Mechanics and Mechanical Engineering
Vol. 12, No. 4 (2008) 279–295

An influence of the dynamic angle of rolling element location on rolling bearing stiffnesses

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Received (11 October 2008)
Revised (24 October 2008)
Accepted (17 December 2008)

In the paper an influence of rolling elements assembly position on stiffnesses of rolling bearings has been considered. Presented models have been developed to determine values of bearing load distribution parameters, individual rolling element loads, as well as to analyze distributions and values of the radial and axial stiffness of bearings as a function of the bearing rolling element distribution and the direction along which the bearing radial load resultant vector acts. Slow–speed, low–accelerations and short–period motions of rolling bearings have been assumed as a characteristic type of motion of rolling kinematic pairs in manipulators.

Keywords: Rolling elements, bearing, stiffnesses

1. Introduction

Analysis of stiffnesses of rolling kinematics pairs is necessary for relevant modeling of machines dynamics. The stiffnesses of rolling bearings depend on position of rolling elements during the 3D bearing system motion [13, 14, 19, 21–23]. In order to investigate the influence of rolling elements position on stiffnesses, it is necessary to analyze distribution of external loads acting on individual rolling elements during the bearing motion. In the literature devoted to the modeling of bearings and rolling bearing systems [6, 11, 12], one finds analysis of the load distribution acting on rolling elements as a function of the external load for a fixed location of bearing rolling elements, which is optimal from the viewpoint of theoretical analysis. These models have been developed on the assumption of a continuous distribution of the load acting on bearing rolling elements. These models have been defined as static cases of analysis of the load distribution of bearing rolling elements, [19, 20]. In the presented algorithm, the above mentioned assumptions have been suppressed.

In the presented paper the considerations deal with an influence of the rolling element distribution with respect to the direction of the resultant vector of the bearing radial loading during the bearing motion on the values of load distribution parameters and on the distribution and values of stiffness of rolling bearings. The cases of bearing with a clearance, without a clearance and with pre–loading have been examined.

2. Assumptions

Let us consider an angular rolling bearing as the most general case and, at the same time, the most often used type of rolling bearings applied in robot kinematic pairs. The considerations included in this section concern also radial bearings and radial bearing systems which are radially or radially and axially loaded, as well as axial bearings and axial bearing systems. Let us assume that bearing rolling elements carry a load only along the normal direction to the main and auxiliary working surfaces and that track deformations occur only in the places of contact of rolling elements with tracks. Let us also assume that rolling elements are symmetrically distributed on the track, at each moment they contact with an inner track and outer track in two opposite points, and that rolling elements do not exhibit any profile errors of the shape.

3. Theoretical introduction

The external radial loads N_α and the external axial loads $F_{a\alpha}$ of the rolling bearing are transferred by means of bearing rings to rolling elements which are non–uniformly loaded. The maximum load is assumed by the rolling element whose center lies on the line the radial force N_α acts. Let us denote an angle of the bearing rolling element load distribution, measured from the line the radial load resultant vector N_α acts (for $\gamma(t) = 0$ from the maximally loaded rolling element [19]), by Ψ, whereas ε stands for ($\varepsilon \geq 0$, theoretically $\varepsilon \in (0, \infty)$) a coefficient of the load distribution angle on rolling elements [11, 12]. The coefficient ε can be expressed by the following relation [11, 12, 19]

$$\varepsilon = \frac{1}{2}[1 - \cos(\psi)] \tag{1}$$

where Ψ is the load distribution angle on bearing rolling elements [rad].

In rolling bearings the load distribution angle Ψ depends, first of all, on a size of the clearance in the bearing and can assume theoretically values in the range $0 \div 180^o$. At the same time, a correctly operating radial or angular bearing, loaded with a radial or longitudinal force, should have the angle Ψ included in the range $(\pi/2 \div \pi)$ [rad], [11]. In practice it has been found and it is assumed [11] that an angular bearing is correctly loaded from the moment it reaches the loading angle $\Psi \geq 90^o$ ($\varepsilon \geq 0.5$).

In a radial bearing, if the load axial component is high enough, then a clearance is eliminated, owing to which the bearing operates like an angular bearing and exactly the same formulas hold. In the case of a radial bearing without axial loading, the radial bearing can be analyzed like an angular bearing loaded radially if we take into account a clearance in the bearing and zero deformation of the bearing along the axial direction ($\delta_a = 0$), [19]. This case corresponds to the model included in [12], in which a zero clearance is assumed in the bearing.

3.1. *Main parameters of the model*

Fig. 1. shows a construction scheme of the rolling bearing. The load of the i-th (any) rolling element is a resultant of the radial and axial load. Under the influence of the external load acting on the bearing and resulting from it deformations in the places of contact of the rolling elements with the tracks, the displacement of the shaft center with respect to the mounting longitudinal axis occurs. The place of contact, whose center lies on the line the radial force N_α acts, is subjected to the strongest deformation–displacement of the rigs with respect to each other, $\lambda_0^{\max}$.

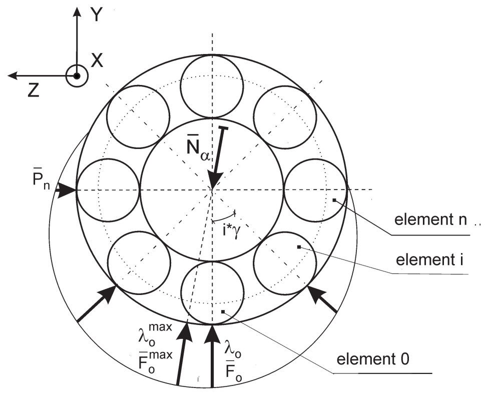

Figure 1 Construction scheme of the rolling bearing

where:

F_i	–	resultant external load of the i-th rolling element of the bearing [N];
$\lambda_0^{\max}$	–	theoretical, maximum value of the displacement of the bearing rings with respect to each other, along the direction in which the resultant radial load acts in the normal direction [mm];
$\lambda_0^{\max}$	–	value of the displacement of the bearing rings with respect to each other in the place of contact of the maximally loaded bearing rolling element in the normal direction –in the case where $\gamma(t) = 0$, then $\lambda_0^{\max}$ [mm].

In general, deformation of each rolling element in the angular or radial rolling bearing to which radial and axial load is applied, as well as in the radial bearing loaded with the radial force (then: $\alpha = 0$, $\delta_a = 0$, $\delta_r = \lambda_o + g/2$) can be described with a trigonometric function by means of the following relation [19] (in the case when $\gamma(t) = 0$, formula (2) assumes the form of formula [11] and corresponds to the case of the static location of bearing rolling elements)

$$\lambda_i = \lambda_o^{\max}\left[1 - \frac{1}{2\varepsilon}(1 - \cos(\gamma(t) + i\gamma))\right] \tag{2}$$

where:
$i = 0, ..., n$
λ_i – magnitude of deformation of the i-th rolling element along the normal direction [mm];
γ – angle between subsequent rolling elements in the bearing circumferential direction [rad];
$\gamma(t)$ – angle between the direction the radial load resultant vector acts and the direction on which the maximally loaded bearing rolling element lies at a given instant of time (Fig. 7). This angle has been introduced in order to account for an effect of the dynamic distribution of rolling elements on the presented considerations [rad];
n – number of rolling elements comprised by the angle of the bearing load region Ψ (taking into account the dynamic distribution of rolling elements) [-];
ε – coefficient of the load distribution angle on rolling elements [-].

In relation (4) the coefficient of the load distribution angle on rolling elements can be expressed by the formula, [19]

$$\varepsilon = \frac{1}{2}\left[1 + \frac{\delta_a}{\delta_r}\tan(\alpha_l) - \frac{g}{2\delta_r}\right] \tag{3}$$

If we assume that the value of the angle $\gamma(t) = 0$ in Eq. (3), then an analysis of the bearing stiffnesses refers to the case of a constant distribution of rolling elements. This case, met in the literature, has been called a static analysis of the bearing stiffnesses. If a variability of the angle $\gamma(t)$ is taken into account, then the analysis of the stiffnesses is called dynamic [19], as it accounts for a distribution of rolling bearing elements during motion with respect to the direction along which the resultant radial load acts. A load distribution angle on rolling elements is determined from the relation (on the basis of Eqs (1) and (3))

$$\psi = \arccos\left[-\frac{\delta_a}{\delta_r}\tan(\alpha_l) + \frac{g}{2\delta_r}\right] \tag{4}$$

where:
g – bearing radial clearance [μm];
δ_a, δ_r – axial and radial mutual displacements (deformations) of the bearing rings under axial and radial loading, respectively, depending on a type and design of the bearing, a way the bearing system is assembled (with pre–loading, with a clearance, or with a zero clearance), a character of the bearing external loading.

In the case of a zero clearance in the bearing, the quantities δ_r, δ_a are the nominal radial and axial displacements of the bearing rings, respectively, and can be determined from [11]. The axial displacement reaches its nominal value (δ_a^*) in the case when only the axial component of the external load acts on the bearing. The radial displacement equals $\delta_r = 0$. In turn, the nominal radial displacement of the bearing rings (δ_r^*) is determined when the axial displacement is assumed to be $\delta_a = 0$ ($\varepsilon = 0.5$). The axial stiffness is infinitely high then.

In the case of a preload or a clearance in the bearing, the radial and axial displacements are defined by

$$\begin{aligned} \delta_r &= \beta_r \delta_r^* \\ \delta_a &= \beta_a \delta_a^* \end{aligned} \tag{5}$$

in which δ_r^*, δ_a^* are the nominal displacements of the bearing rings (nominal radial and axial displacements of the bearing shaft), while β_r, β_a are the bearing radial and axial stiffness coefficients, respectively.

Expression (5) defines the relation of the radial and axial load and the bearing operation angle with respect to each other and it is a measure of bearing stiffness.

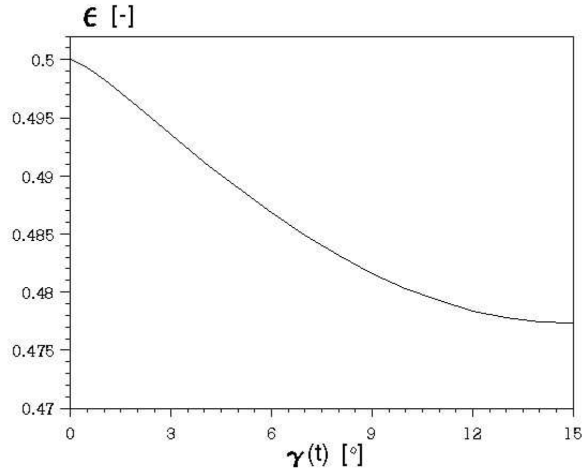

Figure 2 Load distribution angle coefficient as a function of the dynamic angle $\gamma(t)$

When dynamic distribution of the rolling elements has been considered, a phenomenon of variability in the values of distribution parameters of the load acting on the bearing rolling elements as a function of the angle $\gamma(t)$ has been found. Therefore, it is necessary to analyse and to take into account in calculations the changes in the distribution parameters $\varepsilon, \psi, z_k, z_k'$ and the magnitude of the load acting on individual rolling elements, and bearing stiffness coefficients as a function of the angle $\gamma(t)$. In Fig. 2 a sample result of the analysis of the value of the load distribution angle coefficient acting on rolling elements as a function of the angle $\gamma(t)$ (dynamic rolling element distribution) for the *7206 BE* rolling bearing [18] and the data shown in Table I is presented.

Figure 2 has been plotted for the case when the angular bearing load assumes the values: $N_\alpha = 500$ N, $F_{a\alpha} = 1.20713 N_\alpha \tan(\alpha_l)$.

While the value of the angle $\gamma(t)$ is changing, the values of the load angle coefficient ε are read. A numerical algorithm for determination of the load distribution parameters acting on rolling elements can be found in [19]. A variability of the load distribution parameters ψ, z_k, z_k' (18, 20) as a function of the angle $\gamma(t)$ follows from the presented dependence of the load distribution coefficient ε on the angle $\gamma(t)$, Fig. 2.

Formulas (3), (4) allow for determination of parameters of the rolling element loading region in the general case of the bearing load.

4. Bearing load and deformation

The relations describing the resultant forces the individual bearing rolling elements are loaded with (Fig. 1, [19]) have form

$$F_i = F_o^{\max}\left[1 - \frac{1}{2\varepsilon}\left(1 - \cos\left(\gamma(t) + i\gamma\right)\right)\right]^m \tag{6}$$

where: The resultant load F_i of the i-th rolling element is composed of the radial

$i = 1, ..., n$, m	–	coefficient which describes of type of contact area between rolling element and rolling tracks, [1, 2, 4, 8-10, 23-26]
F_i	–	resultant loading of the i-th bearing rolling element [N]
$F_o^{\max}$	–	maximum resultant loading acting on a single rolling element of the bearing [N].

load P_i and the axial load P_{oi}.

The external force $N_\alpha$can be described by,[19]

$$N_\alpha = \sum_{i=-n1}^{+n2} F_o^{\max}\left[1 - \frac{1}{2\varepsilon}\left(1 - \cos(\gamma(t) + i\gamma)\right)\right]^m \cos(\gamma(t) + i\gamma)\cos(\alpha_l) \tag{7}$$

analogously, for axial force

$$F_{a\alpha} = \sum_{i=-n1}^{+n2} F_o^{\max}\left[1 - \frac{1}{2\varepsilon}\left(1 - \cos(\gamma(t) + i\gamma)\right)\right]^m \sin(\alpha_l) \tag{8}$$

where:

$$\begin{aligned} i &= -n_1,\ ...,\ 0,\ ...,\ +n_2 \\ n_1 &= (\psi + \gamma(t))/\gamma \\ n_2 &= (\psi - \gamma(t))/\gamma \end{aligned}$$

The resultant reaction acting on the i-th rolling element is specified in the form [19]

$$F_i = \frac{N_\alpha}{z_k}\left[1 - \frac{1}{2\varepsilon}\left(1 - \cos(\gamma(t) + i\gamma)\right)\right]^m \tag{9}$$

where

$$z_k = \sum_{i=-n1}^{+n2}\left[1 - \frac{1}{2\varepsilon}\left(1 - \cos(\gamma(t) + i\gamma)\right)\right]^m \cos(\gamma(t) + i\gamma)\cos(\alpha_l) \tag{10}$$

or by the formula equivalent to Eq. (8)

$$F_i = \frac{F_{a\alpha}}{z_k'}\left[1 - \frac{1}{2\varepsilon}\left(1 - \cos(\gamma(t) + i\gamma)\right)\right]^m \tag{11}$$

where

$$z_k' = \sum_{i=-n1}^{+n2}\left[1 - \frac{1}{2\varepsilon}\left(1 - \cos(\gamma(t) + i\gamma)\right)\right]^m \sin(\alpha_l) \tag{12}$$

Employing Eqs. (3), (5), (7), (8), (10), (12) and formulas determining the values of δ_r^*, δ_a^* the coefficients of radial stiffness β_r and axial stiffness β_a of the bearing can be represented as

$$\beta_r = \frac{1}{2\varepsilon}\left(\frac{z_k(\varepsilon = 0.5)}{z_k}\right)^{\frac{1}{m}} \qquad \beta_a = \frac{2\varepsilon - 1}{2\varepsilon}\left(\frac{z\sin(\alpha_l)}{z_k'}\right)^{\frac{1}{m}} \tag{13}$$

The values of the stiffness coefficients β_r, β_a depend on the fact if we carry out a static or dynamic analysis of the bearing rolling element location (through a variability of the rolling element load distribution parameters ε, Ψ). If we divide the radial stiffness coefficient β_r by the axial stiffness coefficient β_a, Eq. (13), we get

$$\varepsilon = \frac{1}{2}\left[1 + \frac{\beta_a}{\beta_r}\left(\frac{z_k(\varepsilon = 1/2)}{z_k}\right)^{\frac{1}{m}}\left(\frac{z_k'}{z\sin(\alpha_l)}\right)^{\frac{1}{m}}\right] \tag{14}$$

Equation (14) can be presented in form, [19]

$$\varepsilon = \frac{1}{2}\left[1 + \frac{\beta_a}{\beta_r}\left(\frac{F_{a\alpha}}{N_\alpha\tan(\alpha_l)}\right)^{\frac{1}{m}}\left(\frac{z_k(\varepsilon = 1/2)}{z\cos(\alpha_l)}\right)^{\frac{1}{m}}\right] \tag{15}$$

Equation (15) is used to assign the approximate value of the angle coefficient ε.

Using expression (7), we get

$$\frac{\delta_a}{\delta_r}\tan(\alpha_l) = \frac{\beta_a\delta_a^*}{\beta_r\delta_r^*}\tan(\alpha_l) \tag{16}$$

After substituting Eq. (13) into Eq. (16), one obtains

$$\frac{\delta_a}{\delta_r}\tan(\alpha_l) = \frac{(2\varepsilon - 1)\delta_a^*}{\delta_r^*}\left(\frac{z\sin(\alpha_l)z_k}{z_k(\varepsilon = 1/2)z_k'}\right)^{\frac{1}{m}}\tan(\alpha_l) \tag{17}$$

Equation (17) allows one to analyze the relative radial and axial displacements of the bearing rings as a function of the bearing load distribution.

The nominal displacements for the model under consideration at a zero clearance in the bearing are in the form, [20]

$$\delta_r^* = \frac{\lambda_0^{\max}}{\cos(\alpha_l)}\ \delta_a^* = \frac{\lambda_0^{\max}}{\sin(\alpha_l)} \tag{18}$$

Relation between force and deformation for the rolling element can be presented as a function of the stiffness of the bearing rolling elements–raceways contact, Fig. 1.

$$F_0^{\max} = K(\lambda_0^{\max})^m \tag{19}$$

It has been assumed that in the unloaded bearing, a point or linear contact under load transforms into a region of a circular or elliptic contour and a rectangular or trapezoid region, correspondingly. The features of the contact have been determined on the basis of classical Hertzian formulas [4, 5, 7-9, 17]. Eventually the nominal displacements, Eq. (13) take the form

$$\delta_r^* = \frac{K_r}{\cos(\alpha_l)}\left[\frac{N_\alpha}{z_k(\varepsilon = 0.5)}\right]^{1/m} \qquad \delta_a^* = \frac{K_r}{\sin(\alpha_l)}\left[\frac{F_{a\alpha}}{z\sin(\alpha_l)}\right]^{1/m} \tag{20}$$

The stiffness coefficient $K_r$can be described by

$$K_r = \frac{1}{K^{\frac{1}{m}}} = \left[\frac{1}{\left(\frac{1}{K_i}\right)^{1/m} + \left(\frac{1}{K_o}\right)^{1/m}}\right]^m \tag{21}$$

where K_o and K_i are the stiffnesses of contacts of rolling elements with the outer and inner raceway. Employing Eqs. (18), (3), (7-8), (10) and (12) and the formulas that determine the nominal displacements $\delta_r^*, /, \delta_a^*$ we obtain the coefficients of radial and axial stiffness

$$\beta_r = \frac{1}{2\varepsilon}\left(\frac{z_k(\varepsilon = 0.5)}{z_k}\right)^{1/m} \qquad \beta_a = \frac{2\varepsilon - 1}{2\varepsilon}\left(\frac{z\sin(\alpha_l)}{z_k'}\right)^{1/m} \tag{22}$$

The coefficients in Eq. (22) take into account the kinematics of rolling elements and raceways and its influence on the parameters of the load distribution acting on rolling elements. The radial and axial stiffness of rolling bearings can also be defined as a ratio between the corresponding load increment and the deflection increment. Thus, the dimensional stiffness is as follows

$$c_r = \frac{N_\alpha}{\delta_r} = \frac{c_r^*}{\beta s_r} \qquad c\_a = \frac{F_{a\alpha}}{\delta_a} = \frac{c_a^*}{\beta_a} \tag{23}$$

where the nominal radial and axial stiffness (referred to a zero clearance in the bearing, i.e., the boundary, desirable state) is as follows

$$c_r^* = \frac{N_\alpha}{\delta_r^*} \qquad c_a^* = \frac{F_{a\alpha}}{\delta_a^*} \tag{24}$$

Rolling bearing operation angle at which the resultant reaction of the i-th rolling element acts can be calculated from

$$\alpha_l = \arccos\frac{P_i}{F_i} \tag{25}$$

The formulas that can be found in the literature [11, 12, 15, 16] describe nominal displacements and stiffnesses of bearings for the case of a static or quasi–static position of rolling elements and a continuous load distribution between rolling elements and raceways. This approximation can be used for high–rotary kinematic pairs. As can be seen in the above equations, stiffnesses depend also on the fact if we assume static or dynamic position of bearing rolling elements (by a variability in parameters of the load distribution of rolling elements ε, ψ). Knowing the bearing external load distributions and the load values acting on individual rolling elements as a function of the rolling element location angle $\gamma(t)$ with respect to the direction along which the bearing radial load resultant vector acts, it is possible to analyze an influence of the rolling elements position on stiffnesses coefficients.

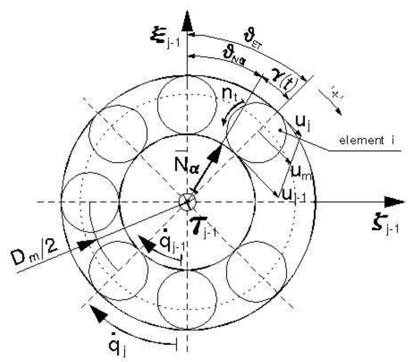

Figure 3 Relation between the resultant vector of the external radial load and bearing rolling element distribution

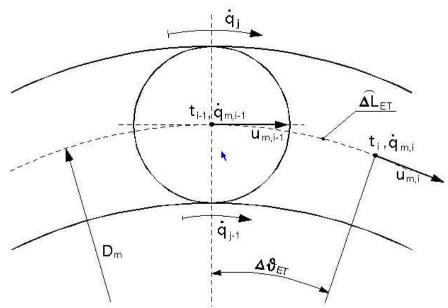

Figure 4 Kinematics of the bearing rolling element

whereas:

γ_o – angle between the direction of the resultant vector of the bearing radial load and the line on which the center of the maximally loaded rolling element lies; this angle is treated as an initial condition [19, 20] for an analysis of the system ($\gamma(t = t_p) = \gamma_o$ [rad]).

4.1. *Relations beetwen the bearing kinematics and position of rolling elements*

The considerations have been carried out on the basis of the kinematics of the motion of the tracks and rolling elements, the bearing external load distribution and the initial conditions of the load distribution, the location and the relative velocity of the tracks and bearing rolling elements. Determination of the relation between a direction along which the radial load resultant vector acts and a location of the maximally loaded bearing rolling element as a function of the rolling element motion character is a factor affecting the bearing stiffnesses.

The direction of the resultant vector of the radial load N_α is expressed as a function of the angle $\nu_{N\alpha}$, Fig. 3, by the expression

$$\vartheta_{N\alpha} = A - B \arcsin\left(\frac{F^{\zeta}_{p\alpha}}{N_\alpha}\right) \tag{26}$$

where
$F^{\zeta}_{p\alpha}$ – is the vector of the resultant radial load in the direction of the ζ axis, Fig. 3. whereas

$F^{\zeta}_{p\alpha}$	$F^{\xi}_{p\alpha}$	A	B
≥ 0	> 0	0	-1
> 0 ≤ 0	≤ 0 < 0	π	1
< 0	≥ 0	2π	-1

In the case the dynamic distribution of bearing rolling elements is taken into account, a relation allowing for calculation of the angular velocity of the rolling element center as a function of the relative angular velocity of the bearing tracks should be determined. Assuming that the inner ring rotates with the angular velocity $\dot{q}_{j-1}$, and the outer ring with the velocity $\dot{q}_j$ and that there is no slide on the tracks, then the circumferential velocity of the rolling element center, (Fig. 3), is

$$u_m = \frac{D_m}{4}\left[\dot{q}_{j-1}\left(1 - \frac{2r_i\cos(\alpha_l)}{D_m}\right) + \dot{q}_j\left(1 + \frac{2r_i\cos(\alpha_l)}{D_m}\right)\right] \tag{27}$$

The angular velocity of the rolling element assembly with a cage with respect to the bearing axis is

$$\dot{q}_m = \frac{1}{2}\left[\dot{q}_{j-1}\left(1 - \frac{2r_i\cos(\alpha_l)}{D_m}\right) + \dot{q}_j\left(1 + \frac{2r_i\cos(\alpha_l)}{D_m}\right)\right] \tag{28}$$

where
u_m – circumferential velocity of the rolling element center [m/s]
D_m – bearing pitch diameter [m]
r_i – rolling radius of the i-th rolling element [m]

As a result of the analysis of Fig. 3, a relation describing the angle between the direction on which the center of the maximally loaded bearing rolling element lies and the direction of the resultant vector of the radial load at a given moment of time, has been formulated

$$\gamma(t) = \vartheta_{ET} - \vartheta_{N\alpha} \tag{29}$$

where

$\vartheta_{N\alpha}$ – angle between the direction of the bearing radial load resultant vector and the axis ξ_{j-1} [rad]

ϑ_{ET} – angle between the line on which the center of the maximally loaded bearing rolling element lies and the axis ξ_{j-1} [rad]

Knowing, as the initial condition, the angle γ_o and the angle along which the resultant radial loading at the initial moment $t = t_p$ acts, we can write the following relation on the basis of Fig.3

$$\vartheta_{ET}(t = t_p) = \vartheta_{N\alpha}(t = t_p) + \gamma_o \tag{30}$$

The angle of the direction along which the resultant vector of the bearing radial load acts with respect to the axis ξ_{j-1} of the local reference frame ξ_{j-1}, ζ_{j-1}, τ_{j-1} and the angle of the direction of the location of the center of the maximally loaded rolling element with respect to the axis ξ_{j-1} of the local reference frame ξ_{j-1}, ζ_{j-1}, τ_{j-1} have been described by (Fig. 3)

$$\begin{aligned} \vartheta_{N\alpha} &= \vartheta_{N\alpha}(t = t_p) + \Delta\vartheta_{N\alpha} \\ \vartheta_{ET} &= \vartheta_{ET}(t = t_p) + \Delta\vartheta_{ET} \end{aligned} \tag{31}$$

Using Eqs. (31), (30) and Fig. 3 we get, [19]

$$\Delta\gamma = \vartheta_{N\alpha}(t = t_p) - \vartheta_{N\alpha} + \Delta\vartheta_{ET} \tag{32}$$

Eventually, the value of the angle describing the relation between the resultant radial load direction and the location of the maximally loaded rolling element can be expressed by the relation

$$\gamma(t) = \gamma_o - \vartheta_{N\alpha} + \vartheta_{N\alpha}(t = t_p) + \Delta\vartheta_{ET} \tag{33}$$

A change in the angle ΔL_{ET} has been expressed as a function of the change in the rolling element pitch center path in the following form, Fig. 4

$$\Delta L_{ET} = \frac{D_m}{2}\Delta\vartheta_{ET} \tag{34}$$

where ΔL_{ET} – change in the rolling element pitch center linear path (along the arc) during the simulation $\Delta t = (0,\, t)$ [m].

Having substituted Eq.(34) into Eq. (33), we obain

$$\gamma(t) = \gamma_o - \vartheta_{N\alpha} + \vartheta_{N\alpha}(t = t_p) + \frac{2\Delta L_{ET}}{D_m} \tag{35}$$

As the rolling element motion is non–uniformly variable in the general case and the simulation of the bearing motion is conducted by means of numerical iterative methods, let us consider the rolling element kinematics in the time interval $\Delta t = (t_{i-1},\, t_i)$ of the motion. Then, depending on the fact if we have an analytical form of the linear velocity $u_m(t)$ or only the values of numerical computations in subsequent computational steps, the following relations hold

$$\Delta L_{ET} = \sum_{t_p}^{t} \left(\int\limits_{t_{i-1}}^{t_i} u_m dt \cong \frac{1}{2}\Delta t(u_{m,i} + u_{m,i-1})|_{\Delta t \to 0} \right) \tag{36}$$

where

t_p	–	initial time of the motion simulation of the rolling element distribution, e.g. $t_p = 0$ [s])
$\Delta t = (t_i - t_{i-1})$	–	time interval of the bearing simulation step [s]
$u_{m,i},\, u_{m,i-1}$	–	linear velocities, respectively, at the moment t_i, t_{i-1} of the time interval Δt of the simulation of the rolling element center motion with respect to the bearing axis [m/s].

The integral $\int u_m dt$ in Eq. (36) can be obtained using Eq. (27) and knowing the forms of the functions describing the time characteristics of the bearing outer and inner track angular velocities (i.e. $\dot{q}_j$, $\dot{q}_{j-1}$). The case when the time t_p (time t_p, e.g. $t_p = 0$, at which the motion of the rolling assembly starts; static initial conditions) is assumed as the beginning of the simulation of the dynamic rolling element distribution and the case when the simulation of the dynamic distribution of rolling elements starts after the rolling assembly motion lasts for a certain time t (so–called dynamic initial conditions) have been distinguished. Finally

$$\gamma(t) = \gamma_o - \vartheta_{N\alpha} + \vartheta_{N\alpha}(t = t_p) + \sum_{t_p}^{t} \frac{\Delta t}{D_m}(u_{m,i-1} + u_{m,i}) \tag{37}$$

where:

$\Delta t \to 0$, $|\gamma(t)| \le 1/2\,\gamma$ – period of changes in the angle $\gamma(t)$ is equal to $1/2\gamma(t)$

γ – circumferential angle between the centers of two subsequent rolling elements versus the bearing axis [rad]; The above mentioned angular condition distinguishes the cases when the angle $\gamma(t)$ exceeds the zero value and the 'boundary' values are equal to $\pm 1/2\,\gamma$. When the "boundary" value is exceeded, a change in the maximally loaded rolling element and, thus, a change in the value of the angle $\gamma(t)$ (taking into consideration a periodicity of its changes) is accounted for automatically.

5. Numerical simulations

In order to conduct an exemplary simulation, the radial and axial load have been assumed as

$$\begin{aligned} N_\alpha &= J[\Xi + \sin(\omega_2 t)] \\ F_{a\alpha} &= F_{a\alpha o} + 1.67 N_\alpha \tan(\alpha_l) \\ F_{a\alpha o} &= Q \sin(\omega_3 t + \phi) \end{aligned} \tag{38}$$

where
J, Q – denote the amplitude of the external radial and axial load, correspondingly,
$F_{a\alpha o}$ – value of the external axial load of the bearing,
ω_2, ω_3 – frequency of changes in the external radial and axial load,
φ – phase shift angle of the external load components,
Ξ – parameter of the external load range.

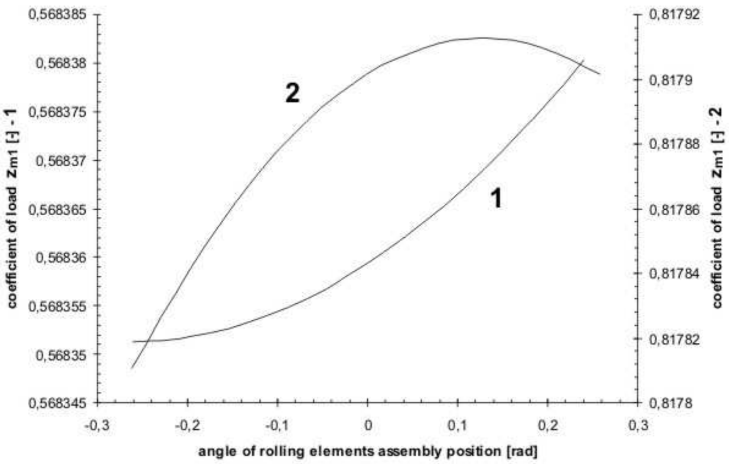

Figure 5 Exemplary coefficients of the bearing load distribution in function of the rolling elements position

Excitement parameters	Value	Excitement parameters	Value
J [N]	250	ω_2 [rad/s]	1
Q [N]	50	ω_3 [rad/s]	1
G [rad/s]	15	γ_o [rad]	$-\pi/18$
Ξsgη [-]	1.5, 2	ϕ [rad]	0
ω_1 [rad/s]	5	$\vartheta_{N\alpha p}$ [rad]	$\pi/4$
$\vartheta_{N\alpha} = \pi/4 + time/10$ [rad]; bearing with a clearance $g = 0$.			

A relative velocity of the rotational motion between the journal and the bush has been expressed by the relation (the value of the angular velocity of the bearing inner ring has been assumed to be zero)

$$\dot{q} = G[\eta - \sin(\omega_1 t)] \quad (39)$$

where:

G	–	amplitude of the relative velocity [rad/s]
η	–	parameter of the bearing motion character [-]
ω_1	–	angular velocity of changes in the relative motion [rad/s]

In Fig. 5 two exemplary coefficients of the bearing load for assumed case of the bearing external load, Eq. (38), are presented. A value of the bearing load distribution angle coefficient ε, Fig. 2, also varies, depending on the angle of the bearing rolling element location $\gamma(t)$. In turn, changes in individual rolling element loads exert an influence on the bearing stiffnesses corresponding to them.

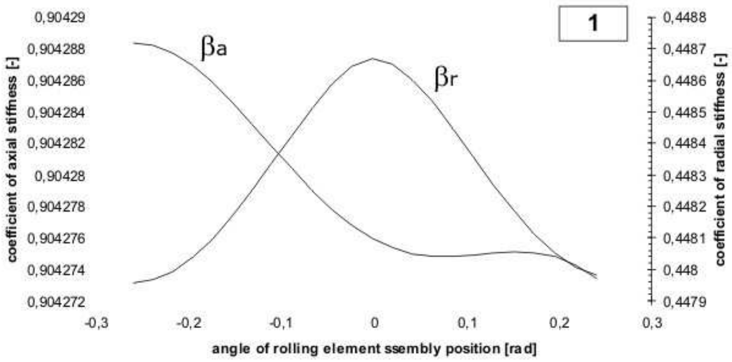

Figure 6 Nondimensional coefficients of stiffnesses in function of the rolling elements position for 1^{st} case of the load distribution coefficient, see Fig. 5.

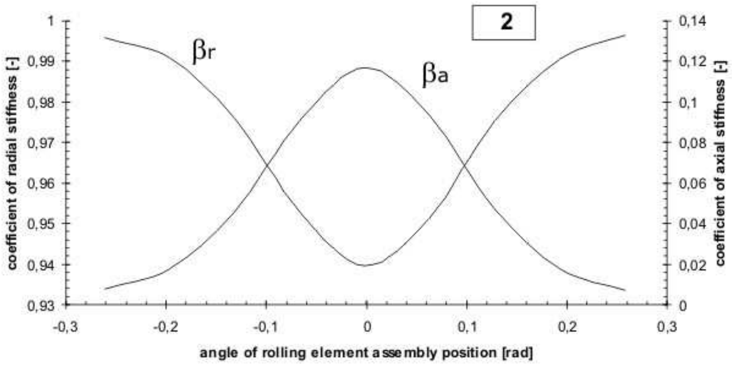

Figure 7 Nondimensional coefficients of stiffnesses in function of the rolling elements position for 2^{nd} case of the load distribution coefficient, see Fig. 5.

Corresponding to Fig. 5 the bearing radial and axial nondimensional and dimensional stiffnesses coefficients, in function of the rolling element position, are shown in Figs 6–9.

In the presented figures an influence of the rolling elements position on the stiffnesses coefficients for different coefficients of the bearing load is visible.

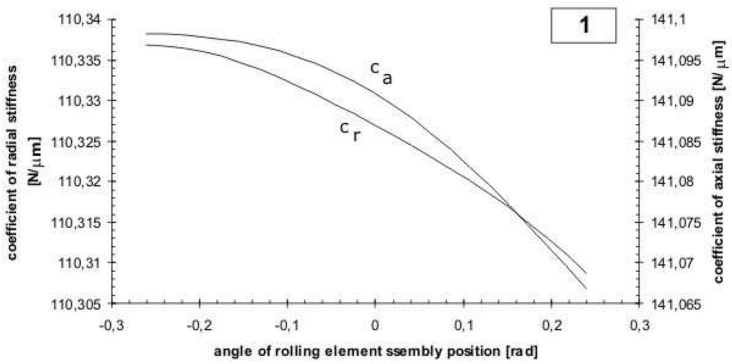

Figure 8 Dimensional coefficients of stiffnesses in function of the rolling elements position for 1^{st} case of the load distribution coefficient, see Fig. 5.

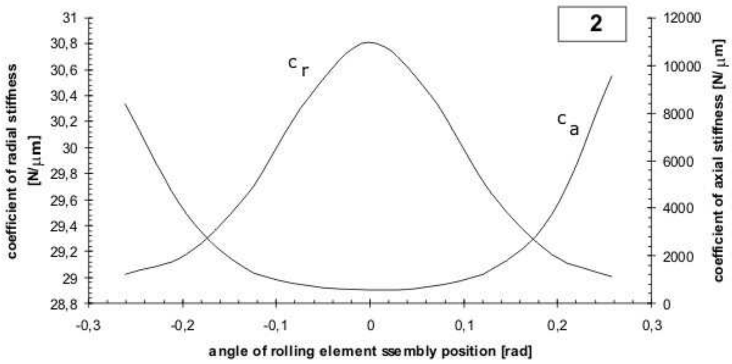

Figure 9 Dimensional coefficients of stiffnesses in function of the rolling elements position for 2^{nd} case of the load distribution coefficient, see Fig. 5.

6. Conclusions

In the paper, an analytical model which allows to analyze an influence of disribution of rolling elements on their loading, and on the stiffnesses coefficients has been developed.The considerations have been carried out on the basis of the kinematics of the motion of the tracks and rolling elements, the bearing external loads distribution and the location rolling elements during motion.

The present paper discusses the method which is based on the verified and extended method for the analysis of the rolling bearing load distribution and whose foundations (for the static or quasi–static rolling element distribution and when an approximation in the form of the continuous load distribution is assumed) are included in [6, 11, 12].

The presented model takes into consideration both the cases of floating and preloaded bearing systems. Slow–speed and short–period motions of rolling bearings have been assumed as a characteristic type of the motion of rolling kinematic pairs in manipulators and robots.

Some selected data referring to the *7206 BE* angular bearing are included in Table III.

Parameter:	Value:
Number of rolling elements z [-]	12
Radius of rolling element r_i [m]	0.00475
Diameter of rolling bearing groove D [m]	0.0365
Angle between rolling elements γ [$^\circ$]	30
Bearing operation angle $\alpha_{\_}$ [$^\circ$]	40
Pitch diameter D_m [m]	0.046
Inner diameter D_{m1} [m]	0.043
Outer diameter D_{m2} [m]	0.051

The remaining data concerning the *7206 BE* bearing are to be found in [18].

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The investigations were supported by the State Committee for Scientific Research, Poland, within the research project No. N501 05132/3516.