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Active control of plates using functionally graded piezocomposite layers

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The paper is aimed at active suppression of flexural vibration of thin rectangular symmetrically laminated plates by means of piezoelectric sensor and actuator layers, which operate in a constant-gain velocity feedback. A new class of the actuator layer made of piezocomposite functionally graded material (FGM) and equipped with interdigitated surface electrodes (IDEs) is applied. A change of electromechanical properties through the actuator layer thickness is obtained by stacking laminae with the piezoceramic (PZT) fibers distributed according to the assumed variation of the PZT volume fraction. The effective material parameters are derived and partial differential equation of the active plate motion is formulated based on the classical laminated plate theory. The dynamic analysis refers to the steady-state vibration of the globally orthotropic plate. The results of numerical simulation show the distributions of both the effective piezoelectric constants and stiffness parameters in the thickness direction of the actuator layer, and also the effect of the applied functionally graded actuator on the plate dynamic response depending on the PZT fiber configuration.

Keywords: Active laminated plates, multi-layer actuator, functionally graded piezocomposite

1. Introduction

Active composites with embedded piezoceramic PZT (lead-zirconate-titanate) fibers have found a relevant role in vibration control of advanced elastic plate structures. The PZT fibers aligned in a polymer matrix increase the resistance to damage comparing with monolithic piezoelectric material and create an anisotropic behaviour with a sufficiently high electromechanical efficiency. The performance of traditionally electroded piezoelectric fiber composites (PFCs) can be improved by applying axially polarized fibers and the interdigitated electrode (IDE) system, which provides electric field in the fiber direction and involves the dominant longitudinal actuation effect, cf [1, 2, 3 and 4]. The interaction between the active layers and the main structure creates (for the high driving voltage) an imperfection of the edge delamination due to the interfacial stress concentration. One of the ways of reducing the damage danger is to use piezocomposite layers with varied electromechanical properties through their thickness [5]. A newly proposed piezoelectric actuators made of functionally graded material (FGM) can produce large strains while minimizing the tangential stress concentration and improve the reliability and lifetime of the actuators. The idea of FGMs, which are commonly made from a mixture of ceramic and metal, has been applied as a thermal barrier to reduce the high thermal stress field at the interface between ceramic and metal, and prevent reduction of strength and stiffness of the structure. Active control of FGM plate with monolithic piezoelectric sensor/actuator layers is discussed in [6]. The dynamic stability analysis of FGM plate under the time-dependent thermal load is presented in [7]. The transverse displacements and stress field in FGM piezoelectric laminates is analysed in [8].

The concept of FGM can be applied to active laminated plates by stacking composite layers of different piezoelectric fillers and a matrix. In the presented paper the multi-layered actuator being a set of integrated unidirectional PFC laminae with PZT fibers distributed according to the rectangular packing pattern is proposed. The properties modification is obtained by the PZT material volume fraction graded through the total thickness of the actuator layer. The piezocomposite FGM actuators are applied for the active vibration reduction of the rectangular cross-ply laminate. The dynamic analysis is based on the classical laminated plate theory with the Kirchhoff assumptions adapted for active laminates by Lee [9] and Reddy [10], and concerns the steady-state vibration in the case of simply supported boundary conditions.

2. Model of the system

The considered symmetrically laminated plate is designed of classic layers (e.g. graphite-epoxy) and piezoelectric layers, which properties and configuration let us to assume a globally orthotropic behaviour. The control action is produced by the piezocomposite actuator layers and the homogeneous PVDF (polyvinylidene fluoride) sensor layers due to the constant-gain velocity feedback. Naturally, to generate the control bending load, the midplane symmetric actuators are both FGM with either opposite polarization or opposite applied electric field. For example, the laminate geometry presented in Fig. 1 is considered.

The transverse vibration $w(x,y,t)$ of the active laminate subjected to the external distributed load $q(x,y,t)$ is described as follows

$$D_{11}w_{,xxxx}+2\left(D_{12}+2D_{66}\right)w_{,xxyy}+D_{22}w_{,yyyy}+\rho t_c w_{,tt}=q\left(x,y,t\right)-p\left(x,y,t\right) \quad (1)$$

where D_{ij} $(i,j=1,\ 2,\ 6)$ denote the elements of the stiffness matrix, which are complex for the viscoelastic material, t_c and ρ are the total thickness and equivalent density of the plate, respectively, $p(x,y,t)$ is the loading produced by the control system. Viscoelastic behaviour of the laminate is approximated according to the Voigt-Kelvin model and is based on the elastic-viscoelastic correspondence principle to predict complex moduli for fiber-reinforced composites. In general case the complex modulus can be written in the form

$$\tilde{Y}=Y\left(1+j\mu\omega\right) \quad (2)$$

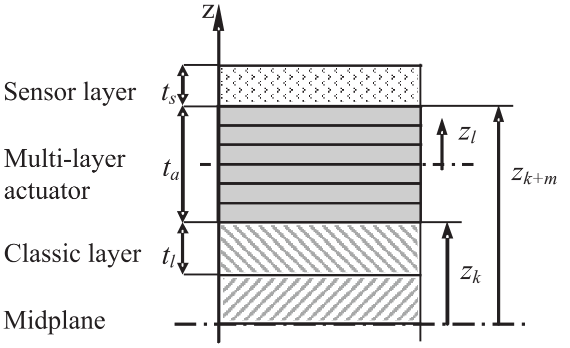

Figure 1 The laminate cross-section

where Y is the elastic modulus, μ denotes the retardation time parameter.

The viscoelastic properties of classic layers and active layers (piezocomposite and monolithic) are considered separately and the complex stiffness matrices for each layer are derived.

The solution to the governing equation Eq.(1) for the steady-state case is obtained in terms of the frequency response function assuming simply supported boundary conditions.

2.1. *Sensor relations*

The sensor equation is formulated due to the constitutive low describing the direct piezoelectric effect, which for the transversally polarized layer with the material axes 1, 2, 3 parallel to the plate reference axes x, y, z, respectively, after eliminating the external electric field has the form

$$D_3 = \mathbf{e}^T \varepsilon \tag{3}$$

where D_3 is the electric displacement in the 3-axis direction, ε represents the strain, $\varepsilon = [\varepsilon_x, \varepsilon_y, \gamma_{xy}]^T$, $\mathbf{e}$ is the piezoelectric coefficient matrix, $\mathbf{e} = [e_{31}\ e_{32}\ 0]^T$, the superscript T denotes matrix transposition.

The voltage $V_s(t)$ generated by the sensor layer in-plane deformation is obtained after integrating the charge stored on the surface electrodes. Using the standard equation for capacitance, and the geometric relation between strain and transverse displacement leads to the following formula (more details one can find in [2])

$$V_s = -\frac{t_s z_0^s}{\in_{33} A_s} \mathbf{e}^T \int_0^a \int_0^b [w_{,xx}, w_{,yy}, 2w_{,xy}]^T \lambda_s(x,y)\, dx\, dy \tag{4}$$

where a and b are the plate in-plane dimensions, $\lambda_s(x,y)$ and A_s are the sensor polarization pattern and effective electrode area, respectively, t_s denotes the sensor layer thickness, $\in_{33}$ is the permittivity constant, z_0^s is the distance of the sensor from the laminate midplane.

2.2. *Actuator relations*

The considered multi-layered actuator consists of the unidirectional PFC laminae with PZT fibers distributed according to the rectangular packing pattern and polarized longitudinally. To produce the electric field along the fibers the interdigitated electrode designed with finger like sections of alternating polarity is implemented. The modification of the electromechanical properties is obtained by the PZT material volume fraction (number of PZT fibers), which is constant for each lamina and graded through the total thickness t_a of the actuator. As an example of the FGM-type actuator the set of distinct PFC laminae with the outer ply equipped with the interdigitated electrode is shown in Fig. 2. In this figure the axes of material properties are also indicated. It should be pointed that for the considered actuator layer the material axes 3, 1 and 2 are parallel to the plate axes x, y and z, respectively. The uniform field method based on the rule of mixtures is applied to obtain the effective properties of the two-phases composite material of each PFC lamina. Due to the method the representative volume element is separated and divided into sub-elements. They represent the uniform field cases between the electrode sections, where an approximately uniform electric field in the fiber direction exists, and the electrical properties near the electrode, which are sensitive to the distance between the electrode and the fiber. The method requires recognizing the independent field variables, which are equal in the both material phases (piezoceramic and polymeric), and combining the dependant variables according to the fraction of PZT phase. The equivalent stiffness matrix and the effective piezoelectric coefficient matrix are formulated after returning to the original form of the constitutive equations formulated in the normal three-dimensional mode. The details and explanations of modelling of the IDEPFCs one can find in [1, 2, 3 and 4].

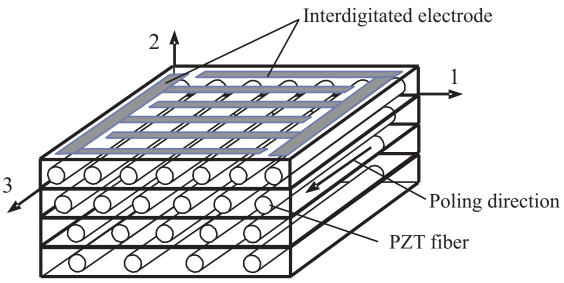

Figure 2 Configuration of the FGM multi-layered actuator with the interdigitated electrode

The constitutive equations of the reverse piezoelectric effect reduced to the actuation in the 3-1 (or $x - y$) plane for each PFC ply can be written in the form

$$\sigma = \mathbf{c}^{ef}\,\varepsilon - \mathbf{e}^{ef}E_3 \tag{5}$$

where σ is the stress representation, $\sigma = [\sigma_x, \sigma_y, \tau_{xy}]^T$, $\mathbf{c}^{ef}$ is the effective stiffness matrix, $\mathbf{e}^{ef}$is the effective piezoelectric constant matrix, $\mathbf{e}^{ef} = \left[e_{33}^{ef},\ e_{31}^{ef},\ 0\right]^T$, E_3 is the electric field in the 3-axis direction.

The effective properties represented by both the mechanical and electric constants depend on the PZT volume fraction. Besides, for the considered multi-layered actuator the effective piezoelectric constants strongly depend on the electrode surroundings, i.e. the electric parameters of the matrix material, and also the distance between the electrode and the PZT fiber. When the rectangular packing pattern is applied, the linear fractions of piezoceramic material v_1^p and v_2^p measured in the 1-axis and 2-axis directions, respectively, differ proportionally to the distance between fibers. The linear volume fraction component v_3^p, which represents the matrix separating the electrode and the fiber, can be expressed as the ratio of the electrode spacing (equal to the length of the fiber between the electrode sections) to the total electric path length between the electrode sections. The path length differs for the particular laminae and increases for the lamina located further from the electrode. For the one-side electroded actuator composed of the laminae of the same thickness t_a^*, the path length can be approximated as

$$l_p^{(i)} = s + (2i-1)\ \left(1 - v_2^p\right)\ t_a^* \tag{6}$$

where s is the electrode spacing, i denotes the lamina number in the sequence from the electrode to the opposite face, $i = 1,\ 2,\ldots,\ m$, and m is the total number of laminae in the FGM multi-layered actuator.

The control moment resultant $\mathbf{M}$ for the FGM actuator composed of m laminae, can be written in the form

$$\mathbf{M} = \left[M_x,\ M_y,\ M_{xy}\right]^T = \lambda\,(x,y) \sum_{i=1}^{m} \left(\mathbf{e}^{ef}\right)^{(i)}\ t_a^* z_0^{(i)} E_3^{(i)} \tag{7}$$

where $\lambda(x,y)$ is the pattern function of the activated area, $z_0^{(i)}$ is the distance of the i-th component lamina from the plate midplane, $\left(\mathbf{e}^{ef}\right)^{(i)}$ denotes the effective piezoelectric constant matrix of the i-th lamina, which for the assumed orientation refers to the x, y plate axes.

The portion of the electric field $E_3^{(i)}$ supplying the i-th component lamina can be expressed in relation to the actuator voltage $V_a(t)$ due to the electric field-voltage formula $E_3^{(i)} = V_a(t)\Big/ l_p^{(i)}$ and becomes smaller with increasing the electric field path $l_p^{(i)}$, e.g. for the laminae located farther from the electrode. The voltage V_a driving the actuator is induced by the PVDF sensor and transformed due to the velocity feedback control.

Finally, the control loading $p(x,y,t)$ produced by two actuator layers located symmetrically about the middle of the plate is

$$p(x,y,t) = 2\left(M_{x,xx} + M_{y,yy} + 2M_{xy,xy}\right) \tag{8}$$

In the considered case, a zero skew angle between the plate axes and the piezocomposite material axes occurs, hence, the twisting moment component M_{xy} vanishes and the two-dimensional bending actuation is realised.

3. Results of simulation

Calculations are carried out for a simply supported laminated plate of dimensions 400×400×2 mm. The plate consists of classic graphite-epoxy layers of thickness

t_l = 0.15mm, PVDF sensors of thickness t_s = 0.1mm, and FGM actuator layers of thickness t_a = 0.6mm. The layers are stacked according to the following symmetric sequence $[S/\mathrm{A}/0°/90°]_s$, where the symbols "S" and "A" indicate the sensor and actuator, respectively. The FGM actuator is composed of six distinct piezocomposite laminae of the same thickness and having constant effective properties throughout the thickness. The amount of PZT fibers increases for the lamina located at a greater distance from the laminate midplane and near the actuator face covered by the IDE electrode. The stiffness parameters of the graphite-epoxy composite are assumed as: Y_{11} = 150 GPa, Y_{22} = 9 GPa, G_{12} = 7.1 GPa, and the equivalent mass density is equal ρ = 1600kg/m^3. The properties of PVDF material are following: Y = 2 GPa, ρ = 1780 kg/m^3 and piezoelectric constants d_{31}= 2.3·10^{-11} mV^{-1}, d_{32}= 3·10^{-12} mV^{-1}.
The electromechanical properties of piezocomposite components listed in Table 1 are given in [1].

Table 1 Properties of PFC components

Parameter	ρ kgm^{-3}	c_{11} GPa	c_{12} GPa	c_{13} GPa	c_{33} GPa	G GPa	e_{31} Cm^{-2}	e_{33} Cm^{-2}	$\in_{33}$ / $\in_0$
PZT-5H	7650	127	80.2	84.7	117	36.3	-4.42	15.5	1392
Matrix	1200	8.15	4.01	4.01	8.15	2.33	0	0	11.2

Internal damping is modelled due to the Voigt-Kelvin description with the following parameters: $\mu_1 = 10^{-6}$ s, $\mu_2 = \mu_{12} = 4{\cdot}10^{-6}$ s for orthotropic graphite-epoxy layers, $\mu_p = 2{\cdot}10^{-6}$ s for mechanically isotropic PVDF layers and $\mu_m = 8{\cdot}10^{-6}$ s for matrix material in piezocomposite layers. Energy dissipation in ceramic PZT fibers is ignored. For external excitation the uniformly distributed harmonic load of the amplitude q_0 = 1 Nm^{-2} is applied. Results presented in terms of amplitude-frequency characteristics are calculated at the reference point $x = y$ = 100 mm. The goal of simulation is to recognize the influence of the PZT fiber arrangement within the actuator layer on its effective electromechanical properties and the dynamic behaviour of the active laminate in the aspect of structural vibration control. Two actuator layers with a different distribution of the PZT fiber volume fraction ($v_f^p = v_1^p v_2^p$) through the thickness are considered. In the first actuator the PZT fraction decreases from $v_f^p = 0.64$ in the outer lamina to $v_f^p = 0.24$ in the inner one with a constant volume gain $\Delta v_f^p = 0.08$ from lamina to lamina. The second actuator is designed assuming a relatively high average value of the PZT fraction changing smoothly toward the middle of the plate from $v_f^p = 0.64$ to $v_f^p = 0.56$ with the five times smaller gain $\Delta v_f^p = 0.016$. For the both cases the rectangular packing scheme with the constant PZT volume fraction in the thickness direction $v_2^p = 0.8$ is applied. Hence, the distance between fibers modifies the amount of PZT material in the distinct lamina.

Figures 3 and 4 show the in-plane effective stiffness parameters and the longitudinal piezoelectric constant of the FGM actuator, respectively, versus the local coordinate z_l measured from the midplane of the actuator layer (see Fig. 1). Markers on the plots indicate the values relating to the particular component lamina. The

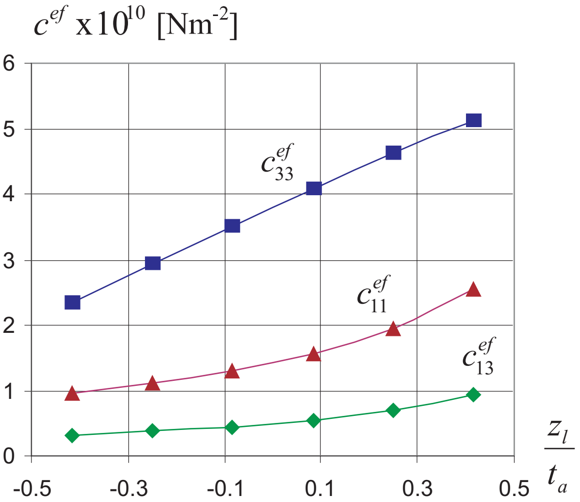

Figure 3 Changes of the effective stiffness parameters through the actuator thickness, $\Delta v_f^p = 0.08$

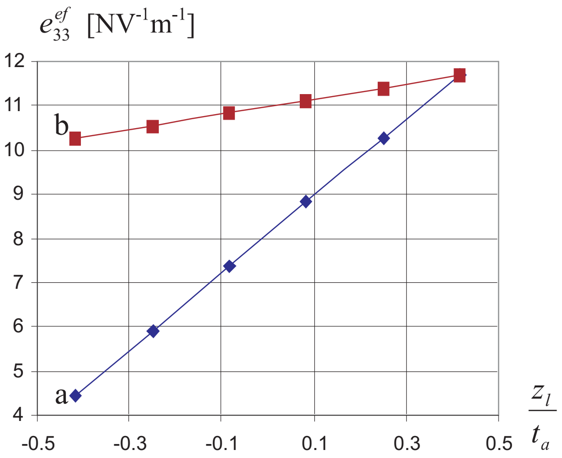

Figure 4 Changes of the effective piezoelectric constant through the actuator thickness,
a – $\Delta v_f^p = 0.08$, b – $\Delta v_f^p = 0.016$

effective stiffness c_{33}^{ef} in the fiber direction strongly depends on the PZT volume fraction comparing with the other stiffness parameters (c_{11}^{ef} and c_{13}^{ef}). The effective piezoelectric constant e_{33}^{ef} changes almost linearly within the actuator layer. Naturally, a greater gain of the PZT volume fraction increases the range of the piezoelectric coefficient value significantly (compare the plots a and b in Fig. 4).

Results of the active vibration reduction are compared in Fig. 5. The frequency response functions are calculated assuming the greater ($\Delta v_f^p = 0.08$) and smaller ($\Delta v_f^p = 0.016$) changes of the PZT volume fraction through the layer thickness

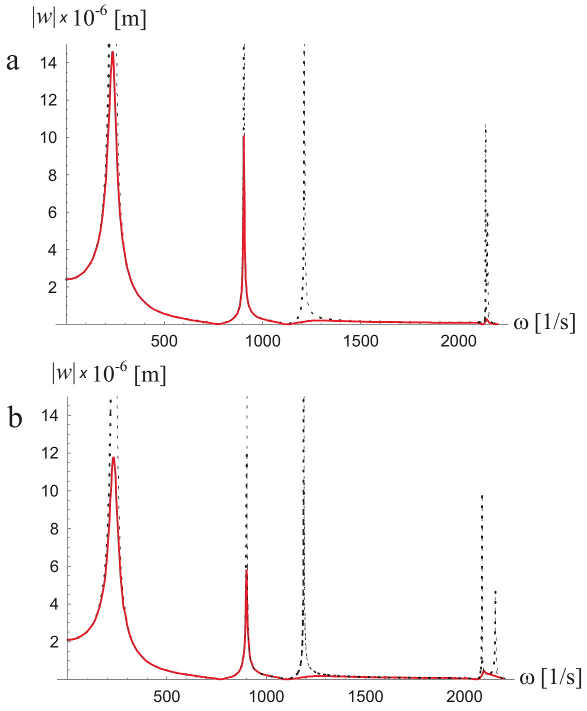

Figure 5 Active vibration reduction for the laminate with FGM actuators. Effect of the PZT volume fraction distribution, a – $\Delta v_f^p = 0.08$, b – $\Delta v_f^p = 0.016$. Dotted line refers to the uncontrolled plate

starting with the same value $v_f^p = 0.64$. The uncontrolled responses relating to the considered composite configurations are drawn using dotted lines. Because of a great average amount of PZT fibers the active damping effect obtained for the volume fraction gain $\Delta v_f^p = 0.016$ (Fig. 5b) is stronger comparing with the effect relating to the PZT volume distribution of the gain $\Delta v_f^p = 0.08$ (Fig. 5a). The laminate configuration with the PZT fibers aligned in the x-axis direction results in an extreme reduction of the 3-1 vibration mode. The amplitudes of the higher modes 3-3 and 1-5 are strongly reduced due to both the active damping and the passive form of energy dissipation relating to the applied material damping model.

4. Final remarks

The model of multi-layered actuator with functionally graded mechanical and piezoelectric properties has been formulated and applied for transverse vibration control of laminated plates. The concept of the FGM actuator is based on the piezoelectric fiber composite with a gradation of the number of PZT fibers realized during the manufacturing process. The variation of the piezoelectric material fraction can be adapted to the desired change of electromechanical properties in the thickness direction. The results of simulation show that this type of actuator layers equipped with the interdigitated electrodes offer a satisfying effectiveness of the control system. Therefore, they can be use to minimize the hazard of damage by the edge delamination which is caused by the interface stress concentration characteristic for traditional actuators.

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