Mechanics and Mechanical Engineering
Vol. 12, No. 2 (2008) 171–184

Thermal Instability in a Rotating Micropolar Viscoelastic Fluid Layer Under the Effect of Electric Field

Mohamed I. A. Othman
Sayed A. Zaki
Faculty of Education, Department of Mathematics,
Salalah-211, P.O. Box 2801, Sultanate of Oman
e-mail: m_i_othman@yahoo.com

Received (18 April 2007)
Revised (31 May 2007)
Accepted (8 November 2008)

The problem of onset of convective instability in a dielectric micropolar viscoelastic fluid (Walters' liquid B') heated from below confined between two horizontal plates under the simultaneous action of the rotation of the system, vertical temperature gradient, one relaxation time and vertical electric field is considered. Linear stability theory is used to derive an eigenvalue of twelve order, and an exact eigenvalue equation for a neutral instability is obtained. Under somewhat artificial boundary conditions, this equation can be solved exactly to yield the eigenvalue relationship from which various critical values are determined in detail. Critical Rayleigh heat numbers and wave number for the onset of instability are presented graphically as a function of rotation at a certain value of the Prandtl number, for various values of the relaxation time, the Rayleigh electric number, the elastic parameter and micropolar parameters.

Keywords: Instability, viscoelastic, rotation, micropolar, the power series method.

1. Introduction

In recent years, using the theory of micropolar fluids developed by Eringen [1,2], several authors [3-5] have investigated problems related to stability and turbulence. As the theory of micropolar fluids encompass a wide variety of fluids (for example: liquid crystals, polymers, animal blood, etc.), in which randomly oriented bar like elements, dumbbell molecules or spherical particles are present, and as each volume element of the fluid undergoes translation as well as rotation, the analysis of the problems of stability revealed a number of interesting physical phenomena which are unseen in Newtonian fluids.

Initiating the study of thermal instability of a micropolar fluid layer heated from below, Ahmadi [6] has shown that there exists cellular convection at the onset of instability. Assuming that the boundaries are free from shear stress and microrotation, he has obtained an analytical solution in the case of free boundaries. His analysis shows that the micropolar fluids are more stable than Newtonian ones. Datta and Sastry [7] have extended the analysis of Ahmadi to the case of heat conducting micropolar fluids. They have found that the heat induced by microrotation causes instability of the layer, whether the fluid is heated from below or above. The instability for heating from above is quite a novel phenomenon as it does not have analogous in Newtonian fluid. While analysing the problem of convective instability of a micropolar fluid layer confined between rigid boundaries, Walzer [8] has mentioned that the analysis of the instability finds applications in the area of Geophysics, for example, in understanding the phenomenon of rising of valconic liquid with bubbles, and convective process inside the earth's mantle. However, he has concluded his analysis without any calculation of eigenvalue. Rama Rao [9] has examined the onset of instability in a heat conducting micropolar fluid layer confined between rigid boundaries. On obtaining a numerical solution of the eigenvalue problem, he has shown that, in the case of adverse temperature gradient, the convective cells at the onset of instability are more elongated than those in the case of positive temperature gradient.

The effect of rotating on thermal convection in micropolar fluids is important in certain chemical engineering and biochemical situations. Sharma and Kumar [10] studied the effect of uniform rotation on thermal instability micropolar fluid. They found that the present of coupling between thermal and micropolar effect might introduce oscillatory motion in the system.

In technological field there exists an important class of fluids, called non-Newtonian fluids, which are also being studied extensively because of their practical applications. One such fluid is called viscoelastic fluid. Walters [11] and Beard and Walters [12] deduced the governing equations for the boundary flow for a prototype viscoelastic fluid which they have designated as liquid B' when this liquid has a very short memory. Singh and Singh [13] have studied the magneto-hydrodynamic flow of a viscoelastic fluid past an accelerated plate. Othman [14] has studied the stability of a rotating layer of viscoelastic dielectric liquid (Walters' liquid B') heated from below. Othman [15] investigated the convective stability of a horizontal layer of viscoelastic conducting liquid (Walters' liquid B') heated from below and rotating about a vertical axis in the presence of a magnetic field and thermal relaxation. In these works, more general model of magneto-hydrodynamic free convection flow which also includes the relaxation time of heat convection and the electric permeability of the electromagnetic field are used. The inclusion of the relaxation time and electric permeability modify the governing thermal and electromagnetic equations, changing them from parabolic to hyperbolic type, and there by eliminating the unrealistic result that thermal disturbance is realized instantaneously everywhere within a fluid.

An important stability problem is the thermal convection in a horizontal thin layer of fluid heated from below. A detailed account of thermal convection in a horizontal thin layer of Newtonian fluid heated from below, under varying assumptions of hydrodynamics, has been given by Chandrasekhar [16]. Othman [17] analyzed the problem of the onset of stability in a horizontal layer of viscoelastic dielectric fluid (Walters' liquid B') under the simultaneous action of a vertical ac electric field and a vertical temperature gradient without rotation.

In the present paper our object is to study the thermal instability of a rotating heat conducting micropolar viscoelastic fluid layer confined between rigid boundaries in the presence of ac electric field and thermal relaxation. Hear, we employ the basic equations of the heat conducting micropolar viscoelastic fluid referred to a rotating frame.

2. Formulation of the problem

We consider an incompressible, dielectric and infinite micropolar viscoelastic fluid layer confined between two horizontal surfaces separated by a distance L. Choosing the origin on the lower boundary, let us introduce the Cartesian co-ordinate system x, y, z in which z is measurement at right angles to the boundaries. Let the system be rotating (round the z-axis) with a uniform angular velocity $\Omega = (0, 0, \Omega)$. The lower bounding surface at $z = 0$ and the upper bounding surface at $z = L$ are maintained at constant temperatures T_0 and T_1, respectively. The lower surface is grounded and the upper surface is kept at high alternating (60 HZ) potential whose root-mean-square value is ϕ_1.

Under the foregoing assumptions the basic equations can be written as Othman [17]

$$\frac{\partial v_i}{\partial x_i} = 0 \tag{1}$$

$$\begin{aligned} \rho\left[\frac{\partial v_i}{\partial t} + v_k\frac{\partial v_i}{\partial x_k}\right] &= \rho g_i - \frac{\partial P}{\partial x_i} + \eta_0\frac{\partial^2 v_i}{\partial x_k \partial x_k} - f_e \\ &\quad -K_0[\frac{\partial}{\partial t}\left(\frac{\partial^2 v_i}{\partial x_k \partial x_k} +\right) v_m\frac{\partial^3 v_i}{\partial x_m \partial x_k \partial x_k} \\ &\quad -\frac{\partial v_i}{\partial x_m}\frac{\partial^2 v_m}{\partial x_k \partial x_k} - 2\frac{\partial v_m}{\partial x_k}\frac{\partial^2 v_i}{\partial x_m \partial x_k}] \\ &\quad +2e_{ijk}\rho v_j \Omega_k + k e_{ijk}\frac{\partial \sigma_i}{\partial x_j} \end{aligned} \tag{2}$$

$$\rho j\left[\frac{\partial}{\partial t} + (v.\nabla)\right]\sigma = (\alpha+\beta)\nabla(\nabla.\sigma) + \gamma\nabla^2\sigma + k(\nabla\wedge v) - 2k\sigma \tag{3}$$

$$\rho C_v\left[\frac{\partial}{\partial t} + (v.\nabla)\right]T = k_v\nabla^2 T + \delta\nabla T.(\nabla\wedge\sigma) + \rho C_v \tau_{\frac{\partial}{\partial t}}\left[\frac{\partial}{\partial t} + v\nabla)\right]T \tag{4}$$

$$\nabla.(\varepsilon E) = 0\,, \tag{5}$$

and

$$\nabla\wedge E = 0 \text{ or } E = -\nabla\phi\,. \tag{6}$$

where, f_{ei} is the force of electric origin which may be expressed as Landau and Lifshitz [18]

$$f_{ei} = \rho_e E_i - \frac{1}{2}E^2\frac{\partial\varepsilon}{\partial x_i} + \frac{1}{2}\frac{\partial}{\partial x_i}\left(\rho\frac{\partial\varepsilon}{\partial\rho E^2}\right) \tag{7}$$

taking into account the fact that the free charge density ρ_e is zero.

If we replace the pressure

$$P^* = P - \frac{1}{2}\rho\frac{\partial\varepsilon}{\partial\rho}E^2 \tag{8}$$

The electrostriction term disappear from Eq. (**??**) which can be rewritten in the form:

$$\rho\left[\frac{\partial\, v_i}{\partial t} + v_k\frac{\partial v_i}{\partial\, x_k}\right] = \rho g_i - \frac{\partial\, P^*}{\partial x_i} + \eta_0\frac{\partial^2 v_i}{\partial x_k \partial x_k} - \frac{1}{2}E^2\frac{\partial\varepsilon}{\partial x_i} - K_0\left[\frac{\partial}{\partial t}\,\frac{\partial^2 v_i}{\partial x_k \partial x_k} + \right.$$

$$\left. v_m\frac{\partial^3 v_i}{\partial x_m \partial x_k \partial x_k} - \frac{\partial v_i}{\partial x_m}\frac{\partial^2 v_m}{\partial x_k \partial x_k} - 2\frac{\partial v_m}{\partial x_k}\frac{\partial^2 v_i}{\partial x_m \partial x_k}\right] +$$

$$2e_{ijk}\rho v_j \Omega_k + k e_{ijk}\frac{\partial \sigma_i}{\partial x_j}\,. \tag{9}$$

The mass density and the dielectric constant are assumed to be functions of temperature as follows:

$$\rho = \rho_0\left[1 - \alpha_0(T - T_0)\right]\,, \quad \alpha_0 > 0 \tag{10}$$

$$\varepsilon = \varepsilon_0\left[1 - e(T - T_0)\right]\,, \quad e > 0 \tag{11}$$

where α_0 and e are usually positive.

It is clear that there exist the following steady solutions (denoted by an over bar):

$$\bar{u} = \bar{v} = \bar{w} = 0\,, \tag{12}$$

$$\bar{\sigma} = 0\,, \tag{13}$$

$$\overline{T} = T_0 - \beta_0 z\,, \tag{14}$$

$$\bar{\rho} = \rho_0[1 + \alpha_0\beta_0 z]\,, \tag{15}$$

$$\bar{\varepsilon} = \varepsilon_0[1 + e\beta_0 z]\,, \tag{16}$$

$$\bar{E}_x = 0, \quad \bar{E}_y = 0, \quad \bar{E}_z = \frac{E_0}{1 + e\beta_0 z}\,, \tag{17}$$

$$\bar{\phi} = -\frac{E_0}{e}\log(1 + e\beta_0 z)\,. \tag{18}$$

where,

$$\beta_0 = \frac{T_0 - T_1}{L}\,, \tag{19}$$

$$E_0 = -\frac{\phi e \beta_0}{\log(1 + e\beta_0 z)}\,. \tag{20}$$

are the adverse temperature gradient and the root mean square value of the electric field at $z = 0$. If necessary, the modified pressure $\bar{P}^*$ can be determined from

$$0 = \bar{\rho} g_i - \frac{\partial \bar{P}^*}{\partial x_i} - \frac{1}{2}\bar{E}_z^2\frac{\partial\varepsilon}{\partial x_i}\,. \tag{21}$$

Let this initial steady state be slightly perturbed where the simple relation $\psi = \bar{\psi} + \psi'$ can be expressed any physical quantities ψ after perturbation and prime refers to perturbed quantities. Following the usual steps of linear stability theory we can obtain the following main equations:

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0\,, \tag{22}$$

$$\begin{aligned}\left(\frac{\partial}{\partial t} - \upsilon\nabla^2\right)\nabla^2 w' - \alpha g + \frac{\varepsilon_0 e^2 E_0^2 \beta}{\rho_0}\nabla_1^2 T' - \frac{\varepsilon_0 e E_0 \beta}{\rho_0}\nabla_1^2 \frac{\partial \phi'}{\partial z'} \\ + \frac{K_0}{\rho_0}\frac{\partial}{\partial t'}\nabla^4 w' + 2\Omega\frac{\partial \zeta}{\partial z} - \frac{k}{\rho_0}\nabla^2 \Omega_3' &= 0\,,\end{aligned} \tag{23}$$

$$(\frac{\partial}{\partial t'} - \upsilon\nabla^2)\zeta = 2\Omega\frac{\partial w'}{\partial z} - \frac{K_0}{\rho_0}\frac{\partial}{\partial t'}\nabla^2\zeta\,, \tag{24}$$

$$\rho_0 j\frac{\partial \Omega_3}{\partial t'} = \gamma\nabla^2\Omega_3 - k[\nabla^2 w' + 2\Omega_3]\,, \tag{25}$$

$$\rho_0 c_v\left(1 + \tau_0\frac{\partial}{\partial t'}\right)\left[\frac{\partial T'}{\partial t'} - \beta_0 w'\right] = k_v\nabla^2 T' - \delta\beta_0\Omega_3\,, \tag{26}$$

$$\nabla^2\phi' + eE_0\frac{\partial T'}{\partial z'} = 0],. \tag{27}$$

where, $\zeta = \frac{\partial v'}{\partial x'} - \frac{\partial u'}{\partial y'} = \nabla \wedge v_z$ is the z-component of vorticity.

$$\Omega_3 = \frac{\partial \sigma_2'}{\partial x'} - \frac{\partial \sigma_1'}{\partial y'} = \nabla \wedge \sigma_z\,, \quad \nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\,, \quad \nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}\,.$$

The boundary conditions appropriate for the problem are given by [10]

$$w' = \frac{\partial^2 w'}{\partial z'^2} = T' = \varphi' = \frac{\partial \zeta}{\partial z'} = \Omega_3 = 0 \text{ at } z' = 0, L\,. \tag{28}$$

Now, introducing the nondimensional variables given by L, $\frac{k_v}{L}$, $\frac{L^2}{k_v}$, βL, $\frac{k_v}{L^2}$, $eE_0\beta_0 L^2$, $\frac{k_v}{L^3}$, and $\Omega = \frac{k_v}{L^2}$ as units of length, velocity, time, temperature, vorticity, electro potential, microrotation and rotation of the fluid respectively, we obtain the equations governing the disturbances as:

$$\begin{aligned}\left(P_r^{-1}\frac{\partial}{\partial t} - \nabla^2\right)\nabla^2 w - (R_H + R_E)\nabla_1^2 T - R_E\nabla_1^2\frac{\partial \phi}{\partial z} \\ + K_0^* P_r^{-1}\frac{\partial}{\partial t}\nabla^4 w + 2\Omega\frac{\partial \zeta}{\partial z} - K\nabla^2\Omega_3 &= 0\,,\end{aligned} \tag{29}$$

$$\left(\frac{\partial}{\partial t} - P_r\nabla^2\right)\zeta = 2P_r\Omega\frac{\partial w}{\partial z} - K_0^*\frac{\partial}{\partial t}\nabla^2\zeta\,, \tag{30}$$

$$\bar{j}\frac{\partial \Omega_3}{\partial t} =_0 \nabla^2\Omega_3 - K_1[\nabla^2 w + 2\Omega_3]\,, \tag{31}$$

$$\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} - \nabla^2 T = \left(1 + \tau_0\frac{\partial}{\partial t}\right)w - \bar{\delta}\Omega_3\,, \tag{32}$$

$$\nabla^2\phi + \frac{\partial T}{\partial z} = 0\,. \tag{33}$$

where,

$$R_H = \frac{\alpha g \beta L^4}{k_v \nu}$$

is the Rayleigh heat number,

$$R_E = \frac{\varepsilon_0 e^2 E_0^2 \beta^2 L^4}{\rho_0 k_v \nu}$$

is the Rayleigh electric number,

$$P_r = \frac{\nu}{k_v}$$

is the Prandtl number,

$$\bar{j} = \frac{j}{L^2}\,, \quad C_0 = \frac{\gamma}{\rho_0 L^2 k_v}\,, \quad K_1 = \frac{k}{\rho_0 k_v}\,, \quad \bar{\delta} = \frac{\delta}{\rho_0 c_v L^2}\,.$$

3. Normal mode analysis

Following the normal mode analysis we assume that the solutions of Eqs. (29-33) are given by:

$$[w, \zeta, T, \phi, \Omega_3] = [W(z), Z(z), \Theta(z), \Phi(z)\,, G(z)] \exp\left[ct + i(ax + by)\right] \tag{34}$$

where, $\lambda = \sqrt{a^2 + b^2}$ is the wave number and c is the stability parameter which is, in general, a complex constant. For solutions having the dependence of the form (34), Eqs. (29–33) yield

$$\left[P_r^{-1}c - D^2 - \lambda^2)\right](D^2 - \lambda^2)W + \lambda^2(R_H + R_E)\Theta + \lambda^2 R_E D\Phi +$$

$$K_0^* P_r^{-1} c(D^2 - \lambda^2)^2 W + 2\Omega DZ - K(D^2 - \lambda^2)G = 0\,, \tag{35}$$

$$\left[c - P_r(D^2 - \lambda^2\right] Z = 2P_r\Omega DW - K_0^* c(D^2 - \lambda^2)\zeta\,, \tag{36}$$

$$\left[(\ell c + 2A) - (D^2 - \lambda^2\right] G = -A(D^2 - \lambda^2)W\,, \tag{37}$$

$$\left[c(1 + \tau_0 c) - (D^2 - \lambda^2)\right] \Theta = (1 + \tau_0 c)W - \bar{\delta} G\,, \tag{38}$$

$$(D^2 - \lambda^2)\Phi + D\Theta = 0\,. \tag{39}$$

where

$$\ell = \frac{\bar{j}A}{K_1}\,, \quad A = \frac{K_1}{C_0}\,, \quad D = \frac{d}{dz}\,. \tag{40}$$

In seeking solutions of these equations we must impose certain boundary conditions at the lower surface $z = 0$ and the upper surface $z = 1$. The most realistic boundary conditions may be written as

$$W = DW = \Theta = \Phi = Z = G = 0\,, \text{ at } z = 0, 1\,. \tag{41}$$

In this paper, however, we shall use somewhat different boundary conditions given by [21]

$$W = D^2W = \Theta = \Phi = DZ = G = 0 \text{ at } z = 0, 1\,. \tag{42}$$

This case, although admittedly an artificial one to consider, is of importance since its exact solution is readily obtained. Furthermore, from past experience with problems of this kind (see for example, Chandrasekhar [16] and Turnbull [20]), one may feel fairly confident that the general features of the physical situation will be disclosed by a discussion of this case equally as well as by a discussion of solutions satisfying less artificial boundary conditions.

Eliminating Z, Θ, Φ and G from Eqs. (35–39), we obtain:

$$\begin{gathered}\big\{[c+(K_0^*c-P_r)(D^2-\lambda^2)][\ell c+2A-(D^2-\lambda^2)][P_r^{-1}c-(D^2-\lambda^2)][c(1+\tau_0 c)\\ -(D^2-\lambda^2)](D^2-\lambda^2)^2+\lambda^2(R_H+R_E)(1+\tau_0 c)x[c+(K_0^*c-P_r)(D^2-\lambda^2)]\,.\\ [\ell c+2A-(D^2-\lambda^2)](D^2-\lambda^2)+\bar{\delta}\lambda^2 A(R_H+R_E)[c+K_0^*c-P_r)(D^2-\lambda^2)]\,.\\ (D^2-\lambda^2)^2+K_0^*P_r^{-1}c[c+(K_0^*c-P_r)(D^2-\lambda^2)][c(1+\tau_0 c)-(D^2-\lambda^2)]\,.\\ [\ell c+2A-(D^2-\lambda^2](D^2-\lambda^2)^3-\lambda^2 R_E(1+\tau_0 c[c+(K_0^*c-P_r)(D^2-\lambda^2)]\,.\\ [\ell c+2A-(D^2-\lambda^2)]D^2-\bar{\delta}\lambda^2 AR_E[c+(K_0^*c-P_r)(D^2-\lambda^2)](D^2-\lambda^2)D^2\\ +4\Omega^2 P_r[\ell c+2A-(D^2-\lambda^2)][c(1+\tau_0 c)-(D^2-\lambda^2)](D^2-\lambda^2)D^2\\ +KA[c+(K_0^*c-P_r)(D^2-\lambda^2)][c(1+\tau_0 c)-(D^2-\lambda^2)](D^2-\lambda^2)^3\big\}\,W=0\,.\end{gathered} \tag{43}$$

It can be shown from equation (43) that all even order derivatives of W vanish on the boundaries. The proper solution for W characterizing the lowest mode is:

$$W=W_0\sin\pi z\,, \tag{44}$$

where W_0 is a constant. Substituting (44) in (43) and putting $\pi^2+\lambda^2=b$, we obtain:

$$\begin{gathered}\lambda^2 R_H\left\{\bar{\delta}b^2A[c-b(K_0^*c-P_r)]-b(1+\tau_0 c)[c-b(K_0^*c-P_r)][\ell c+2A+b]\right\}\\ =\lambda^2 R_E\left\{b(1+\tau_0 c)[c-b(K_0^*c-P_r)][\ell c+2A+b]-\bar{\delta}b^2A[c-b(K_0^*c-P_r)]\right.\\ \left.-\pi^2(1+\tau_0 c)[c-b(K_0^*c-P_r)][\ell c+2A+b]+\bar{\delta}bA\pi^2[-b(K_0^*c-P_r)]\right\}\\ +K_0^*b^3P_r^{-1}c[c-b(K_0^*c-P_r)][c(1+\tau_0 c)+b][\ell c+2A+b]+KAb^3[c-b(K_0^*c-P_r)]\,.\\ [c1+\tau_0 c)+b]-b^2[c-b(K_0^*c-P_r)][\ell c+2A+b][cP_r^{-1}+b][c(1+\tau_0 c)+b]\\ -4\pi^2\Omega^2P_rb[\ell c+2A+b][c(1+\tau_0 c)+b]\,.\end{gathered} \tag{45}$$

4. Overstability motions

Since c is, in general, a complex constant we put $c=c_r+i\omega$, where c_r and ω are real. The marginal state is reached when $c_r=0$; if $\omega=0$, one says that principle of exchange of stabilities is valid otherwise we have overstability and then $c=i\omega$ at marginal stability.

Putting $c=i\omega$ in equation (44), the real and imaginary parts of equation (45) yield:

$$R=X+i\omega Y \tag{46}$$

There, X and Y are real-valued functions of P_r, τ_0, λ, Ω, R_E, A, K, $\bar{\delta}$, K_0^*, ℓ and ω, and explicit expansions for these functions are follows:

$$X = \frac{C_1 C_3 + \omega^2 C_2 C_4}{\lambda^2 (C_1^2 + C_2^2)}\,, \tag{47}$$

$$Y = \frac{C_1 C_4 + C_2 C_3}{\lambda^2 (C_1^2 + C_2^2)}\,. \tag{48}$$

where

$$\begin{aligned} C_1 \;=\;& \left[P_r(\bar{\delta}A - 1) - \omega^2 \tau_0 K_0^*\right] b^3 + \left[\omega^2 \tau_0 - \omega^2 K_0^*(\ell + 2A\tau_0) + P_r(\ell\omega^2\tau_0 - 2A)\right] b^2 \\ & + \omega^2(\ell + 2A\tau_0) b\,, \end{aligned} \tag{49}$$

$$\begin{aligned} C_2 \;=\;& \left[-K_0^*(\bar{\delta}A - 1) - P_r\tau_0\right] b^3 + \left[(\bar{\delta}A - 1) + K_0^*(2A - \ell\omega^2\tau_0) - P_r(\ell + 2A\tau_0)\right] b^2 \\ & + (\ell\omega^2\tau_0 + 2A) b\,, \end{aligned} \tag{50}$$

$$\begin{aligned} C_3 = {}& (\omega^2 K_0^{*2} P_r^{-1} - P_r) b^6 + \big[A P_r(K-2) - \omega^2 K_0^*(2P_r^{-1} + 2 + \omega^2 K_0^* \tau_0 P_r^{-1} \\ & -2AK_0^* P_r^{-1} + 2\ell) + \omega^2 P_r \tau_0\big] b^5 + \big[\omega^4 K\omega_0^*(2\ell\,\tau_0 - K\omega_0^* \ell P_r^{-1} - 2K\omega_0^* A P_r^{-1}\tau_0 + 2P_r^{-1}\tau_0) \\ & -4\omega^2 K\omega_0^* A(1 + P_r^{-1}) + \omega^2 P_r(\ell + 2A\tau_0) + \omega^2(1 + 2\ell) + \omega^2 AK(K\omega_0^* - P_r\tau_0)\big] b^4 \\ & + \big[\lambda^2 r_E(P_r - \bar{\delta}AP_r + \omega^2 K\omega_0^{*2}\tau_0) + 2\omega^4 K\omega_0^* P_r^{-1}(\ell + 2A\tau_0) - \omega^4 \ell\tau_0(1 + P_r^{-1}) \\ & + \omega^2(4A - \omega^2 P_r^{-1}\tau_0 + 2P_r^{-1}A - K\omega A - 4\pi^2\omega^2 P_r\big] b^3 + \big\{\lambda^2 r_E\,[\,\omega^2 K\omega_0^*(\ell + 2A\tau_0) \\ & -\omega^2\tau_0(1 + P_r\ell) + 2AP_r - \pi^2(P_r + \omega^2 K\omega_0^*\tau_0 - \bar{\delta}AP_r)\big] - \omega^4 P_r^{-1}(\ell + 2A\tau_0) \\ & + 4\pi^2\Omega^2 P_r(\omega^2\tau_0 - 2A)\big\} b^2 + \big\{\lambda^2 r_E\left[\omega^2\pi^2(\tau_0 - K_0^*\ell - 2KA\tau_0 + P_r\ell\tau_0)\right. \\ & \left. -\omega^2(\ell + 2A\tau_0) - 2\pi^2 P_r A\right] + 4\pi^2\omega^2\Omega^2 P_r(\ell + 2A\tau_0)\big\} b + \pi^2\lambda^2\omega^2 r_E(\ell + 2A\tau_0)\,, \end{aligned} \tag{51}$$

$$\begin{aligned} C_4 = {}& 2K_0^{*2} b^6 + \left[(1 + \ell)(\omega^2 K\omega_0^{*2} P_r^{-1} - P_r) + K\omega_0^{*2}(4A - 2\omega^2\tau_0 - K\omega A) - 2\right] b^5 \\ & + \big[\omega^2 K\omega_0^{*2} P_r^{-1}(2A - \omega^2\ell\,\tau_0) - \omega^2 K\omega_0^*(2\ell + 2P_r^{-1} + \ell P_r^{-1} - K\omega A\tau_0) + \omega^2(2\tau_0 - \ell P_r^{-1} \\ & + \ell P_r\tau_0) + A(K - KP_r - 2P_r - 4) - (1 + P_r^{-1})\big] b^4 + \big[\lambda^2 r_E(P_r\tau_0 - K\omega_0^* + \bar{\delta}K_0^* A) \\ & + 2\omega^2 K\omega_0^* P_r^{-1}(\omega^2\ell\tau_0 - 2A) + \omega^2(2P_r^{-1}\ell + \ell + 4A\tau_0 + P_r^{-1} - K\omega A\tau_0)\big] b^3 \\ & + \big\{\lambda^2 r_E\left[1 + P_r(\ell + 2A\tau_0) - \bar{\delta}A + \pi^2(K\omega_0^* - P_r\tau_0) - \pi^2 K\omega_0^*\bar{\delta}A + K\omega_0^*(\omega^2\ell\tau_0 - 2A)\right] \\ & + \omega^2 P_r^{-1}(2A - \omega^2\tau_0) - 4\pi^2\Omega^2 P_r(1 + \ell)\big\} b^2 + \big\{\lambda^2 r_E\left[2A - \omega^2\ell\tau_0 + \pi^2\,\bar{\delta}A - 1\right. \\ & \left. + 2K\omega_0^* A - \omega^2 K\omega_0^* \ell tau_0 - P_r\ell - 2P_r A\tau_0)\right] + 4\pi^2\Omega^2 P_r(\omega^2\ell\tau_0 - 2A)\big\} b \\ & + \pi^2\lambda^2 r_E\omega^2\ell\tau_0 - 2A\,. \end{aligned} \tag{52}$$

It is apparent from Eq. (45) that for arbitrary assigned values of P_r, r_E, τ_0, λ, Ω, K_0^*, A, K, $\bar{\delta}$, ℓ and ω, R_H will be complex but the physical meaning of R required it to be real.

Consequently, from the condition that R must be real, so we have either

$$R_H = X \quad \text{and} \quad \omega = 0 \tag{53}$$

or

$$R_H = X \quad \text{and} \quad Y = 0. \tag{54}$$

From Eq. (53) we obtain the eigenvalue equation for a natural stationary instability,

$$R_H = \frac{C_3}{\lambda^2 C_1}\,. \tag{55}$$

In this case

$$C_1 = P_r(A\bar{\delta} - 1)b^3 - 2AP_r b^2 \tag{56}$$

$$\begin{aligned} C_3 \;=\;& -P_r b^6 + AP_r(K\omega - 2)b^5 + \left[\lambda^2 r_E P_r(1 - \bar{\delta}A) - 4\pi^2\omega^2 P_r\right] b^3 \\ &+ \left[\lambda^2 r_E P_r(\pi^2\bar{\delta}A - 2A - \pi^2) - 8\pi^2\omega^2 P_r A\right] b^2 - 2\pi^2 P_r \lambda^2 r_E A b\,. \end{aligned} \tag{57}$$

For Newtonian viscous fluid $R_E = A = K = \bar{\delta} = \omega = \omega = 0$, Eq. (55) reduces to

$$R_H = \frac{b^3}{\lambda^2}\quad . \tag{58}$$

which agrees with the classical result (Chandrasekhar [16]). Equation (55) will give the critical Rayleigh heat number R_{HC} for the onset of stationary instability.

On the other hand Eq. (49) leads,

$$R_H = \frac{C_1 C_3 + \omega^2 C_2 C_4}{\lambda^2(C_1^2 + C_2^2)}\,, \tag{59}$$

and

$$C_1 C_4 - C_2 C_3 = 0\,. \tag{60}$$

For assigned values of P_r, K_0^*, τ_0, Ω, A, K, $\bar{\delta}$, ℓ and R_E – Eqs. (59) and (60) define R_H as a function of λ, the minimum of this function determines the critical Rayleigh number R_{HC} for the onset of oscillatory convection (i.e. overstability) should be compared with that the onset of stationary convection (i.e. ordinary instability). The type of instability, which takes place in practice, will be that corresponding to the lower value of the critical Rayleigh heat number.

5. Numerical results

In order to determine the conditions under which instability sets in overstability P_r, K_0^*, τ_0, Ω, A, K, $\bar{\delta}$, ℓ and R_E were assigned fixed values, and the value of ω was evaluated numerically from Eq.(60). Using this value of ω, the value of r_H was evaluated numerically from Eq.(59). The procedure was then repeated for various values of λ in order to locate the minimum of R_H. The critical Rayleigh heat number R_{HC} obtained for both stationary instability and overstability is shown in Figs.1–4.

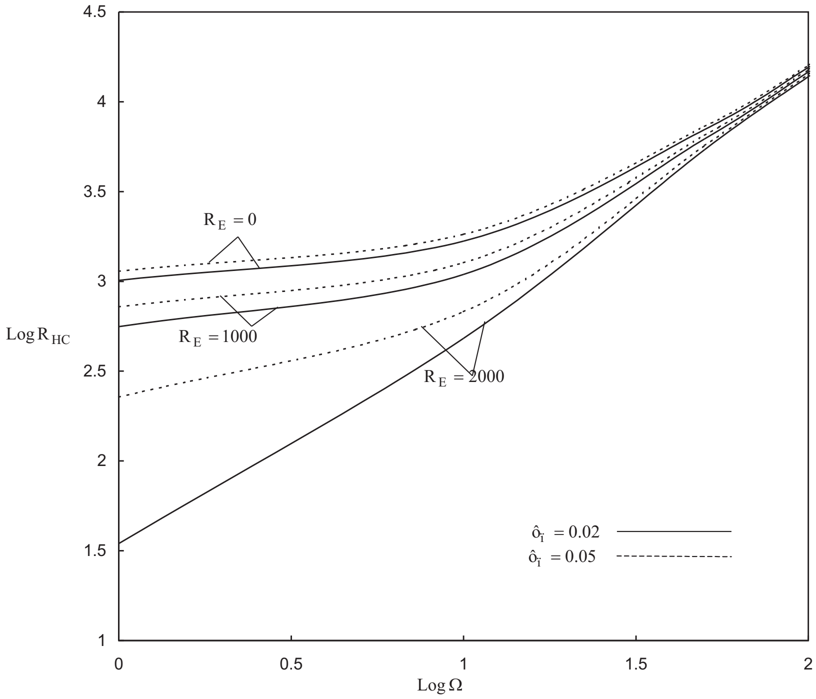

Figure 1 Represents the critical Rayleigh heat number R_{HC} as a function of Ω for various values of τ_0 and R_E at $P_r = 100$, $A = 0.2$, $\ell = 1$, $K = 1$, $\overline{\delta} = 1$, $\omega = 5$ and $K_0^* = 0.1$

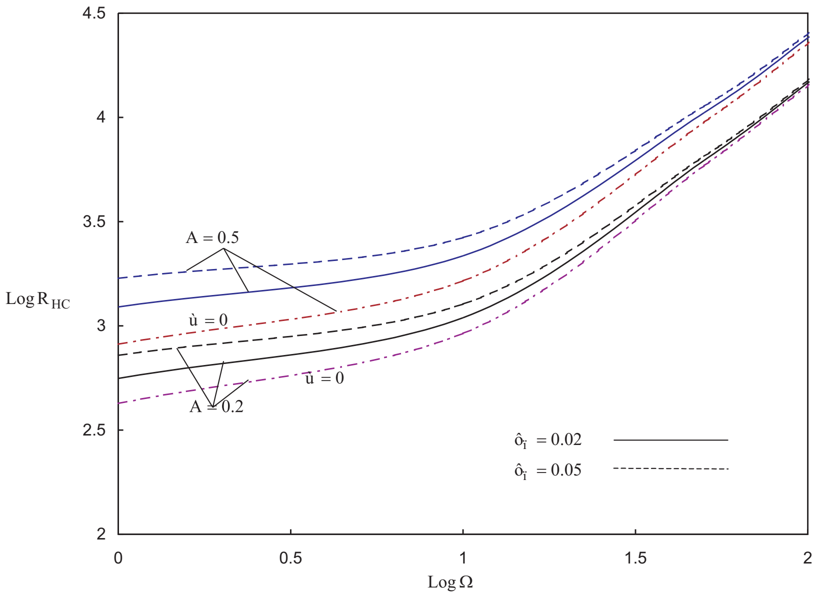

Figure 2 Represents the critical Rayleigh heat number R_{HC} as a function of Ω for various values of τ_0 and A at $P_r = 100$, $A = 0.2$, $\ell = 1$, $K = 1$, $\overline{\delta} = 1$, $\omega = 5$ and $R_E = 1000$. $\omega = 0$ represents the onset of stationary convection

We have plotted the variation of the critical Rayleigh heat number R_{HC} with the rotation ω using Eq.(59) satisfying (60) for the onset of over stable case for values of the dimensionless parameters $P_r = 100$, $\bar{\delta} = 1,\ 0.5,\ 0.1$, $K_0^* = 0.1\ 0.8$, $K = 1$, $\ell = 1$, $\tau_0 = 0.02,\ 0.05$ and $A = 0.2,\ 0.5$. Figure 1 represents the dependence of R_{HC} on ω for three values of $R_E = 0,\ 1000,\ 2000$, $\tau_0 = 0.02,\ 0.05$, $\bar{\delta} = 1$, $K_0^* = 0.1$ and $A = 0.2$. Figure 2 represents the dependence of R_{HC} on ω in the case of $R_E = 1000$. Figure 3 represents the dependence of R_{HC} on ω in the case of $R_E = 1000$, $\tau_0 = 0.02,\ 0.05$, $A = 0.2$, $K_0^* = 0.1,\ 0.8$ and $\bar{\delta} = 1$. Figure 4 represents the dependence of R_{HC} on ω in the case of $R_E = 1000$, $A = 0.2$, $\tau_0 = 0.02,\ 0.05$, $K_0^* = 0.1$ and $\bar{\delta} = 0.1,\ 0.5$. The flow is stable if $R_H < R_{HC}$ and otherwise unstable.

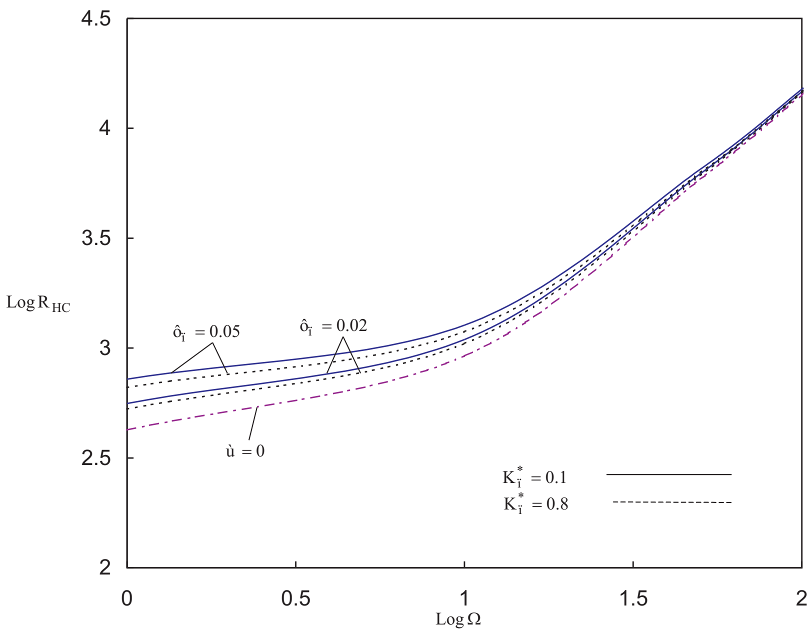

Figure 3 Represents the critical Rayleigh heat number R_{HC} as a function of Ω for various values of τ_0 and K_0^* at $P_r = 100$, $A = 0.2$, $\ell = 1$, $K = 1$, $\bar{\delta} = 1$, $\omega = 5$ and $R_E = 1000$. $\omega = 0$ represents the onset of stationary convection

Figures 1–4 reveal that the critical Rayleigh heat number R_{HC} decreases with an increase the Rayleigh electric number r_E and the elastic parameter K_0^*, while R_{HC} increases with an increase the relaxation time τ_0, the rotation ω and the parameters A, $\bar{\delta}$ (i.e. the onset of stability is delayed as r_E and K_0^* increase, while the onset of instability is delayed as τ_0, ω, $\bar{\delta}$ and A increase). The value of R_{HC} for an oscillatory instability is greater than that of a stationary instability.

In Figure 5 we have exhibited the dependence of critical wave number λ_C on ω for three values of $\bar{\delta} = 1,\ 0.5,\ 0.1$, $\tau_0 = 0.02,\ 0.05$, $K_0^* = 0.1$, $R_E = 1000$ and

$A = 0.2$. Figure 5 reveals that the critical wave number λ_C decreases or increases as $\bar{\delta}$ or τ_0 increases. This implies that the width of the cell at the onset of instability increases with the heat imparted by microrotation, while it reduces as the relaxation time τ_0 increases.

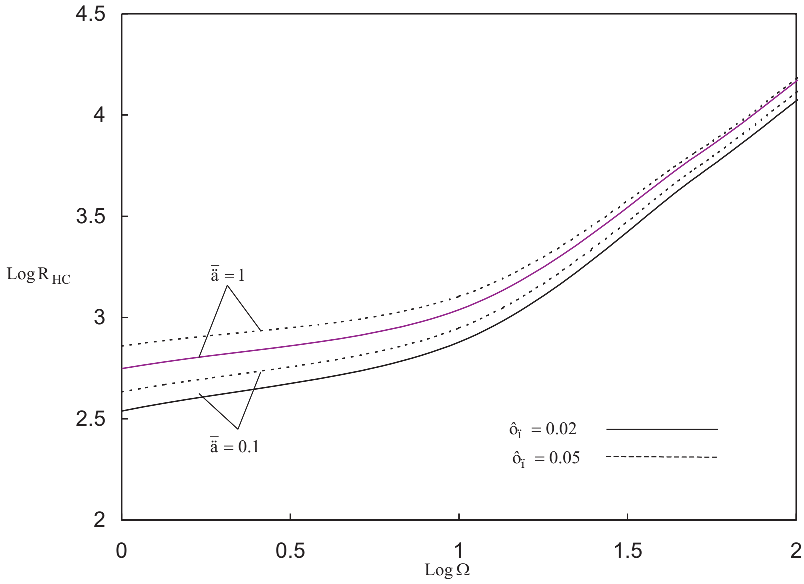

Figure 4 Represents the critical Rayleigh heat number R_{HC} as a function of Ω for various values of τ_0 and $\bar{\delta}$ at $P_r = 100$, $A = 0.2$, $\ell = 1$, $K = 1$, $A = 0.2$, $\omega = 5$ and $R_E = 1000$. $\omega = 0$ represents the onset of stationary convection

In the case of a Newtonian fluid, it is well known that the rotation introduces vorticity into the fluid. Then, the fluid moves in the horizontal plates with higher velocity. On account of this motion the velocity of the fluid perpendicular to the plates reduces, thus the onset of convection is inhibited. In the case of a micropolar fluid, free from the rotation of the system, it is apparent that a part of vorticity of the fluid is spent in inducing rotation to the micropolar additives. This apparent increase in the viscosity of the fluid reduces the velocity of the fluid, and hence delays the onset of instability. When the system is subject to low rotation, the microrotation and the rotation of the system have reinforced each other as the net effect of these two agents is to curtail the vertical component of the velocity. On the other hand, in the case of high rotation the motion of the fluid prevails essentially in the horizontal plates. This motion is reduced by the presence of micropolar additives. Thus the component of the velocity perpendicular to the horizontal plates enhances, thereby the system is prone to instability.

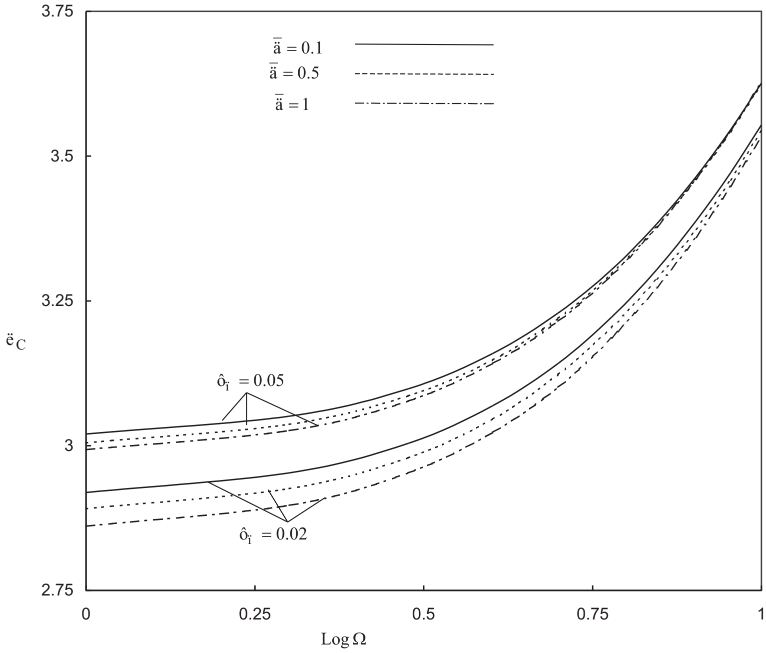

Figure 5 Represents the critical wavenumber λ_C as a function of Ω for various values of τ_0 and $\overline{\delta}$ at $P_r = 100$, $A = 0.2$, $\ell = 1$, $K = 1$, $A = 0.2$, $\omega = 5$ and $R_E = 1000$

6. Conclusion

Natural convection of a rotating micropolar viscoelastic fluid heated from below in the presence of electric field has been analyzed numerically. The study focused on the effect of a rotating micropolar fluid, elastic parameter, electric field and relaxation time on the convection phenomenon. From the above analysis, we conclude that the micropolar additives, the rotation of the system and the relaxation time have stabilizing effect while the elastic parameter and the presence of electric field have distabilising effect. It is also noted from Figs. 1–4 that the critical Rayleigh heat number for overstability is always greater than the critical Rayleigh heat number for stationary convection.

References

[1] **Eringen, AC, Şuhubi, ES**: Nonlinear theory of simple micro-elastic solids, *Int. J. Engng. Sci.*, (1964), **2**, 189-203.

[2] **Eringen, AC**: Theory of micropolar fluids, *J. Math. Mech.*, (1966), **16**, 1-18.

[3] **Jean, SK, Bhattacharyya, SP**: The effect of microstructure on the thermal convection in a rectangular box of fluid heated from below, *Int. J. Engng. Sci.*, (1986), **24**, 69.

[4] **Hsu, TH, Chen, CK**: Natural convection of micropolar fluids in a rectangular enclosure, *Int. J. Engng. Sci.*, (1996), **34**, 407-415.

[5] **Peddiesou, J**: Boundary-layer theory for a micropolar fluid, *Int. J. Engng. Sci.*, (1972), **10**, 23.

[6] **Ahmadi, G**: Stability of micropolar fluid layer heated from below, *Int. J. Engng. Sci.*, (1976), **14**, 81-89.

[7] **Datta, A. B, Sastary, VUK**: Thermal instability of a horizontal layer of micropolar fluid heated from below, *Int. J. Engng. Sci.*, (1976), **14**, 631-637.

[8] **Walzer, U**: *Ger. Bcit. Geo-physik*, (1976), Leipzig, **85**, 137.

[9] **Rama Rao, KV**: Thermal instability in a micropolar fluid layer between rigid boundaries, *Acta Mechanica*, (1979), **32**, 79.

[10] **Sharma, RC, Kumar, P**: Effect of rotation on thermal convection in micropolar fluids, *Int. J. Engng. Sci.*, (1994), **32**, 545-551.

[11] **Walters, K**: *Second-order effects in elasticity, Plasticity and fluid dynamics*, (1964), Pergamon, Oxford, 507.

[12] **Beard, DW, Walters, K**: Elastico-viscous boundary-layer flows, I. Two- dimensional flow near a stagnation point, *Proc. Camb. Phil. Soc.*, (1964), **60**, 667-671.

[13] **Singh, A, Singh, J**: Magnetohydrodynamic flow of a viscoelastic fluid past an accelerated plate, *Nat. Acad. Sci. Lett.*, (1983), **6**, 233-241.

[14] **Othman, MIA**: Electrohydrodynamic instability of a rotating layer of a visco- elastic fluid heated from below, *ZAMP*, (2004), **55**, 1-15.

[15] **Othman, MIA, Zaki, S. A**: The effect of thermal relaxation time on a electrohydrodynamic viscoelastic fluid layer heated from below, *Can. J. Phys.*, (2003), **81**, 779-787.

[16] **Chandrasekhar, S**: *Hydrodynamic and hydromagnetic stability*, (1961), Oxford Univ. Press, London.

[17] **Othman, MIA**: Electrohydrodynamic stability in a horizontal viscoelastic fluid layer in the presence of a vertical temperature gradient, *Int. J. Engng. Sci.*, (2001), **39**, 1217-1232.

[18] **Landau, L, Lifshitz, E**: *Electrohydrodynamics of continuous media*, (1960), Pergamon Press, New York.

[19] **Bhattacharyya, SP, Abbas, M**: On the stability of a hot rotating layer of micropolar fluid, *Lett. Appl. Engng. Sci.*, (1985), **23**, 371.

[20] **Turnbull, RJ**: Effect of dielectrophoretic forces on the Benard instability, *Phys. Fluids*, (1969), **12**, 1809.