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Unsteady MHD Flow in a Circular Pipe of a Dusty Non-Newtonian Fluid

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In this paper, the unsteady magnetohydrodynamic flow of a dusty viscous incompressible electrically conducting non-Newtonian Casson fluid through a circular pipe is investigated. A constant pressure gradient in the axial direction and a uniform magnetic field directed perpendicular to the flow direction are applied. The particle-phase is assumed to behave as a viscous fluid. A numerical solution is obtained for the governing nonlinear momentum equations using finite differences. The effect of the magnetic field, the non-Newtonian fluid characteristics, and the particle-phase viscosity on the transient behavior of the velocity, volumetric flow rates, and skin friction coefficients of both fluid and particle-phases are studied. It is found that all the flow parameters for both phases decrease as the magnetic field increases or the flow index decreases. On the other hand, increasing the particle-phase viscosity increases the skin friction of the particle phase, but decreases the other flow parameters.

Keywords: non-Newtonian fluid, magnetohydrodynamic flow

1. Introduction

The flow of a dusty and electrically conducting fluid through a pipe in the presence of a transverse magnetic field has important applications such as magnetohydrodynamic generators, pumps, accelerators, and flowmeters. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. When the particle concentration becomes high, mutual particle interaction leads to higher particle-phase viscous stresses and can be accounted for by endowing the particle phase by the so-called particle-phase viscosity. There have been many articles dealing with theoretical modelling and experimental measurements of the particle-phase viscosity in a dusty fluid [1-4].

The flow of a conducting fluid in a circular pipe has been investigated by many authors [5-8]. Gadiraju et al. [5] investigated steady two-phase vertical flow in a pipe. Dube and Sharma [6] and Ritter and Peddieson [7] reported solutions for unsteady dusty-gas flow in a circular pipe in the absence of a magnetic field and particle-phase viscous stresses. Chamkha [8] obtained exact solutions which generalize the results reported in [6,7] by the inclusion of the magnetic and particle-phase viscous effects.

A number of industrially important fluids such as muolton plastics, polymers, pulps and foods exhibit non-Newtonian fluid behavior [9]. Due to the growing use of these non-Newtonian materials, in various manufacturing and processing industries, considerable efforts have been directed towards understanding their flow characteristics. Many of the inelastic non-Newtonian fluids, encountered in chemical engineering processes, are fluids exhibiting a yield stress that has to be exceeded before the fluid moves [10]. It is of interest in this paper to study the influence of the magnetic field as well as the non-Newtonian fluid characteristics on the dusty fluid flow properties in situations where the particle-phase is considered dense enough to include the particulate viscous stresses.

In the present study, the unsteady flow of a dusty non-Newtonian Casson fluid through a circular pipe is investigated in the presence of a uniform magnetic field. The carrier fluid is assumed viscous, incompressible and electrically conducting. The particle phase is assumed to be incompressible pressureless and electrically non-conducting. The flow in the pipe starts from rest through the application of a constant axial pressure gradient. The governing nonlinear momentum equations for both the fluid and particle-phases are solved numerically using the finite difference approximations. The effect of the magnetic field, the non-Newtonian fluid characteristics and the particle-phase viscosity on the velocity of the fluid and particle-phases are reported.

2. Governing Equations

Consider unsteady, laminar, axisymmetric horizontal flow of a dusty conducting non-Newtonian fluid through an infinitely long pipe of radius "d" driven by a constant pressure gradient. A uniform magnetic field is applied perpendicular to the flow direction. The magnetic Reynolds number is assumed to be very small and consequently the induced magnetic field is neglected [11]. We assume that both phases behave as viscous fluids [8]. In addition, assume that the volume fraction of suspended particles is finite and constant. Taking into account these and the previously mentioned assumptions, the governing momentum equations can be written as

$$\rho\frac{\partial V}{\partial t} = -\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial V}{\partial r}\right) + \frac{\rho_p\phi}{1-\phi}N(V_p - V) - \sigma B_o^2 V \tag{1}$$

$$\rho_p\frac{\partial V_p}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\mu_p r\frac{\partial V_p}{\partial r}\right) + \rho_p N(V - V_p) \tag{2}$$

where t is the time, r is the distance in the radial direction, V is the fluid-phase velocity, V_p is the particle-phase velocity, ρ is the fluid-phase density, ρ_p is the particle-phase density, $\partial P/\partial z$ is the fluid pressure gradient, ϕ is the particle-phase volume fraction, N is a momentum transfer coefficient (the reciprocal of the relaxation time, the time needed for the relative velocity between the phases to reduce e^{-1} of its original value (Chamkha [8]), σ is the fluid electrical conductivity, B_o is the magnetic induction, μ_p is the particle-phase viscosity which is assumed constant, and μ is the apparent viscosity of the fluid which is given by,

$$\mu = \left(K_c + \sqrt{\frac{\tau_o}{\left|\frac{\partial V}{\partial r}\right|}} \right)^2$$

where K_c is the coefficient of viscosity of a Casson fluid, τ_o is the yield stress, and $|\partial V/\partial r|$ is the magnitude of the velocity gradient which is always positive regardless of the sign of $\partial V/\partial r$. In this work, ρ, ρ_p, μ_p, ϕ and B_o are all constant. It should be pointed out that the particle-phase pressure is assumed negligible and that the particles are being dragged along with the fluid-phase.

The initial and boundary conditions of the problem are given as

$$V(r,0) = 0, \quad V_p(r,0) = 0 \tag{3}$$

$$\frac{\partial V(0,t)}{\partial r} = 0, \quad \frac{\partial V_p(0,t)}{\partial r} = 0, \quad V(d,t) = 0, \quad V_p(d,t) = 0 \tag{4}$$

where "d" is the pipe radius.

Equations (1)-(4) constitute a nonlinear initial-value problem which can be made dimensionless by introducing the following dimensionless variables and parameters

$$\bar{r} = \frac{r}{d}, \quad \bar{t} = \frac{tK_c}{\rho d^2}, \quad G_o = -\frac{\partial P}{\partial z}, \quad k = \frac{\rho_p \phi}{\rho(1-\phi)}, \quad \bar{\mu} = \frac{\mu}{K_c}$$

$$\bar{V}(r,t) = \frac{K_c V(r,t)}{G_o d^2}, \quad \bar{V}_p(r,t) = \frac{K_c V_p(r,t)}{G_o d^2}$$

$\alpha = Nd^2\rho/K_c$ is the inverse Stoke's number,
$B = \mu_p/K_c$ is the viscosity ratio,
$\tau_D = \tau_o/G_o d$ is the Casson number (dimensionless yield stress),
$H_a = B_o d\sqrt{\sigma/K_c}$ is the Hartmann number (Sutton [11]).

By introducing the above dimensionless variables and parameters as well as the expression of the fluid viscosity defined above, Eqs. (1)-(4) can be written as (the bars are dropped),

$$\frac{\partial V}{\partial t} = 1 + \sqrt{\mu}\frac{\partial^2 V}{\partial r^2} + \frac{\mu}{r}\frac{\partial V}{\partial r} + k\alpha(V_p - V) - H_a^2 V \tag{5}$$

$$\frac{\partial V_p}{\partial t} = B\left(\frac{\partial^2 V_p}{\partial r^2} + \frac{1}{r}\frac{\partial V_p}{\partial r}\right) + \alpha(V - V_p) \tag{6}$$

$$\mu = \left(1 + \sqrt{\frac{\tau_D}{\left|\frac{\partial V}{\partial r}\right|}} \right)^2$$

$$V(r,0) = 0, \quad V_p(r,0) = 0, \tag{7}$$

$$\frac{\partial V(0,t)}{\partial r}=0,\quad \frac{\partial V_p(0,t)}{\partial r}=0,\quad V(1,t)=0,\quad V_p(1,t)=0 \tag{8}$$

The volumetric flow rates and skin-friction coefficients for both the fluid and particle phases are defined, respectively, as (Chamkha [8])

$$\begin{array}{l} Q=2\int_0^1 \pi r V(r,t)dr,\quad Q_p=2\int_0^1 \pi r V_p(r,t)dr, \\ C=-\frac{\partial V(1,t)}{\partial r},\quad C_p=-k\frac{\partial V_p(1,t)}{\partial r} \end{array} \tag{9}$$

3. Results and Discussion

Equations (5) and (6) represent a coupled system of nonlinear partial differential equations which are solved numerically under the initial and boundary conditions (6), using the finite difference approximations. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank-Nicolson implicit method (Mitchell [12] and Evans [13]) is used at two successive time levels. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tridiagonal system is solved using the generalized Thomas algorithm (Mitchell [12] and Evans [13]). Computations have been made for $\alpha = 1$ and $k = 10$. Grid-independence studies show that the computational domain $0 < t < \infty$ and $0 < r < 1$ can be divided into intervals with step sizes $\Delta t = 0.0001$ and $\Delta r = 0.005$ for time and space respectively. It should be mentioned that the results obtained herein reduce to those reported by Dube and Sharma [6] and Chamkha [7] for the cases of non-magnetic, inviscid particle-phase, and Newtonian fluid. These comparisons lend confidence in the accuracy and correctness of the solutions.

Figures 1 and 2 present the time evolution of the profiles of the velocity of the fluid V and dust particles V_p, respectively, for various values of τ_D and for $Ha = 0.5$ and $B = 0.5$. Both V and V_p increase with time and V reaches the steady-state faster than V_p for all values of τ_D. It is clear also from Fig. 1 that increasing τ_D decreases both V and V_p while its effect on the steady-state time can be neglected.

Figures 3, 4, 5, and 6 present the influence of the magnetic field parameter Ha on the transient behavior of the fluid-phase volumetric flow rate Q, the particle-phase volumetric flow rate Q_p, the fluid-phase skin friction coefficient C, and the particle-phase skin friction coefficient C_p for various values of τ_D and for $B = 0.5$. Initially, both phases are at rest, and suddenly; they are set to motion through the application of a constant pressure gradient. As a result, the shear stress at the surface of the pipe increases. This explains the obvious increases in Q, Q_p, C, and C_p shown in Figs. 3 through 6, respectively, for all values of the parameter τ_D. These parameters continue to increase until the flow stabilizes and steady-state conditions are attained. The influence of the magnetic field, as shown in Figs. 3 through 6, is to retard the flow of both phases causing their average velocities and wall shear stresses in the pipe as well as their steady-state times to decrease. Also, it can be concluded from the figures that increasing τ_D decreases greatly the parameters Q, Q_p, C, and C_p but slightly increases their steady-state time.

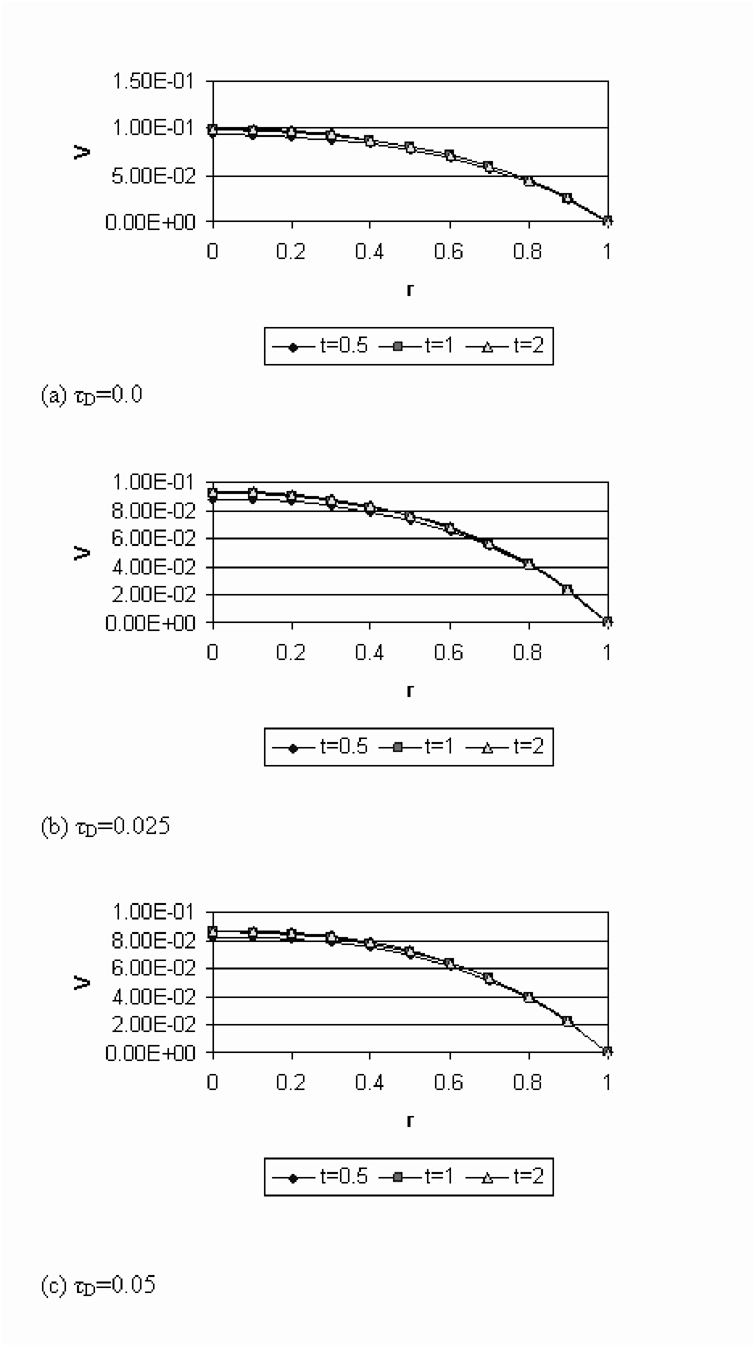


Figure 1 Time development of V for various values of τ_D

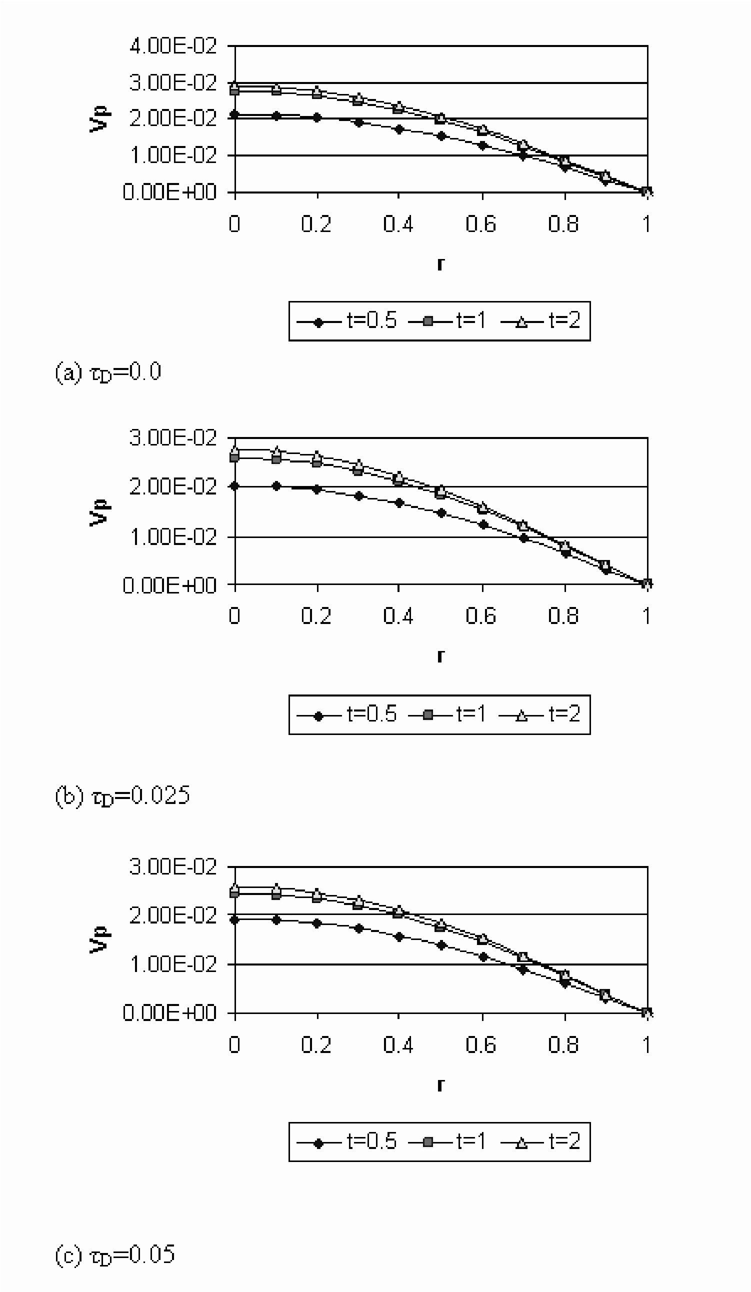


Figure 2 Time development of V_p for various values of τ_D

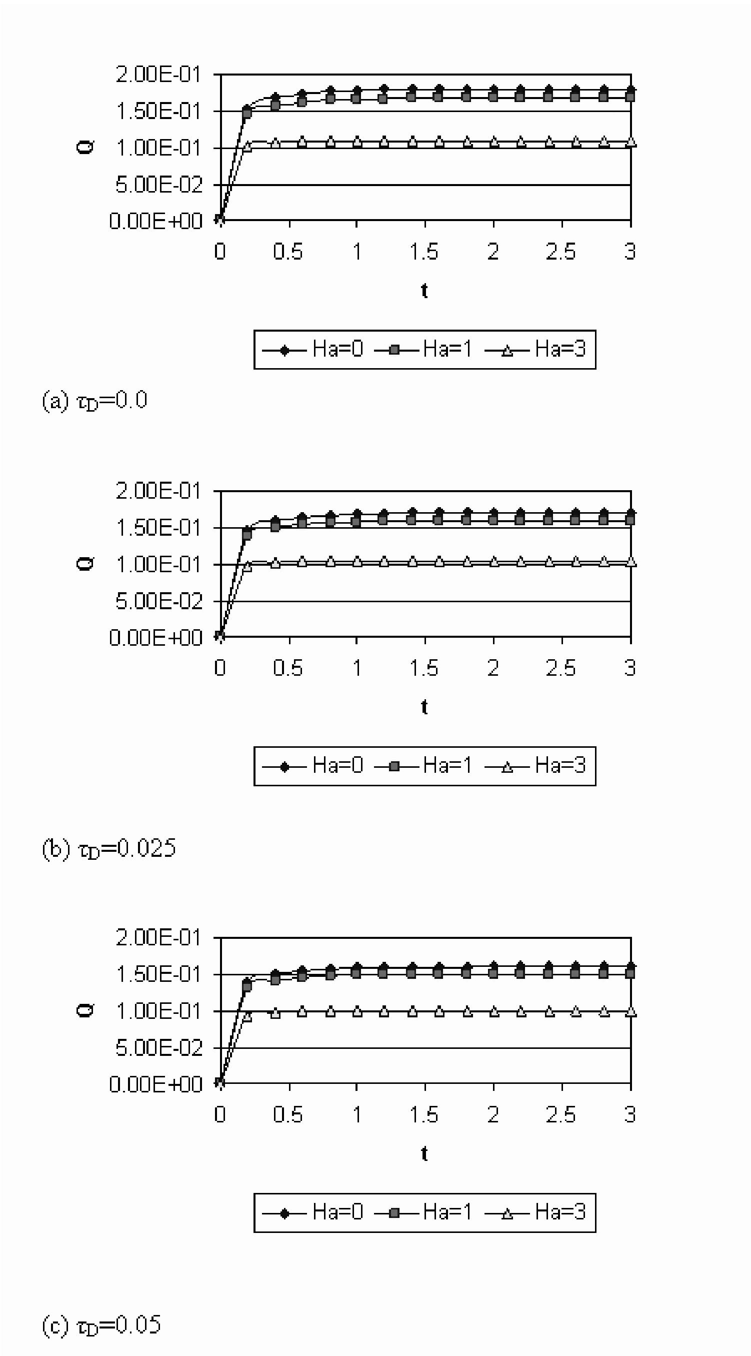


Figure 3 Time development of Q for various values of τ_D and Ha

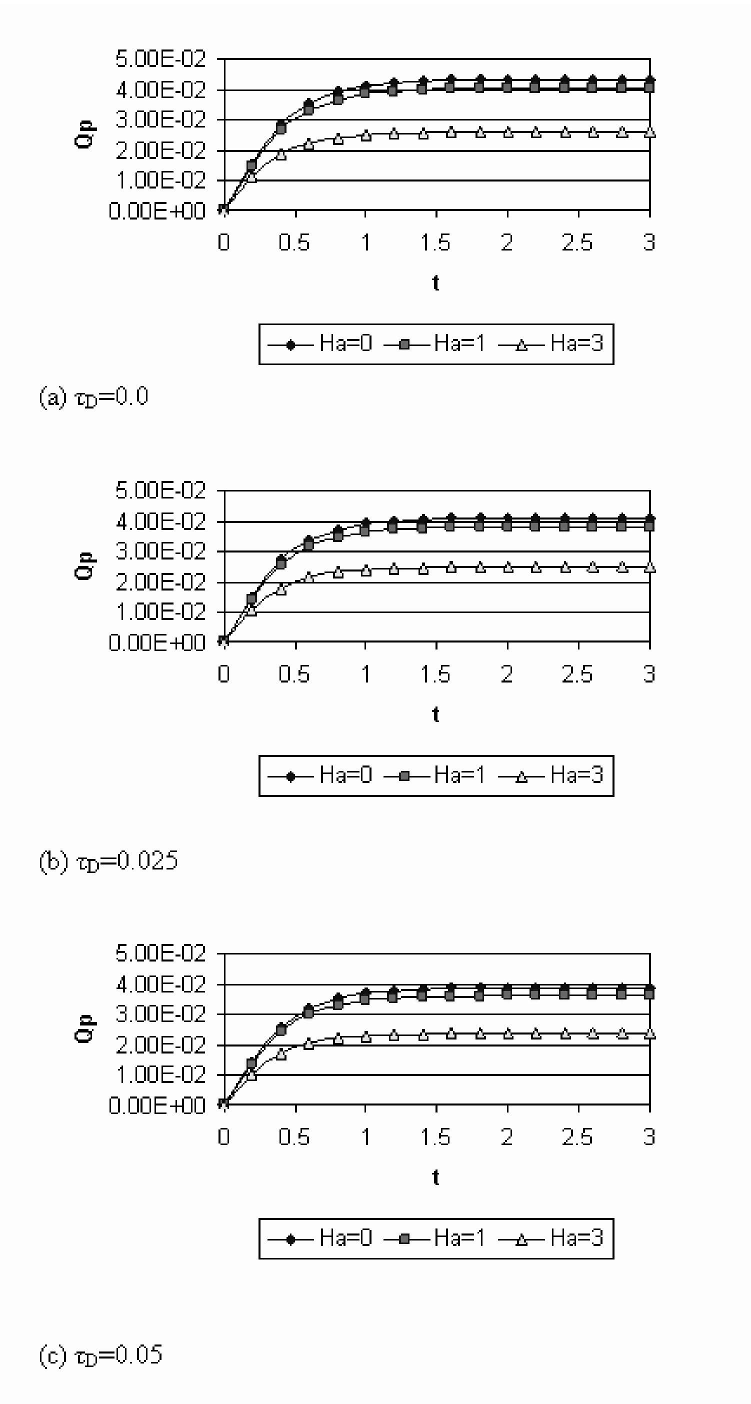


Figure 4 Time development of Q_p for various values of τ_D and Ha

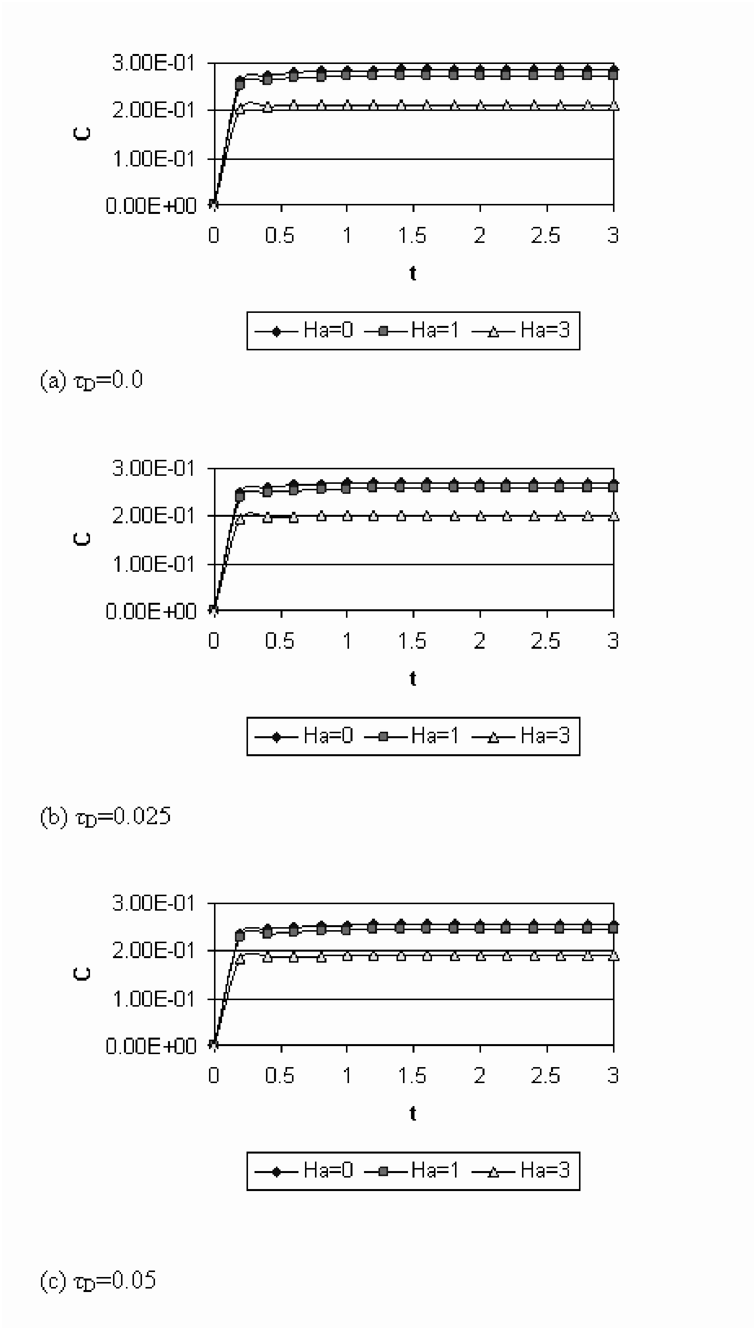


Figure 5 Time development of C for various values of τ_D and Ha

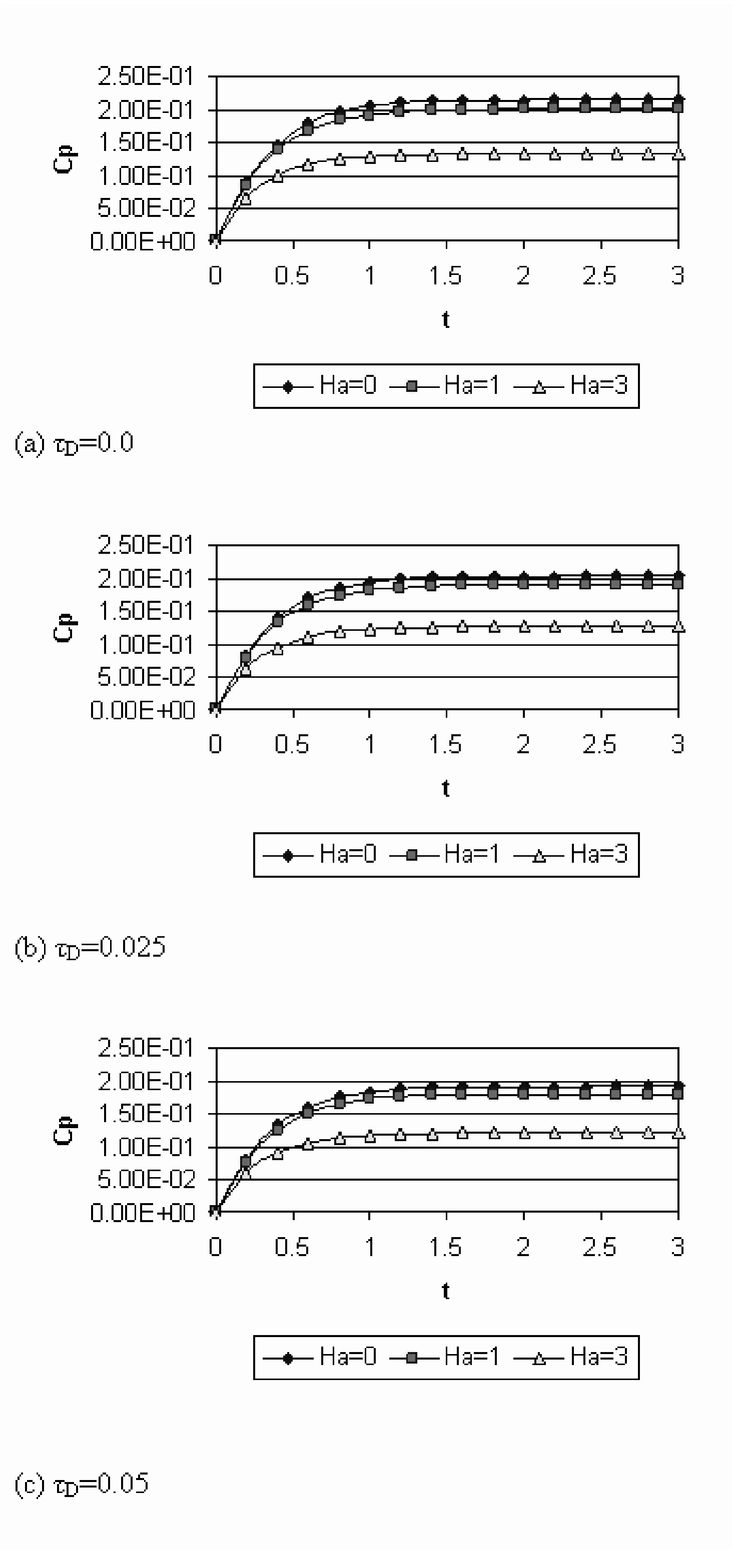


Figure 6 Time development of C_p for various values of τ_D and Ha

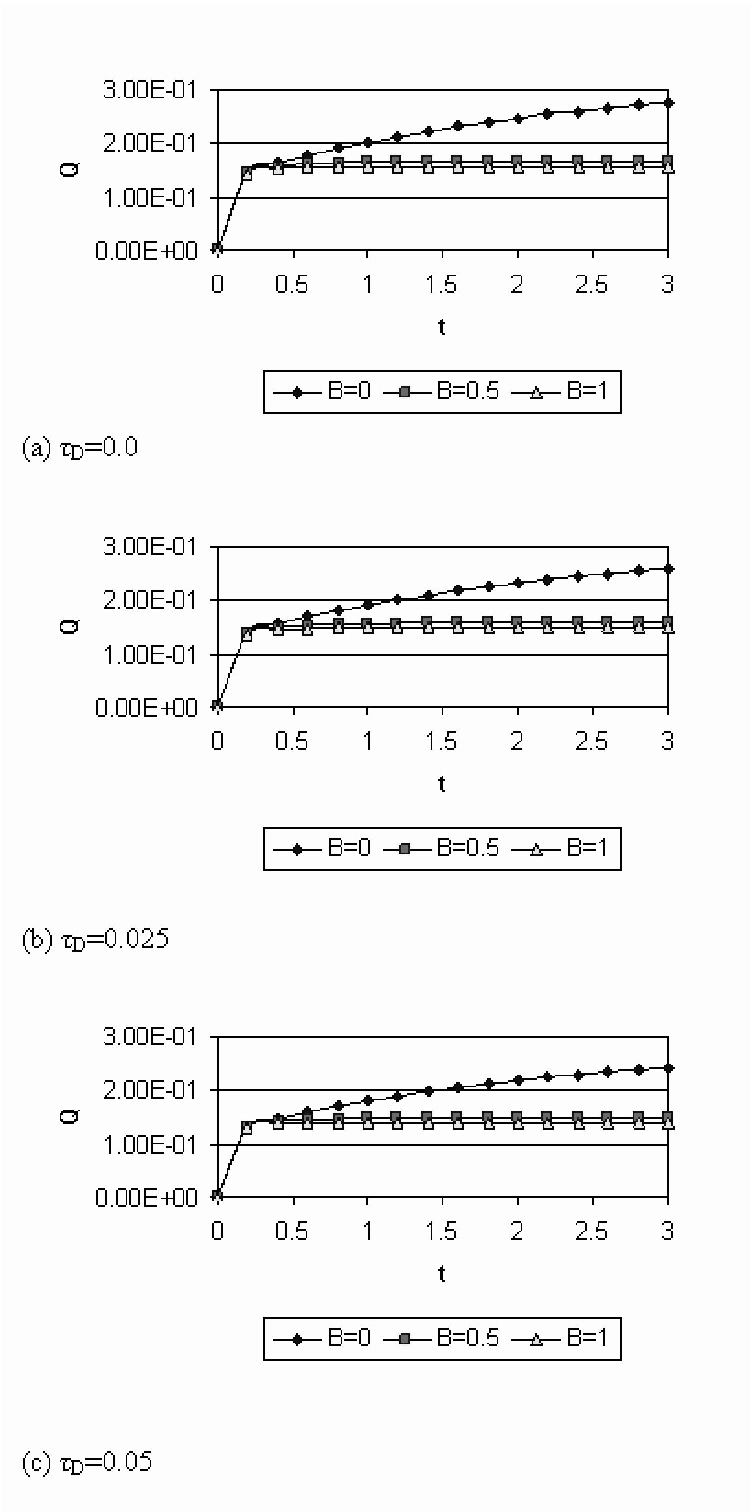


Figure 7 Time development of Q for various values of τ_D and β

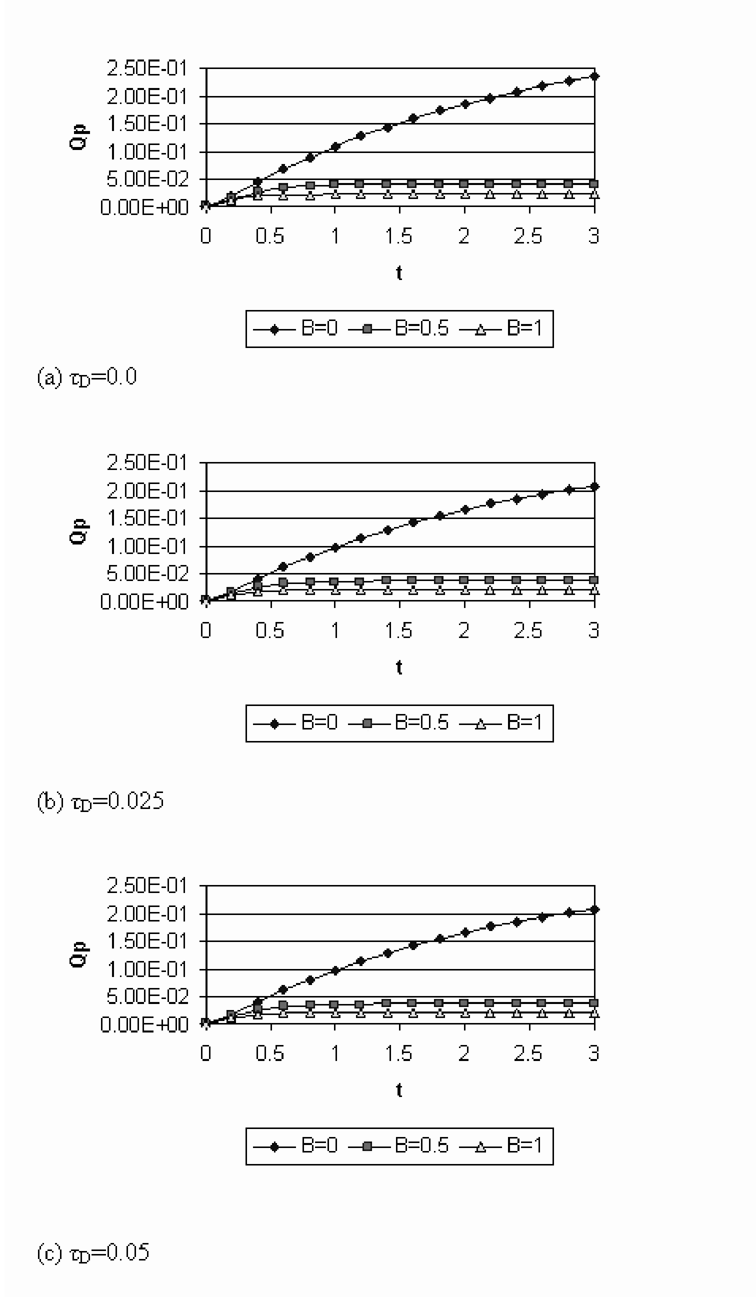
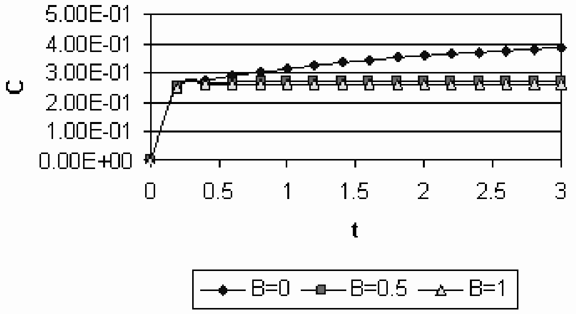
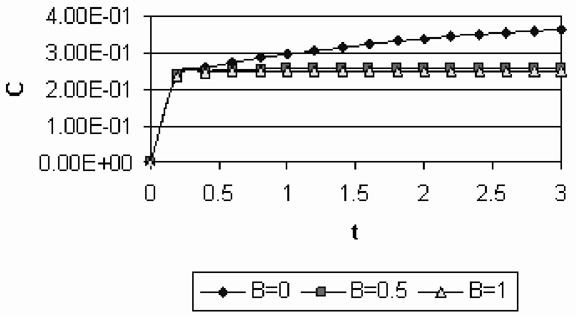


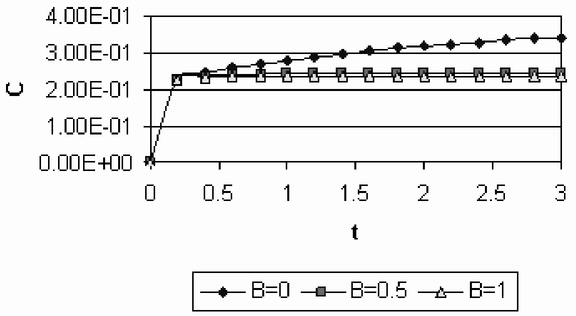
Figure 8 Time development of Q_p for various values of τ_D and β



(a) τ_D=0.0



(b) τ_D=0.025



(c) τ_D=0.05

Figure 9 Time development of C for various values of τ_D and β

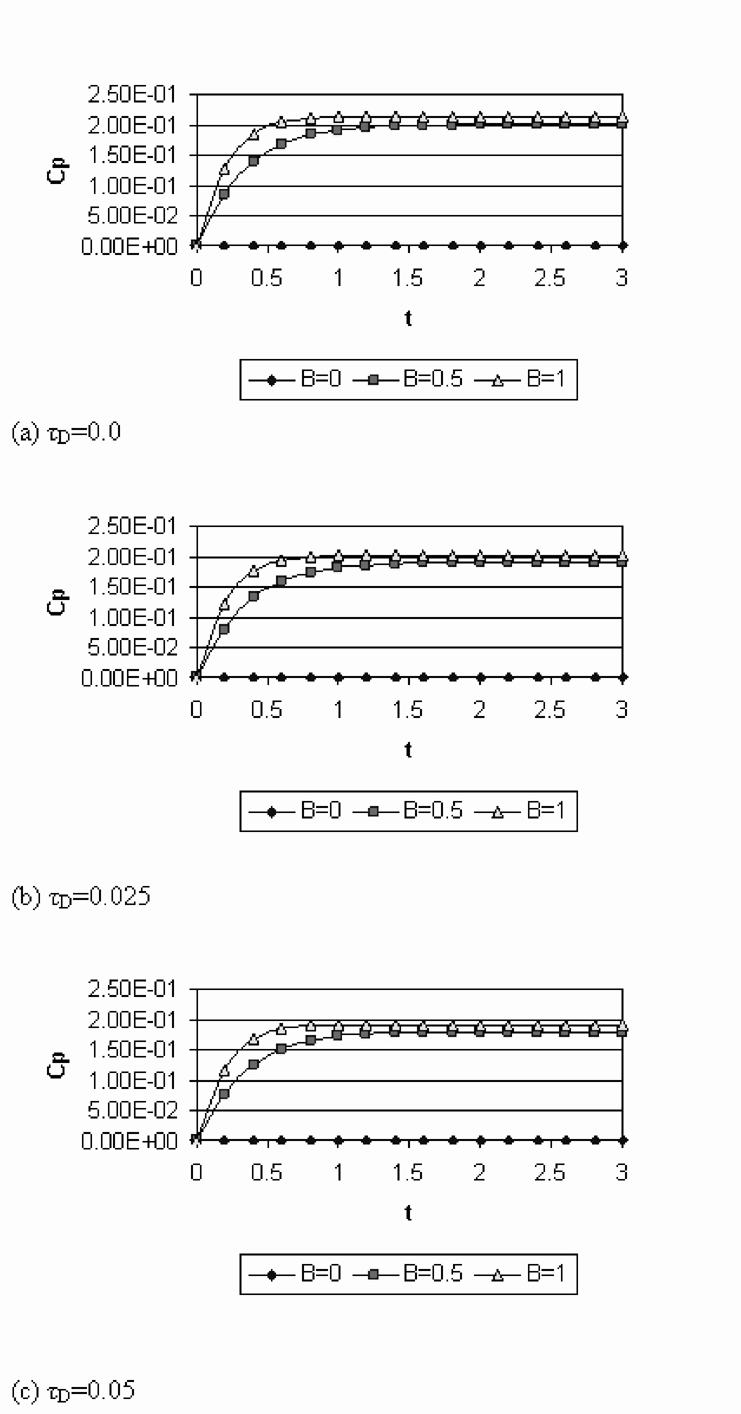


Figure 10 Time development of C_p for various values of τ_D and β

Figures 7, 8, 9, and 10 present the influence of the particle-phase viscosity B on the transient behavior of Q, Q_p, C, and C_p for various values of τ_D and for $Ha = 1$. It is clear from the figures that the inclusion of the particle-phase viscous stresses causes Q, Q_p, C to decrease and C_p to increase (see definition of C_p; Eq. (9)) for all values of τ_D and t. Also, the approach to steady-state conditions is much accelerated than that of the case of inviscid particle-phase ($B = 0$) as clear from Figs. 7, 8, and 9. Figure 10 shows that increasing B increases C_p but decreases its steady-state time.

4. Conclusions

The transient MHD flow of a particulate suspension in an electrically conducting non-Newtonian Casson fluid in a circular pipe with an applied uniform transverse magnetic field is studied. The governing nonlinear partial differential equations are solved numerically. The effect of the magnetic field parameter Ha, the non-Newtonian fluid characteristics (the Casson number τ_D), and the particle-phase viscosity B on the transient behavior of the velocity, volumetric flow rates, and skin friction coefficients of both fluid and particle-phases are studied. It was found that all these parameters decrease as the strength of the magnetic field or the yield stress increases. The particle-phase viscosity has an apparent effect on increasing the skin friction of the particle-phase while decreasing the rest of the parameters. The approach to steady-state conditions is much decreased when increasing B or Ha but it is not greatly affected by changing τ_D.

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