Mechanics and Mechanical Engineering
Vol. 12, No. 2 (2008) 165–176

Unsteady MHD Couette Flow with Heat Transfer in the Presence of Uniform Suction and Injection

Hazem A. ATTIA
Department of Mathematics, College of Science,
King Saud University, (Al-Qasseem Branch),P.O. Box 237, Buraidah 81999, KSA

Received (10 October 2007)
Revised (5 March 2008)
Accepted (8 June 2008)

The unsteady Couette flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel non–conducting porous plates is studied with heat transfer. An external uniform magnetic field and a uniform suction and injection are applied perpendicular to the plates while the fluid motion is subjected to a constant pressure gradient. The two plates are kept at different but constant temperatures while the Joule and viscous dissipations are included in the energy equation. The effect of the magnetic field and the uniform suction and injection on both the velocity and temperature distributions is examined.

Keywords: Couette flow, magnetohydrodynamic (MHD)

1. Introduction

The magnetohydrodynamic flow between two parallel plates, known as Hartmann flow, is a classical problem that has many applications in magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays. Hartmann and Lazarus [1] studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary, and insulated plates. Then, a lot of research work concerning the Hartmann flow has been obtained under different physical effects [2–10].

In the present study, the unsteady Couette flow and heat transfer of an incompressible, viscous, electrically conducting fluid between two infinite non-conducting horizontal porous plates are studied. The upper plate is moving with a constant velocity while the lower plate is kept stationary. The fluid is acted upon by a constant pressure gradient, a uniform suction and injection and a uniform magnetic field perpendicular to the plates. The induced magnetic field is neglected by assuming a very small magnetic Reynolds number [4, 5]. The two plates are maintained at two different but constant temperatures. This configuration is a good approximation of some practical situations such as heat exchangers, flow meters, and pipes that connect system components. The cooling of these devices can be achieved by utilizing a porous surface through which a coolant, either a liquid or gas, is forced. Therefore, the results obtained here are important for the design of the wall and the cooling arrangements of these devices. The equations of motion are solved analytically using the Laplace transform method while the energy equation is solved numerically taking the Joule and the viscous dissipations into consideration. The effect of the magnetic field and the suction and injection on both the velocity and temperature distributions is studied.

2. Description of the Problem

The two non-conducting plates are located at the $y = \pm h$ planes and extend from $x = -\infty$ to ∞ and $z = -\infty$ to ∞. The lower and upper plates are kept at the two constant temperatures T_1 and T_2, respectively, where $T_2 > T_1$. The fluid flows between the two plates under the influence of a constant pressure gradient dP/dx in the$x$–direction, and a uniform suction from above and injection from below which are applied at $t = 0$. The upper plate is moving with a constant velocity U_o while the lower plate is kept stationary. The whole system is subjected to a uniform magnetic field B_o in the positive y–direction. This is the total magnetic field acting on the fluid since the induced magnetic field is neglected. From the geometry of the problem, it is evident that $\partial/\partial x = \partial/\partial z = 0$ for all quantities apart from the pressure gradient dP/dx, which is assumed constant. The velocity vector of the fluid is

$$v(y,t) = u(y,t)i + v_o j$$

with the initial and boundary conditions $u = 0$ at $t \leq 0$, and $u = 0$ at $y = -h$, and $u = U_o$ at $y = h$ for $t > 0$. The temperature $T(y,t)$ at any point in the fluid satisfies both the initial and boundary conditions $T = T_1$ at $t \leq 0$, $T = T_2$ at $y = h$, and $T = T_1$ at $y = -h$ for $t > 0$. The fluid flow is governed by the momentum equation

$$\rho\frac{\partial u}{\partial t} + \rho v_o\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - \sigma B_o^2 u \tag{1}$$

where ρ, μ and σ are, respectively, the density, the coefficient of viscosity and the electrical conductivity of the fluid. To find the temperature distribution inside the fluid we use the energy equation [11]

$$\rho c\frac{\partial T}{\partial t} + \rho c v_o\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_o^2 u^2, \tag{2}$$

where c and k are, respectively, the specific heat capacity and the thermal conductivity of the fluid. The second and third terms on the right–hand side represent the viscous and Joule dissipations, respectively.

The problem is simplified by writing the equations in the non-dimensional form. We define the following non–dimensional quantities

$$\hat{x} = \frac{x}{h},\ \hat{y} = \frac{y}{h},\ \hat{z} = \frac{z}{h},\ \hat{u} = \frac{u}{U_o},\ \hat{P} = \frac{P}{\rho U_o^2},\ t = \frac{tU_o}{h},$$

$S = v_o/U_o$ – is the suction parameter,
$\Pr = \mu c/k$ – is the Prandtl number,
$Ec = U_o^2/c(T_2 - T_1)$ – is the Eckert number,
$Ha^2 = \sigma B_o^2 h^2/\mu$ – where Ha is the Hartmann number,
In terms of the above non–dimensional variables and parameters, the basic eqs. (1)-(2) are written as (the "hats" will be dropped for convenience)

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} - \frac{Ha^2}{Re}u, \tag{3}$$

$$\frac{\partial T}{\partial t} + S\frac{\partial T}{\partial y} = \frac{1}{Re\Pr}\frac{\partial^2 T}{\partial y^2} + \frac{Ec}{Re}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{EcHa^2}{Re}u^2, \tag{4}$$

The initial and boundary conditions for the velocity become

$$t \le 0 : u = 0,\ t > 0 : u = 0,\ y = -1,\ u = 1,\ y = 1 \tag{5}$$

and the initial and boundary conditions for the temperature are given by

3. Numerical Solution of the Governing Equations

Equations (3) and (4) are solved numerically using finite differences [13] under the initial and boundary conditions (5) and (6) to determine the velocity and temperature distributions for different values of the parameters Ha and S. The Crank-Nicolson implicit method is applied. The finite difference equations are written at the mid–point of the computational cell and the different terms are replaced by their second–order central difference approximations in the y–direction. The diffusion term is replaced by the average of the central differences at two successive time levels. Finally, the block tri–diagonal system is solved using Thomas' algorithm. All calculations have been carried out for $dP/dx = -5$, $\Pr = 1$ and $Ec = 0.2$.

4. Results and Discussion

Figures 1–2 presents the velocity and temperature distributions as functions of y for different values of the time starting from $t = 0$ to the steady state. Figures 1 and 2 are evaluated for$Ha = 1$ and $S = 1$. It is observed that the velocity component u and temperature T reach the steady state monotonically and that u reaches the steady state faster than T. This is expected, since u acts as the source of temperature. Figures 3–4 shows the effect of the Hartmann number Ha on the time development of the velocity u and temperature T at the centre of the channel ($y = 0$). In this figure, $S = 0$ (suction suppressed). It is clear from Fig. 3 that increasing the parameter Ha decreases u and its steady state time. This is due to increasing the magnetic damping force on u. Fig. 4 indicates that increasing Ha increases T at small time but decreases it at large time. This can be attributed to the fact that, for small time, u is small and an increase in Ha increases the Joule dissipation which is proportional to Ha and therefore, the temperature increases. For large time, increasing Ha decreases u and, in turn, decreases the Joule and viscous dissipations and, in turn, decreases T. This accounts for crossing the curves of T with time for various values of Ha.

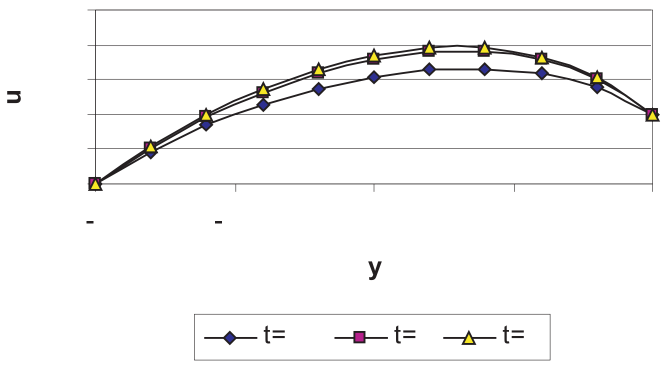

Figure 1 Time development of the profil of u, $Ha = 1$ and $S = 1$

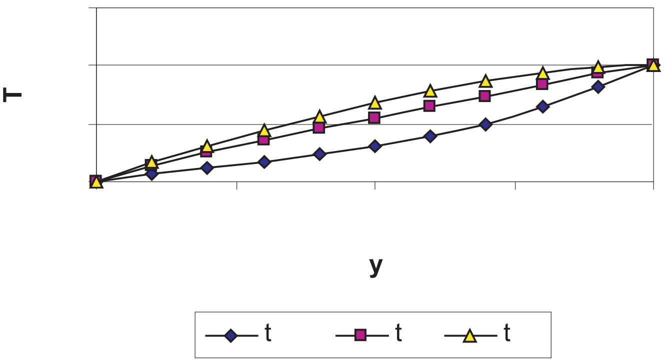

Figure 2 Time development of the profil of T, $Ha = 1$ and $S = 1$

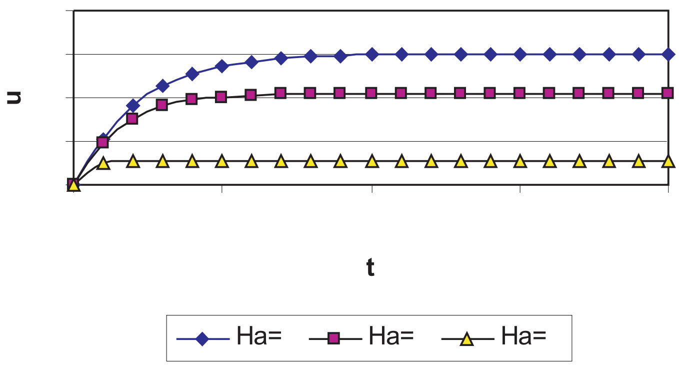

Figure 3 Effect of Ha on the time variation of u at $y = 0$, $S = 0$

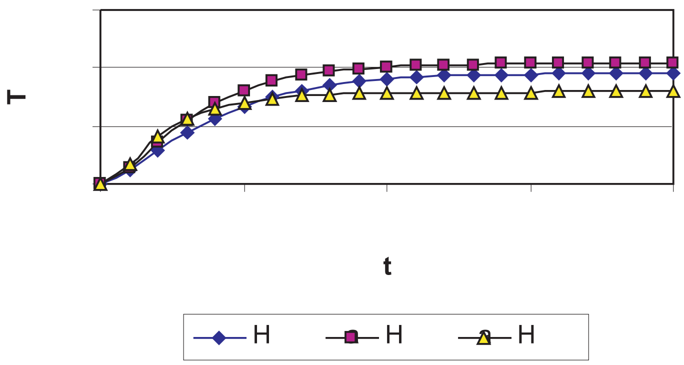

Figure 4 Effect of Ha on the time variation of T at $y = 0$, $S = 0$

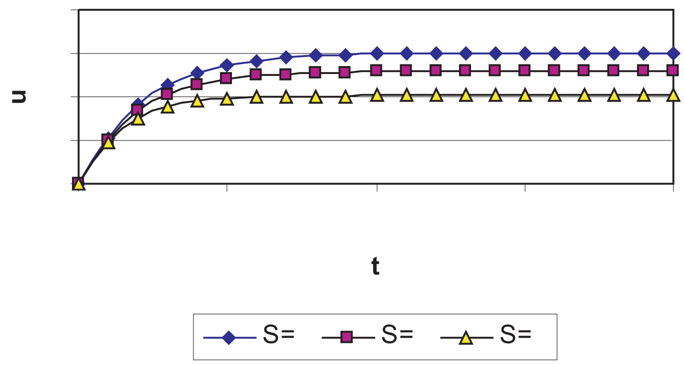

Figure 5 Effect of S on the time variation of u at $y = 0$, $Ha = 0$

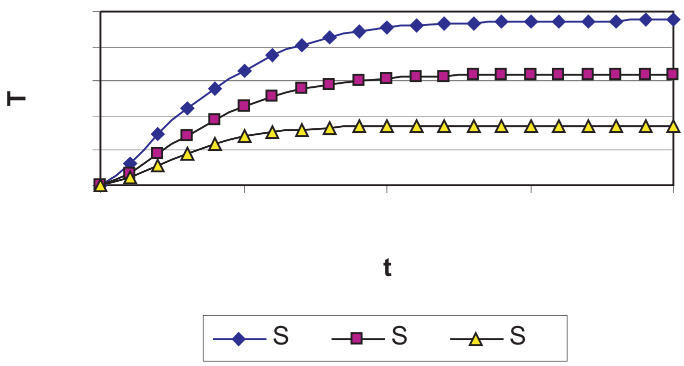

Figure 6 Effect of S on the time variation of T at $y = 0$, $Ha = 0$

Figures 5–6 shows the effect of the suction parameter on the time development of the velocity u and temperature T at the centre of the channel ($y = 0$). In this figure, $Ha = 0$ (hydrodynamic case). In Fig. 5, it is observed that increasing the suction decreases the velocity u at the center and its steady state time due to the convection of fluid from regions in the lower half to the center, which has higher fluid speed. In Fig. 6, the temperature at the center is affected more by the convection term, which pumps the fluid from the cold lower half towards the centre.

$$t \leq 0 : T = 0,\, t > 0 : T = 1,\, y = 1,\, T = 0,\, y = -1. \tag{6}$$

5. Conclusion

The unsteady Couette flow of a conducting fluid under the influence of an applied uniform magnetic field has been studied in the presence of uniform suction and injection. The effect of the magnetic field and the suction and injection velocity on the velocity and temperature distributions has been investigated. It is found that both the magnetic field and suction or injection velocity has a marked effect on both the velocity and temperature distributions. It is of interest to see that the effect of the magnetic field on the temperature at the center of the channel depends on time. For small time, increasing the magnetic field increases the temperature, however, for large time, increasing the magnetic field decreases the temperature.

References

[1] **Hartmann, J. and Lazarus, F.**: Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 15(6,7), **1937**.

[2] **Tao, I. N.**, *J. of Aerospace Sci.* 27, 334, **1960**.

[3] **Alpher, R. A.**,*Int. J. Heat and Mass Transfer*, 3, 108, **1961**.

[4] **Sutton, G. W. and Sherman, A.**: Engineering Magnetohydrodynamics, *McGraw-Hill Book Co.*, **1965**.

[5] **Cramer, K. and Pai, S. I.**: Magnetofluid dynamics for engineers and applied physicists, *McGraw-Hill Book Co.*, **1973**.

[6] **Nigam, S. D. and Singh, S. N.**, *Quart. J. Mech. Appl.*, Math. 13, 85, **1960**.

[7] **Tani, I.**, *J. of Aerospace Sci.* 29, 287, **1962**.

[8] **Soundalgekar, V. M., Vighnesam, N. V. and Takhar, H. S.**, *IEEE Trans. Plasma Sci.*, PS-7(3), 178, **1979**.

[9] **Soundalgekar, V. M. and Uplekar, A. G.**, *IEEE Trans. Plasma Sci.* PS-14(5), 579, **1986**.

[10] **Attia, H. A.**, *Mech. Res. Comm.*, 26(1), 115, **1999**.

[11] 11. H. Schlichting. Boundary layer theory. *McGraw-Hill Book Co.*, **1968**.

[12] **Spiegel, M. R.**: Theory and problems of Laplace transform, *McGraw-Hill Book Co.*, **1986**.

[13] **Ames, W. F.**: Numerical solutions of partial differential equations, 2nd ed., *Academic Press*, New York. **1977**.