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Prediction of Life of Rolling Pairs in Cycloidal Gear Design

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Cycloidal gear design is based on forming geometry and material features of load carrying assembly to fulfil the following criteria of the construction: functionality, reliability and life. Life of cycloidal gear is determined by life of the weakest kinematic pair in power transmission system. That system consists of the following, connected in series, rolling pairs: the central cylindrical roller bearing, the set of rolling bolts in straight-line mechanism and the special meshing of planetary wheel. Dimensions of the gear are determined by combination of three basic parameters: curtailment coefficient of epicycloid, gear eccentric and roller diameter of co-operating wheel. The combination mentioned above should fulfil conditions of internal cycloidal meshing and assure proper reliability in power transmission system. In the paper there are presented results of calculations of fatigue life of rolling pairs in cycloidal gear for different parameters affected gear overall dimensions. These results made possible to find optimal geometry of the gear in fatigue life aspect.

Keywords: Rolling pair, fatigue life, cycloidal gear.

1. Introduction

Cycloidal gear design, known also as Cyclo gear, is based on forming geometry and material features of load carrying assembly to fulfil the following criteria of the construction:

- functionality connected with power, transmission ratio and rotational speed,
- reliability, i.e. probability of correct operation in defined time interval,
- life expressed as total number of hours of operation or accumulated number of rotations until wear occurs.

Life of Cyclo gear is determined by life of the weakest kinematic pair in power transmission system. That system (Fig. 1) is compound of three, connected in series, rolling pairs: the central cylindrical roller bearing (1), the set of rolling bolts in straight-line mechanism (2) and the special meshing of the planetary wheel (3). The possible range of gear ratio is $i = 9 - 87$. The meshing is created by planetary wheels (usually set of two, displaced against each other with π angle) co-operating with stationary set of rollers. Planetary wheels have external teeth with the shape of equidistant of curtate epicycloid [1, 2]. Drive moment M_h is transmitted on planetary wheel from high-speed shaft at speed n_h by means of eccentric, inner ring and rollers of cylindrical roller bearing. Central hole of the planetary wheel serves as the raceway of outer ring of the bearing. Straight-line mechanism is used for transmission the torque moment M_c from planetary wheel onto output shaft. That mechanism consists of bolts rolling away internal holes of planetary wheel. The bolts are fixed in the disk connected with output shaft. The third of moments, M_2, loads rollers of co-operating wheel.

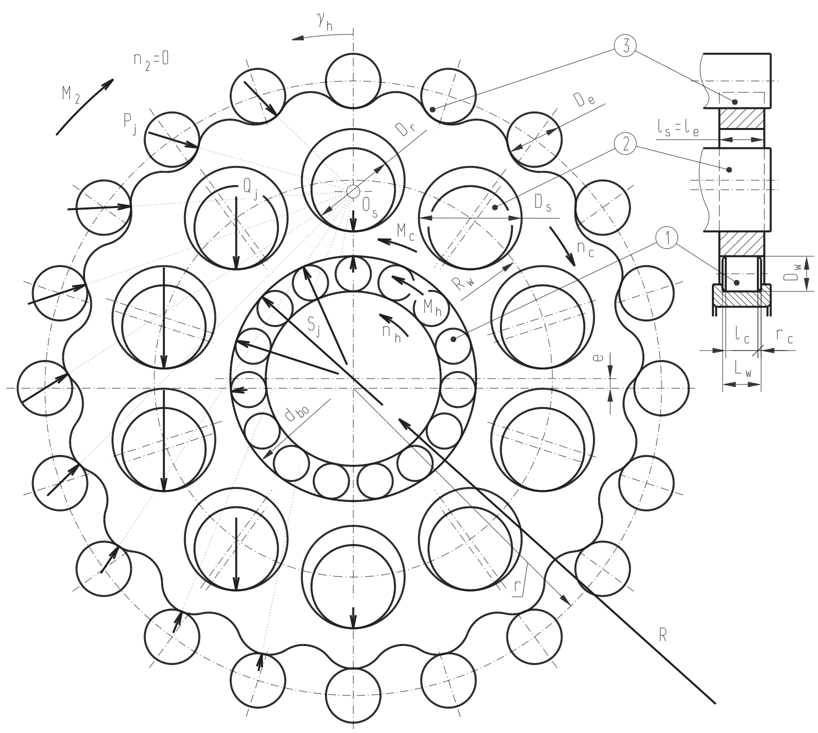

Figure 1 Operation principle and state of load in power transmission system of the cycloidal gear

Dimensions of planetary gear and the whole gear are determined by three parameters:

- eccentricity e, with values range 1–10 mm,

- curtailment coefficient of epicycloid m_e, with values range 0.5–0.85,
- offset of epicycloid equidistant equal to radius of the roller of co-operating wheel $D_e/2$.

Cycloidal gear should have relatively low spacing radius r of the rollers of co-operating wheel (Fig. 1). Life of meshing and other rolling pairs should be equal or higher than assumed one. The value of radius r, at required transmission ratio, power and rotational speed of drive shaft, is the result of proper combination of values e, m_e and D_e:

$$r = e \cdot (i+1)/m_e\,. \tag{1}$$

Simultaneously, the combination of e, m_e and D_e should fulfil the conditions of operation of internal cycloidal meshing and strength conditions defined for all parts in power transmission system.

The major type of wear, determining life of planetary wheels and in the consequence entire gear, is fatigue wear of teeth surface of the planetary wheel and bearing raceway in the central hole. This is the conclusion made after tests of prototype of Cyclo gear [3, 4]. Similar type of wear is supposed to occur in the straight-line mechanism. In theory in each kinematic pair of cycloidal gear rolling friction occurs. Kinematic pairs mentioned above are the rolling pairs more or less similar to kinematic pairs of rolling bearing. All rolling parts in power transmission system are usually made of bearing steel after heat treatment for the typical hardness for raceway in rolling bearings. Lubrication conditions of the gear are advantageous for creation of elastohydrodynamic oil film in contact points of the co-operating parts. Regarding similarity of the phenomena occurring in rolling pairs in cycloidal gear to the phenomena of wear of conventional rolling bearings, the fatigue life theory of rolling bearings to predicting life of these pairs was implemented.

In the paper, it is presented the analyze of influence of the parameters e, m_e and D_e on fatigue life of the rolling pairs of cycloidal gear. The aim of analyze was to determine the most advantageous gear geometry from fatigue life point of view. Analyze was made after implementation the methodology of fatigue life prediction of the rolling pairs in the Cyclo gear and computer programs discussed in the references [5, 6, 7, 8].

2. Subject of tests

Analyze of the influence of the geometrical parameters on fatigue life of the Cyclo gear was made for prototype of the gear [3, 4] with ratio $i = 19$, power $N_p = 3.7$ kW and rotational speed of the shaft $n_h = 750$ rpm. The output torque was $M_1 = 2M_c = 880$ Nm. Main parameters of the gear are presented in Table 1.

3. Hierarchy of life of the rolling pairs in cyclo gear

As the first, calculations of predicted fatigue life of all three rolling pairs for the parameters e, m_e and D_e were made. The parameters were corresponding with real gear prototype (Table 1). For that case the value of the resulting force acting on bearing of the drive shaft was equal $R = 10314$ N. The purpose of the calculation was to find out the weakest rolling pair of the gear, the one that determines gear fatigue life.

Distributions of forces acting on teeth P_j and reaction forces in straight-line mechanism Q_j (Fig. 1) were determined using analytical method and FEM. In the first case, the influence of rotational movement and strains of the gear elements were neglected and the load distributions were calculated for one of the positions of the drive shaft (angle ($\gamma_h = 0$). In the other case, calculations were made for selected positions of the drive shaft. Greatest lower and least upper bound of average of the forces were calculated, basing on the results of distributions of loads P_j, Q_j. Owing to that it was possible to calculate predicted fatigue life using methodology described in reference [3].

Co-operating wheel		
	Number of the co-operating wheel roller	$z_k = 20$
	Diameter of the co-operating wheel roller	$D_e = 17$ mm
	Spacing radius of the co-operating wheel rollers	$r = 96$ mm
Planetary wheel		
	Number of teeth of the planetary wheel	$z_s = 19$
	Planetary wheel width	$l_e = 14.5$ mm
	Central hole diameter	$d_{bo} = 76.5$ mm
	Side hole diameter	$D_s = 32$ mm
	Eccentric	$e = 3$ mm
	Curtailment coefficient of epicycloid	$m_e = 0.625$ mm
Straight-line mechanism		
	Spacing radius of bolts	$R_w = 65$ mm
	Number of bolts	$U = 10$
	Bolt diameter	$D_r = 26$ mm
Central bearing (N 209E, FAG)		
	Number of rollers	$W = 15$
	Roller diameter	$D_w = 11$ mm
	Roller length	$L_w = 12$ mm
	Roller chamfer	$r_w \approx 0.5$ mm
	Radial clearance	$g = 0.045$ mm

Table 1 Main parameters of the prototype of Cyclo gear

Calculations of life were made for nominal, theoretical profile of teeth of the planetary wheel and for real, corrected teeth. The method for generalization of force distributions was described in details in paper [9].

The surface life of the rolling pair at level $\varphi = 0.9$ was assumed as the criterion value in the calculations. Basing on rolling bearings fatigue life theory, it was assumed that the rolling parts life is higher than the surface of rolling life. Taking that assumption into consideration, central cylindrical roller bearing life is determined by inner and outer raceway; in straight-line mechanism life of cylindrical surfaces of the holes in planetary wheels is decisive and meshing life is determined by life of tooth flanks. As the measure of life it was assumed accumulated number of rotations of the drive shaft.

In Fig. 2 it is presented the hierarchy of life of the main rolling pairs typical for Cyclo gear. Data for the diagram were taken from the reference [9].

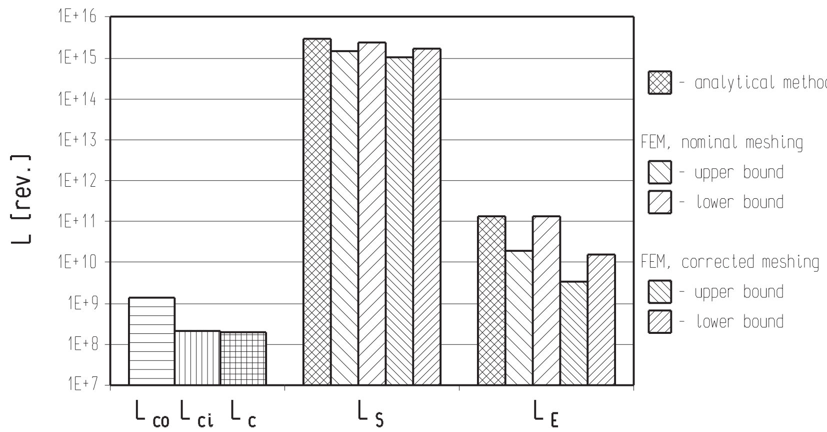

Figure 2 Durability of rolling pairs of Cyclo gear

The life of central cylindrical roller bearing of the tested cycloidal gear is the lowest, as it followed from calculations. In addition, laboratory tests of the gear prototype confirmed this [4]. The life of holes surface in straight-line mechanism is very high, in practice unlimited. Its influence on the life of gear can be neglected. The life of cycloidal meshing is many times higher than the central bearing life. Its value depends on type of loads distribution and teeth (nominal or corrected) were taken into considerations. Predicted life is the lowest for the gear with corrected teeth. In that gear the number of teeth under load is lower than maximal number of teeth to be able to be in contact [3] and values of forces acting on teeth of the planetary wheel are higher than in nominal teeth gear. In the future, it is necessary to modify the gear design to increase nominal life of the central roller bearing. In that case teeth life could be comparable with central bearing life. Meshing life, determined basing on loads distributions in analytical method, is unrealistically overestimated (about 7 times) comparing to life calculated by FEM. However, analytical method, enabling determination of loads distributions quickly, can be useful for research the influence of gear geometry parameters on meshing life.

In Fig. 3 there are presented curves of life of single tooth of the planetary wheel L_e as the function of the co-operating wheel roller radius $D_e/2$.

On the plot there are several examples of the gear with the same parameters as the prototype gear (Table 1) but with various values of eccentric, transmission ratio and, as the consequence of that, curtailment coefficient, number of tooth of the planetary wheel $z_s = i$ and number of the co-operating wheel roller $z_k = i + 1$. Continuous line curves correspond loads distributions from analytical method and dashed line curves distributions calculated using FEM for nominal meshing and for position of drive shaft when angle $\gamma_h = 0$. Each curve reaches maximum for certain roller diameter of the co-operating wheel, which can be considered as optimal from the fatigue life point of view. Simultaneously, as it follows from Fig. 3, position of maximum for curves for the gear with specified ratio and eccentric does not depend on the method of determination the loads distributions. Considering that, analyze

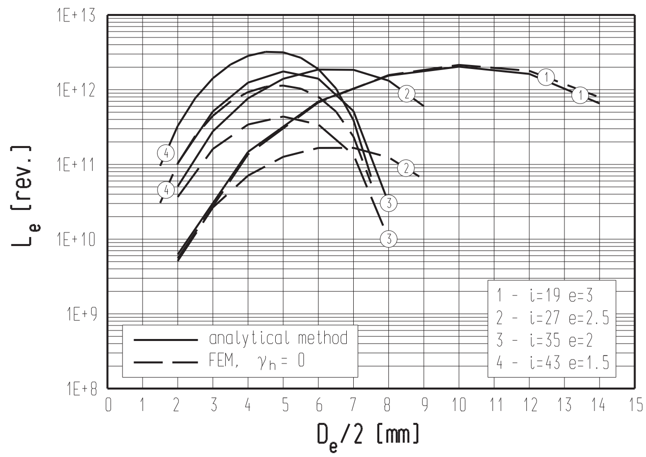

Figure 3 Comparison of fatigue life calculation results of single tooth for load distribution obtained according to analytical and FEM methods

of the influence of the geometry parameters on fatigue life of Cyclo gear presented further in this paper would implement loads distributions determined analytically.

4. Influence of geometry parameters on fatigue life of Cyclo gear

Analyze of the influence of parameters e, m_e and D_e on the fatigue life of rolling pairs in cycloidal gear are presented on the example similar to prototype gear (Table 1) and particularly the same spacing of the rollers of co-operating wheel $r = 96$ mm, but with different transmission ratio, eccentrics and curtailment coefficients (Table 2). It was tested meshing life of the gear with various diameters of the co-operating wheel rollers D_e, that is for the different values of the offset of epicycloid equidistant.

i	Design variant									
	1		2		3		4		5	
	m_e	e[mm]	m_e	e[mm]	m_e	e[mm]	m_e	e[mm]	m_e	e[mm]
19	0.525	2.52	0.575	2.76	0.625	3	0.675	3.24	0.725	3.48
27	0.530	1.82	0.580	1.99	0.630	2.16	0.680	2.33	0.730	2.5
35	0.550	1.47	0.600	1.6	0.650	1.73	0.700	1.87	0.750	2
47	0.550	1.23	0.600	1.34	0.672	1.5	0.700	1.57	0.750	1.68

Table 2 Gear ratio i, eccentric e and curtailment coefficient of epicycloid m_e of tested cycloidal gears

Results of the calculations of fatigue life for single tooth of the planetary wheel (L_e) and for the whole meshing (L_E) of Cyclo gear with ratio $i = 19$ are shown in Fig. 4.

The curves of the life in the plot are presented as the function of radius of co-operating wheel roller $D_e/2$. Similarly, like in Fig. 3, each curve is character-

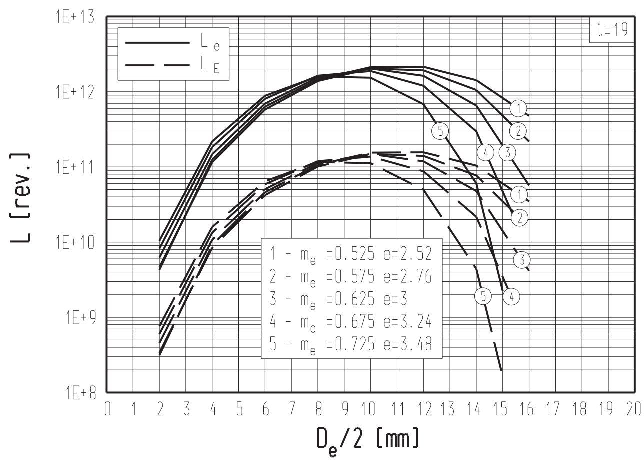

Figure 4 Fatigue life of single tooth L_e of the planetary wheel and the meshing of Cyclo gear L_E in function of radius of the roller of co-operating wheel, $i = 19$

ized by the maximum for certain roller diameter of the co-operating wheel. As is well known, low radius of co-operating wheel roller corresponds high values of the equivalent contact curvature and at the same time higher contact stresses and lower fatigue life of the tooth flank surface [3]. The consequence of higher roller radius is sharpening the teeth of the planetary wheel. It also produces increase of equivalent value of the contact curvature and decreases meshing life. For each couple of the parameters e and m_e it can be found the most advantageous co-operating wheel roller diameter $D_{e\ opt}$ in consideration of fatigue life. Higher values of m_e can also implicate sharpening of teeth therefore optimal value of roller diameter is lower for higher e and m_e values and higher for lower values of these parameters. Similar calculations were made for Cyclo gear with ratio $i = 27$, 35, 43, but this time only for single tooth of the planetary wheel. Life curves for these gears are presented in Figs. 5, 6 and 7.

In cycloidal gear design process it is advantageous to have, at least approximated, choice of selection of optimal co-operating wheel roller diameter without time consuming calculations of the life. Therefore on the base of formula of curtailment coefficient of epicycloid:

$$m_e = \frac{e \cdot z_k}{r} \tag{2}$$

it was inserted notion of curtailment coefficient of epicycloid equidistant:

$$m_{ee} = \frac{e \cdot z_k}{r - \frac{D_e}{2}} \,. \tag{3}$$

For defined eccentric and gear ratio optimal value of co-operating wheel roller diameter corresponds optimal value of the coefficient m_{ee}:

$$m_{ee\ opt} = \frac{e \cdot z_k}{r - \frac{D_{e\ opt}}{2}} \,. \tag{4}$$

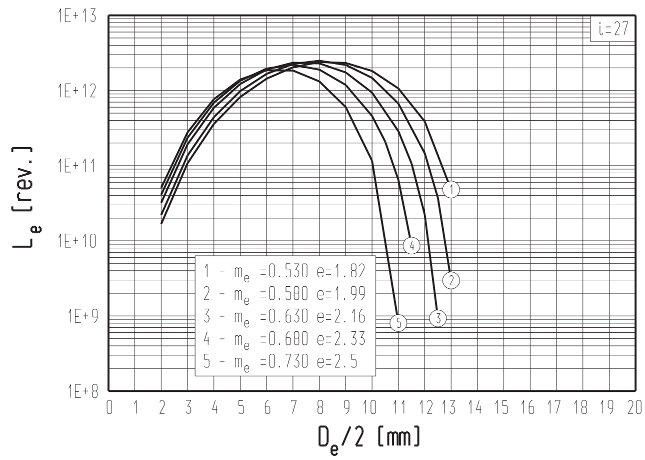

Figure 5 Fatigue life of single tooth L_e of the planetary wheel in function of radius of the roller of co-operating wheel, $i = 27$

Values of $D_{e\ opt}$ and $m_{ee\ opt}$ for various gear ratios and coefficients m_e are shown in Table 3. Fig. 8 shows the illustration of the relationship $m_{ee\ opt}$ against m_e.

$i = 19$			$i = 27$			$i = 35$			$i = 43$		
m_e	m_{eeopt}	D_{eopt} [mm]	m_e	m_{eeopt}	D_{eopt} [mm]	m_e	m_{eeopt}	D_{eopt} [mm]	m_e	m_{eeopt}	D_{eopt} [mm]
0.525	0.593	22.13	0.53	0.582	16.73	0.55	0.591	12.95	0.55	0.598	10.97
0.575	0.648	20.51	0.58	0.633	15.91	0.6	0.641	12.23	0.6	0.649	10.34
0.625	0.699	20.29	0.63	0.683	14.83	0.65	0.69	11.49	0.672	0.723	9.39
0.675	0.749	19.06	0.68	0.733	13.92	0.7	0.743	10.70	0.7	0.755	9.01
0.725	0.799	17.78	0.73	0.782	12.95	0.75	0.792	10.26	0.75	0.805	8.34

Table 3 Optimal diameter of the cooperation wheel roller $D_{e\ opt}$ and optimal value of the curtailment coefficient of epicycloid equidistant $m_{ee\ opt}$

As it can be seen, for given gear ratio that relationship is linear type. Simultaneously, it can be made average of the obtained results by plotting trend line describing the relation $m_{ee\ opt}$ versus m_e for any value of the ratio. From the equation of trend line:

$$m_{ee\ opt} = m_e + 0.056\,, \tag{5}$$

and from the equation (4) it can be calculated approximated value of the most advantageous co-operating wheel roller diameter for given transmission ratio and eccentric of the gear.

Low values of the eccentric accompany load increase of the central bearing and the gear meshing. In Table 4 there are presented values of the resulting force acting on drive shaft bearing for tested construction variants of the Cyclo gear with ratio $i = 19$. In Fig. 9 there is presented relation between central roller bearing life L_c and meshing life L_E for different values of the gear eccentric [8].

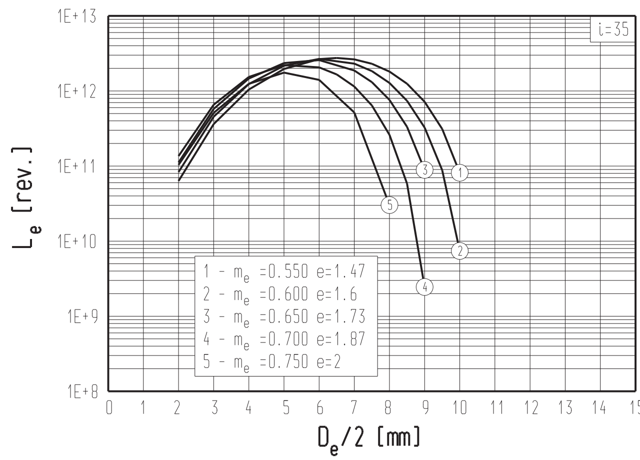

Figure 6 Fatigue life of single tooth L_e of the planetary wheel in function of radius of the roller of co-operating wheel, $i = 35$

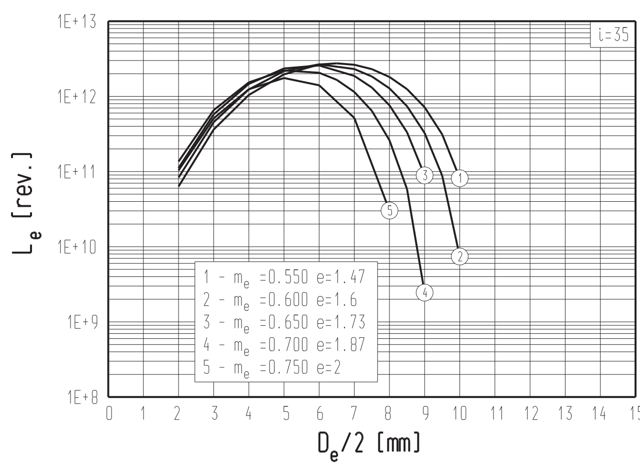

Figure 7 Fatigue life of single tooth L_e of the planetary wheel in function of radius of the roller of co-operating wheel, $i = 43$

e[mm]	2.52	2.76	3.00	3.24	3.48
m_e	0.525	0.575	0.625	0.675	0.725
R[N]	11541	10870	10314	9832	9421

Table 4 Resulting force R acting on drive shaft bearing; $i = 19$

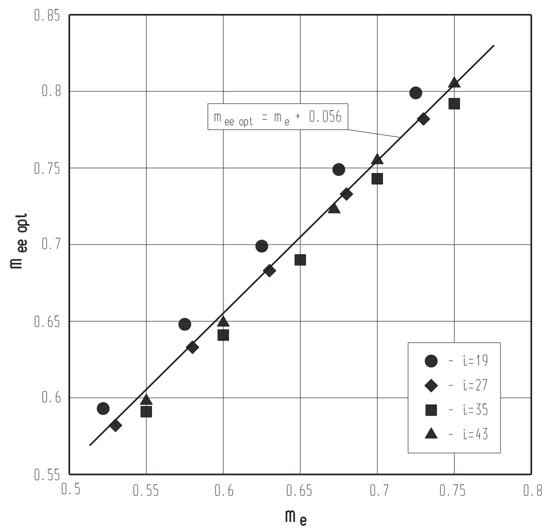

Figure 8 Optimal value of the curtailment coefficient of epicycloid equidistant $m_{ee\ opt}$ versus curtailment coefficient of epicycloid m_e

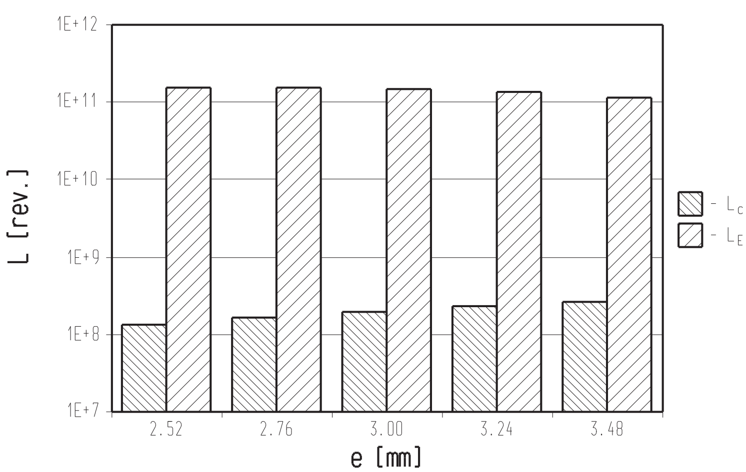

Figure 9 Fatigue life of the central roller bearing L_c and the meshing of the Cyclo gear L_E for different values of e and the most advantageous values of $D_e = D_{e\ opt}$

As it can be seen in Fig. 9, meshing life for most advantageous value D_e increases slightly together with decrease of e and m_e values. At the same time the central bearing life decreases on a little higher level. Regarding the life of entire gear more advantageous are higher values of the eccentric.

5. Conclusions

Results of the fatigue life of rolling pairs in the prototype example of the cycloidal gear presented in the paper allowed for estimation of the life of individual pairs and evaluation of their construction. Calculations revealed the weakest pair of the gear is bearing of the drive shaft. That requires modification.

Important meaning for the gear life has cycloidal meshing. In case of wrong selection of the parameters of the equidistant of curtate epicycloid profiling the planetary wheel teeth, meshing life could be lower than central bearing life. It can determine entire gear life (Figs. 2, 4).

Life of the holes surfaces co-operating with the bolts of straight-line mechanism is in practice unlimited. Its influence on Cyclo gear life can be neglected.

Meshing life based on analytically calculated loads distributions is unreal and overestimated compared to the life calculated by FEM. Analytical method can be implemented on design stage of the gear to define the most advantageous parameters of cycloidal meshing.

For each pair of the parameters: eccentric e and curtailment coefficient m_e, it can be defined the most advantageous value of co-operating wheel diameter $D_{e\ opt}$ regarding fatigue life of meshing. Values of the parameters e, m_e, D_e fixed in design process explicitly determine cycloidal gear size.

Higher values of parameter m_e require lower values of the roller diameter D_e. It minimizes level of sharpening the teeth and at the same time increases fatigue life of meshing.

For applied in practice range of values of $m_e = 0.5 - 0.85$ and constant gear size ($r \cong$ const) predicted meshing fatigue life and central bearing fatigue life in the planetary wheels are practically constant. Central cylindrical roller bearing determines the gear life and regarding that, decreasing gear dimensions by sacrificing increase of the eccentric value would not be recommended.

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