the magnitude of Nu_U for all values of b. Tables 3, 5, and 7 indicate that increasing b decreases T and the magnitude of Nu_U while increases Nu_L for all values of a. Tables 4, 6, and 8 show that, for large values of Ha, increasing b decreases T, Nu_L, and the magnitude of Nu_U for all values of a. Tables 3 and 4, 5 and 6, and, 7 and 8 indicate that the effect of a or b on T is more pronounced for smaller values of Ha. Also, the effect of a and b on the heat transfer at the upper plate Nu_U is more apparent than that at the lower plate.

4. Conclusion

The unsteady Couette-Poiseulle flow of a conducting fluid under the influence of an applied uniform magnetic field has been studied with temperature dependent viscosity and thermal conductivity in the presence of an external uniform magnetic field. It was found that the magnetic field or the viscosity exponent has a damping effect on the velocity component u while the effect of the parameter b on u can be entirely neglected. It is also shown that increasing the magnetic field increases the temperature T, however, increasing the viscosity exponent decreases T. It is of interest to find that the effect of the parameter b on the temperature T depends on time. The magnetic field or viscosity exponent has a marked effect on the axial skin friction coefficients and the Nusselt number at both walls of the channel. The parameter a or b has a marked effect on the Nusselt number at the upper plate more than the lower plate which becomes more pronounced for smaller values of the magnetic field.

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