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Some Aspects Of Dynamic Buckling of Plates Under In–Plane Pulse Loading

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The paper deals with the dynamic response of thin iso- and orthotropic plates subjected to in–plane pulse loading. The problem is investigated on the basis of finite element method using computer code ANSYS8.1. The influence on the dynamic behaviour of plates of following factors: shape of pulse loading, imperfection sensitivity, time of pulse duration and material properties is analysed. The dynamic buckling load is estimated on the basis of different dynamic stability criteria.

Keywords: Dynamic stability, pulse loading, orthotropic plates

1. Introduction

The dynamic behaviour of plates under in–plane pulse loading depends on the quantity of "pulse intensity" – it means on its amplitude and duration. Thus following phenomena may occur:

- impact for pulses of high intensity,
- dynamic buckling for moderate loading amplitudes and pulse duration close to the period of fundamental natural vibrations,
- quasi-static behaviour for pulses of low intensity and/or long duration.

When the second case is considered one should remember that in dynamic buckling the bifurcation load (as it is in static case) does not exist, the phenomenon lies in rapid growth of deflections of imperfect plate. Therefore, it is necessary to define dynamic critical loads on the basis of assumed dynamic buckling criterion and further to determine dynamic load factor DLF as a ratio of dynamic to static buckling load.In world literature one can find many criteria allowing for determining dynamic critical or failure loads. Some of them will be discussed in the following section.

The studies of dynamic buckling of plates started in fifties of the last century. In Volmir's books [14], [15] the solutions, obtained in the analytical–numerical way, were presented for rectangular pulses. The dynamic buckling for pulses of high intensity usually occurred for modes of greater number of half–waves than in static case. The considerations were conducted in unlimited elastic range and under assumption that the shape of initial imperfections is identical as the static buckling mode. Volmir formulated a simple buckling criterion – the dynamic buckling occurs at this mode for which the deflections grow the most rapidly and the dynamic buckling load corresponds to the pulse amplitude at which the maximal deflection equals some chosen value.

The special attention should be paid to the work done by Ari–Gur et al. [3] in which the different approximate criteria allowing for determination of dynamic buckling are discussed and applied. The analysis of dynamic response of columns loaded by a pulse compressive load was conducted theoretically and experimentally. The predominant effect of the initial geometrical imperfections and pulse intensity on the dynamic buckling load was shown while the effect of material properties was found insignificant. The papers of Weller and al. [16], Abramovich and Grunwald [1], Ari–Gur and Simonetta [2] are the further contribution to the problem of dynamic buckling of composite plates. In papers mentioned above the fact that the dynamic buckling loads are not always higher than static ones was strongly underlined.

Cui et al. [4] investigated experimentally the dynamic buckling mechanism of plates under fluid–solid slamming. The effect of boundary conditions was also discussed. In the following paper [5] the experimental results were verified numerically using the computer code ABAQUS.

In a paper of Petry and Fahlbusch [12] the parametric studies of dynamic buckling of isotropic rectangular plates are performed. The large deflection plate equations were solved by Galerkin method using Navier's double Fourier series. The comparison of the dynamic load factor (DLF) determined according to Budiansky-Hutchinson criterion and to the stress failure criterion was presented.

More literature on dynamic buckling of plates can be found in the review paper by Jones [7] and in his book [8].

In this work the dynamic response of thin iso- and orthotropic plates subjected to in–plane pulse loading of different shapes is investigated using computer code ANSYS 8.1.

The influence on the dynamic behaviour of plates of following factors: shape of pulse loading, imperfection sensitivity, time of pulse duration and material properties will be analysed. The dynamic load factor will be estimated basing on different dynamic buckling or failure criteria.

2. Some dynamic buckling/ Failure criteria

Dynamic critical load – it means the amplitude of pulse force which at given duration causes the dynamic buckling - can be very high for plates with small imperfection and much lower for plates with a significant imperfection.

- The simplest criterion was proposed by Volmir [14] – the dynamic critical load corresponds to the amplitude of pulse force (of constant duration) at which the maximal plate deflection is equal to some constant value k ($k =$ one half or one plate thickness).

- In many publications dynamic buckling load is determined on the basis of Budiansky&Hutchinson [6] stability criterion that states: dynamic stability loss occurs when the maximal plate deflection grows rapidly with the small variation of the load amplitude.

The dynamic load factor is then defined as the quotient of the dynamic buckling load and the critical static load for the plate:

$$DLF = \frac{N_{cr}^{dyn}}{N_{cr}^{stat}} \tag{1}$$

- Petry and Fahlbusch [12] presented a dynamic failure criterion for isotropic plates: a dynamic response caused by a pulse load is defined to be dynamic stable if the condition that the effective stress σ_{eff} (found by Huber–Mises formula) is not greater than limit stress σ_L, is fulfilled at every time everywhere in the structure. This criterion seems to be rather conservative as a failure criterion and is valid only for linearly elastic – perfectly plastic materials.

Basing on this dynamic failure criterion one can find the dynamic failure load and then:

$$DLF_{cr} = \frac{N_f^{dyn}}{N_{cr}^{stat}} \qquad \text{or} \qquad DLF_f = \frac{N_f^{dyn}}{N_f^{stat}} \tag{2}$$

where N_f^{stat} - static failure load.

In this paper the authors propose to apply the dynamic failure criterion in case of orthotropic plates. Then the effective stress has to be determined on the basis of Hill's criterion. For plane stress state it can be formulated as follows:

$$a_1 = 1 \tag{3}$$

$$a_2 = \frac{\sigma_{xo}^2}{\sigma_{yo}^2} \tag{4}$$

$$a_3 = \frac{\sigma_{xo}^2}{3\tau_{xyo}^2} \tag{5}$$

$$\sigma_{eff}^2 = a_1\sigma_x^2 + a_2\sigma_y^2 - a_{12}\sigma_x\sigma_y + 3a_3\tau_{xy}^2 \tag{6}$$

where: a_1, $a_2\,a_3$, a_{12}, a_3 - coefficients depending on the uniaxial limit stresses in the principal directions of orthotropy σ_{xo}; σ_{yo} and limit stress in pure shear τ_{xyo} (see Kołakowski&Kowal–Michalska [9]). The effective stress determined by Eq.(2) is compared with the uniaxial limit stress in the direction of loading σ_{xo} .

It should be mentioned that for isotropic materials Hill's condition transforms into well known Huber–Mises formula.

3. Formulation of a problem

In this work the dynamic response of thin rectangular plates subjected to in–plane pulse loading of different shapes is investigated.

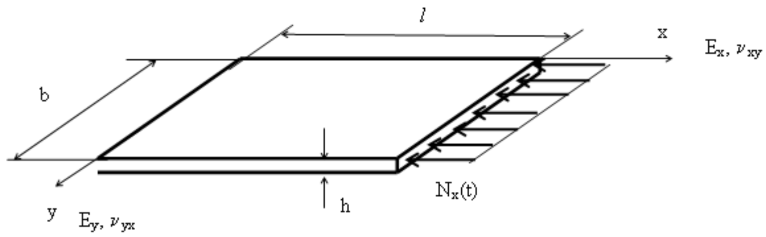

Figure 1 Geometry and loading of a plate

The plates are rectangular with the principal axes of orthotropy parallel to the plate edges (Tab. 1). It is assumed that the all edges are simply supported. The unloaded edges remain straight and parallel during loading. Additionally it is assumed that normal and shear forces disappear along the unloaded edges.

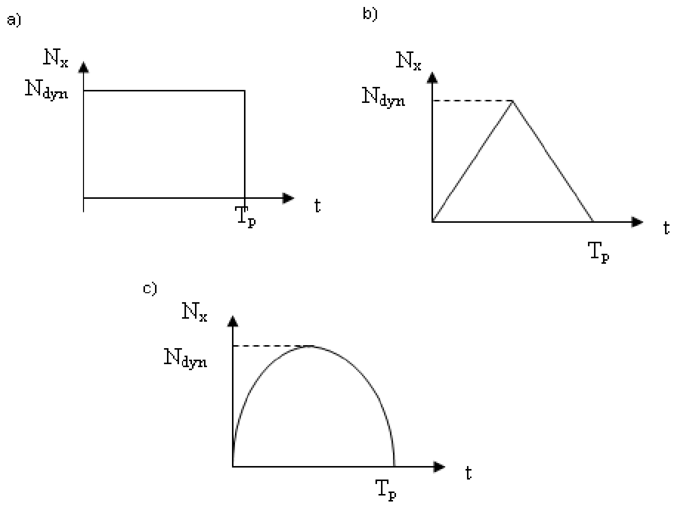

Figure 2 Shapes of pulse loading (in all cases for $t > Tp \rightarrow Nx(t) = 0$)

The shape of initial imperfection should fulfil the boundary conditions along all edges. In numerical calculations different shapes of pulse loading are considered (Fig. 2).

4. Discussion of the results from finite elements analysis

The numerical calculations for square iso- and orthotropic plates of dimensions $b = l = 300\,[\mathrm{mm}]$; $h = 1.5\,[\mathrm{mm}]$ and material properties given in Tab. 1 were conducted using the computer code ANSYS 8.1. The plates were treated as simply supported along all edges, that were kept straight during loading. The simply support boundary conditions in FEM analysis were performed on all edges as zero displacements normal to the surface of the plate and normal to the appropriate edge in the plate surface.

The eight–node shell element SHELL91 of six degrees of freedom at each node was applied. The analysed plates were divided into 50×50 quadrilateral elements in x–y plane.

At the first stage the modal analysis was performed in aim to determine the period of fundamental natural vibrations T (and pulse duration T_p). It was the linear analysis without damping with Block Lanczos extracting method. Next, by linear stability analysis, using eigenvalue method, the critical static load N_{cr} and corresponding buckling mode were determined. The buckling eigen–mode with amplitude w_{0max} was assumed as the initial imperfection shape of a plate. The amplitudes of the pulse force were applied as multiples of the static critical load. The structural dynamic analysis, which allowed to find the response of a plate for pulse loading, was conducted using the "Full Transient Dynamic Analysis" with geometric nonlinearities included (see User's Guide ANSYS [13]). It uses the Newmark time integration scheme with the Newton–Raphson procedure. The integration time step was taken as $1/75 \div 1/50$ of the period T. The initial displacements and velocities were set to zero. For different values of imperfection amplitude the maximal deflection of a plate w_{max} and the values of the effective stress were registered. The results are presented in figures.

Table 1 Material data

Material	$E = E_x$ [GPa]	$\nu = \nu_{xy}$	E_y [GPa]	ν_{yx}	G [GPa]
steel	210	0.3	-	-	-
aluminium	70	0.33	-	-	-
glass-epoxy	53.781	0.25	17.927	0.083	8.964
Material	σ_{x0} [MPa}	σ_{y0} [MPa]	τ_{xy0} [MPa]	ρ [kg/m^3]	
steel	200	-	-	7850	
aluminium	100	-	-	2950	
glass-epoxy	1034	61.7	41.4	2900	

In order to verify the assumed FE model the comparative calculations were conducted. The results obtained were compared with the results found on the basis of two analytical approaches.

The first calculations were conducted for isotropic steel square plate. The plate was loaded by pulse of rectangular shape of duration $T_p = 0.0122\,[\mathrm{s}]$ equal to the period of fundamental natural vibrations of a plate T. For $N_{dyn}/N_{cr} = 2$ and the imperfection $w_{0max}/h = 0.05$ the calculations were performed using ANSYS model and by the analytical–numerical method basing on Koiter's asymptotic solution (for details see: Kowal–Michalska, Kolakowski, Mania [10]). As a first approximation a single mode analysis was applied to the problem and the curve marked by stars was obtained – Fig. 3. The full line curve represents the results obtained by FE analysis (ANSYS).

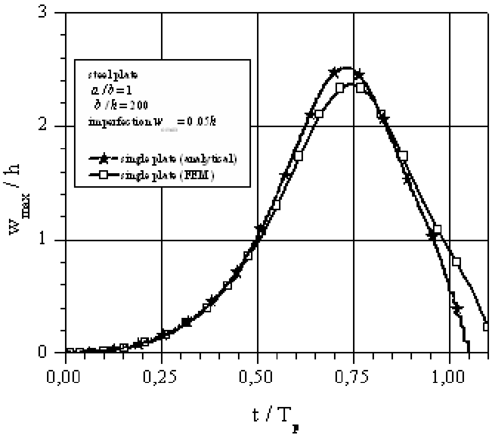

Figure 3 The response of a steel plate for rectangular pulse

Next verifying calculations were made for an aluminium plate loaded by sinusoidal pulse, using the same data as in the paper of Petry and Fahlbusch [12] (Fig. 4).

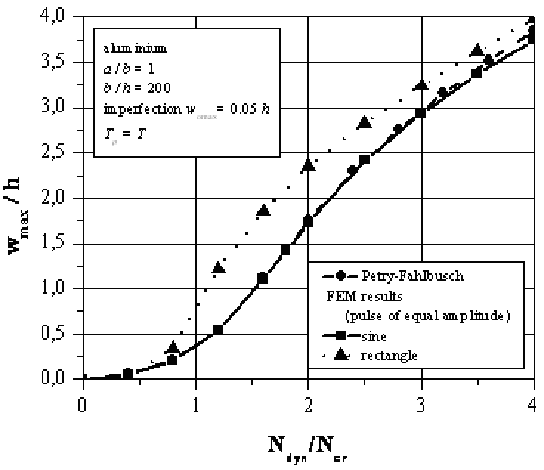

Figure 4 Comparison of the results obtained using ANSYS software and the results obtained by Petry and Fahlbusch

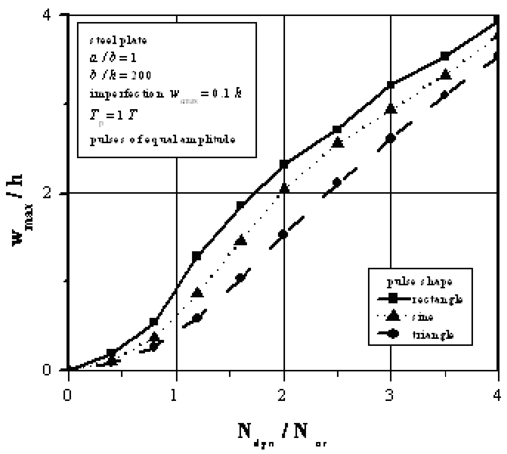

Figure 5 Influence of a pulse shape on the maximal deflection of isotropic plate (pulses of equal amplitude)

As before almost perfect agreement was achieved. Therefore it can be assumed that the proposed model and numerical calculations can be applied to the further dynamic analysis of plate behaviour under pulse loading.

In Fig. 5 the curves showing the maximal dimensionless deflection versus ratio of dynamic load amplitude to the static critical load (N_{dyn}/N_{cr}) for different pulses of equal amplitude and constant time of pulse duration (equal to the period of natural flexural vibrations of a plate). As it can be expected the rectangular impulse causes the fastest growth of the deflection amplitude.

When pulses of equal energy are considered (that means the areas under the rectangular and sinusoidal shapes in Fig. 2 are equal) it can be seen that for pulses of short duration the deflections caused by rectangular loading grow more rapidly but for the duration time equal to the period of natural vibrations ($T_p = T$) larger deflections correspond to the sinusoidal pulse (Fig. 6). The character of dynamic response of a plate changes with the increasing pulse duration T_p.

The fact that the geometric initial imperfections affect the dynamic response of a plate in a great extent (Fig. 7) is well known from the literature. The comparison between the static response of a plate and the dynamic ones for rectangular pulse loading is shown in Fig. 7. It can be seen that for small imperfections ($w_o = 0.005h \div 0.015h$), the curves describing the dynamic deflections intersect the static response and in some range of loading run below it (these results are similar to those obtained by Volmir [15] for very small imperfection amplitude equal 0.001h). With the increase of imperfection amplitudes the dynamic deflections become more pronounced and are much higher than static ones in assumed range of loading.

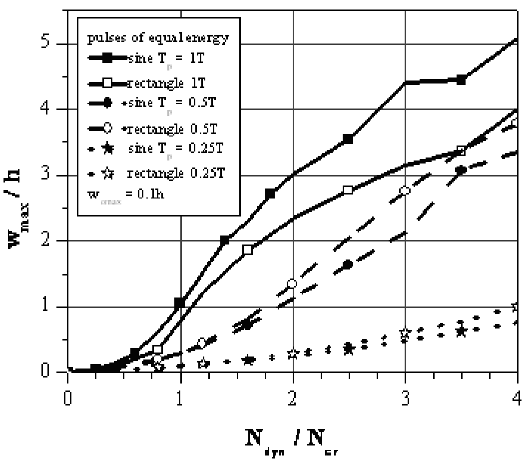

Figure 6 Influence of a pulse shape on the maximal deflection of isotropic steel plate (a/b=1; b/h=200) for different time of pulse duration (pulses of equal energy)

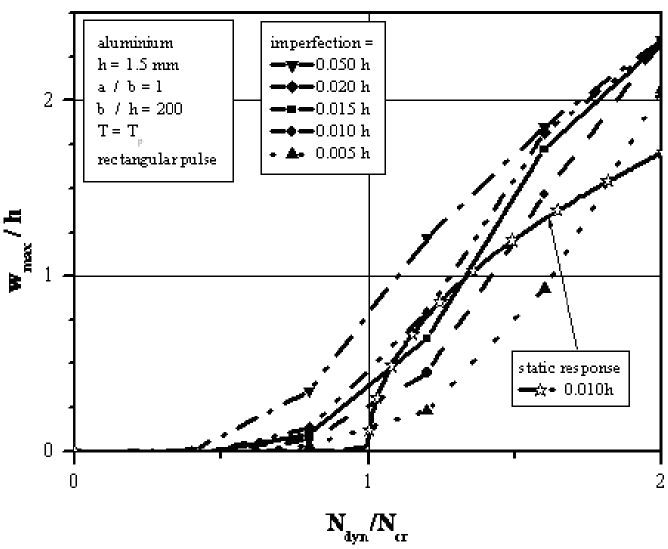

Figure 7 The dynamic response of a plate for a pulse loading for different values of geometric imperfection amplitudes and the static response

According to the dynamic buckling criteria, discussed earlier, the dynamic load factor was determined for a steel plate of material parameters given in Tab. 1. It shows that the values of DLF found on the basis of Volmir's or Budiansky–Hutchinson criterion are very close each other (see Fig. 8). The effective stress reaches the yield limit when the pulse amplitude equals 2.2 of static buckling load.

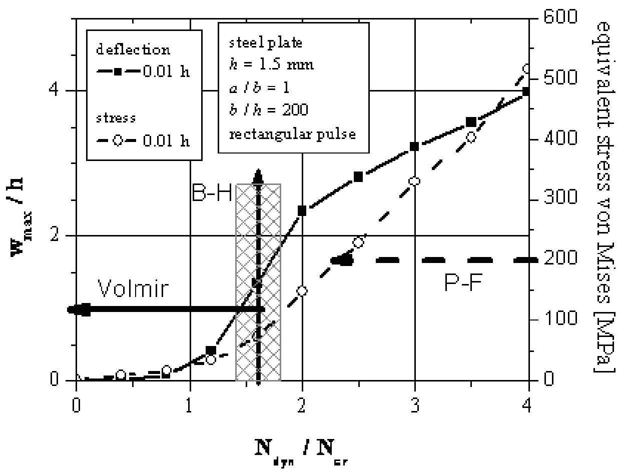

Figure 8 Estimation of DLF value according to assumed dynamic buckling criteria

The similar calculations were conducted for square plates made of orthotropic material of properties given in Table 1. The results presented in Fig. 9 and Fig. 10 were obtained for rectangular pulse loading of duration time T_p equal to the period of natural flexural vibrations of a plate T. It was found that the influence of material parameters on the dynamic response of a plate of the same dimensions and initial imperfections is not significant (see the corresponding curves in Fig. 9 and Fig. 7).

Table 2 Values of dynamic load factor

w_{0max}/h	DLF Volmir's criterion $w_{max} = h$	DLF Budiansky-Hutchinson criterion
0.01	1.45	1.2-1.6
0.05	1.12	0.8-1.2
0.1	1.11	0.8-1.2

From Fig. 9 it can be found that applying Volmir's dynamic stability criterion ($w_{max} = h$) the dynamic buckling load is always greater than static one. Following

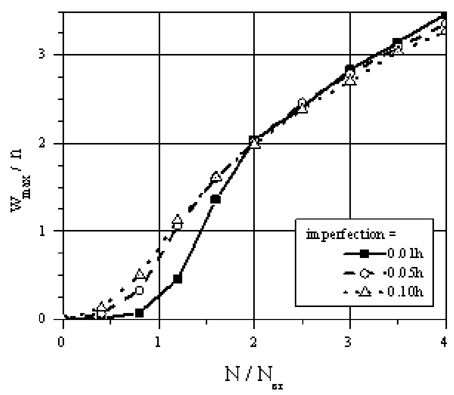

Figure 9 Influence of geometric imperfections on dynamic response of orthotropic plate

Budiansky–Hutchinson's criterion it can be observed that for small imperfection amplitude ($w_o = 0.01h$) the value of DLF (Eq.1) is greater than one but for larger imperfections the dynamic buckling load can be estimated as smaller than static one (see also Table 2).

The non–dimensional effective stress σ_{eff}/σ_{x0} (Eq.3), denoted as the stress ratio in Fig. 10, was determined for the orthotropic plate of initial imperfection $w_{0max} = 0.05h$. This relation allows one to estimate the pulse amplitude for which the effective stress reaches the yield limit in the direction of loading ($N_{dyn} = 1.6\,N_{cr}$).

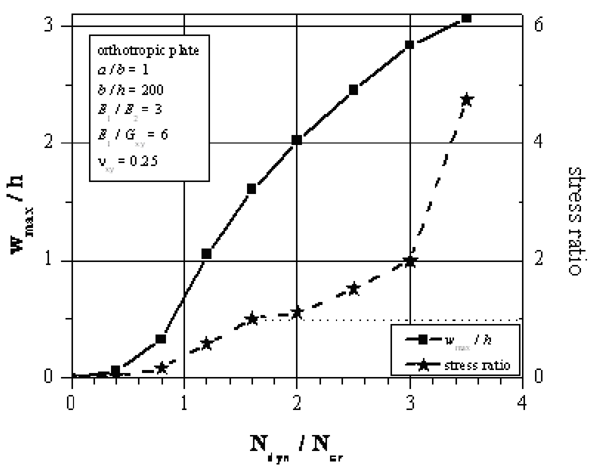

Figure 10 Maximal deflection and stress ratio versus amplitude of pulse loading

5. Final conclusions

The calculations conducted in this paper confirmed the facts well known from the subject literature – the geometric imperfections, shape and duration time of pulse loading are the factors that strongly affect the dynamic behaviour of plates. In most works the influence of pulse shape was investigated under the assumption of equal amplitude at constant duration time for different pulses and then the rectangular pulse always causes the largest deflections. In this paper the pulses of equal energy were also compared. Then it showed that for pulses of short duration the deflections caused by rectangular loading grow more rapidly but for the duration time equal to the period of natural vibrations larger deflections correspond to the sinusoidal pulse.

It should be also noticed that usually the analysis of dynamic stability is performed under the assumption of unlimited elastic range. Taking into account the dynamic failure criterion it can be easily seen that the limit state (determined by the moment when the effective stress reaches the limit stress) appears for rather low values of pulse amplitude.

The influence of material properties on the curves showing the maximal deflections versus non–dimensional pulse amplitude is not significant, so the values of dynamic load factor estimated according to Volmir's or Budiansky–Hutchinson criterion stay close for considered materials (isotropic and orthotropic). As it was shown here the relations describing the effective stress as a function of pulse amplitude and the values of DLF found on the basis of dynamic failure criterion depend strongly on the applied failure criterion and material data.

The purpose of the analysis conducted by FEM was to verify the applied model with the results known from the literature in aim to analyse in future complex thin–walled structures under dynamic pulse loading with application of different dynamic buckling criteria.

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