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Strain–rate Effect in Dynamic Buckling of Thin–walled Isotropic Columns

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The aim of this paper is the analysis of strain–rate sensitivity in dynamic stability of thin–walled isotropic columns of closed rectangular cross–section, subjected to in–plane pulse loading of finite duration. The analysis is performed with the FE Method application. The full Lagrange strain tensor is assumed and various material characteristics are applied. The First Shear Deformation Theory displacement field is employed for the solution as well as the viscoplasticity constitutive model for material behavior under high strain rate loading. In the performed analysis the strain–rate effect influence on the dynamic buckling load is examined and also the initial imperfections of walls, pulse shape and pulse duration. The applications of some dynamic criteria are compared as well.

Keywords: Dynamic buckling, strain–rate sensitivity, FEM

1. Introduction

The demand for the car body safety has become a matter of considerably concern of producers and is a contradictory requirement to the need of the car body weight reduction. These both require optimum design of thin–walled car body structures to achieve the desired mechanical behavior at low and high strain rates. Dependently of strain rate the loading can be of various types and also of static or dynamic character. Especially in thin–walled members it may cause buckling which may be an effect of suddenly applied loads. This dynamic loading could be discrete type compression of finite duration. In literature the structure response for this loading is called dynamic buckling. The purpose of this paper is the analysis of dynamic viscoplastic behavior of isotropic columns of closed rectangular cross-section, built of thin-walled rectangular plates subjected to in–plane pulse loading of finite duration (Fig. 1). The analysis of dynamic stability of isotropic plated structures under in–plane pulse loading depends on pulse characteristic – it means pulse duration, pulse shape and magnitude of its amplitude [6]. The dynamic impulse buckling occurs when the loading process is of an intermediate amplitude and the pulse finite duration is close to the period of fundamental natural flexural vibrations. Usually the effects of dumping are neglected in such cases [6]. The dynamic behavior of a column, consisting of rectangular thin–walled plates, under in–plane loads involves rapid deflections growth of walls, which are initially not flat but imperfect. There is no buckling load and there is no bifurcation point over the loading path, as in the static case. Therefore the dynamic critical load is defined on the basis of an assumed dynamic buckling criterion. Very popular and most often used, adopted from shell structures to plated columns, Budiansky–Hutchinson [4] criterion assumes that dynamic stability loss occurs when the maximal plate deflection grows rapidly with the small variation of the load amplitude. The other, Petry–Fahlbusch [14] failure criterion states that a dynamic response caused by a pulse load is defined to be dynamic stable if the condition that the effective stress σ_{eff} is not greater than limit stress σ_L is fulfilled at every time everywhere in the structure. Ari Gur and Simoneta [1] analyzed laminated plates behaviour under impulse loading and formulated own criteria of dynamic buckling, two of them of collapse–type conditions.

In [6] wide review of literature devoted to dynamic buckling of thin–walled structures is presented as well as own original research results of dynamic buckling of plated structures – columns with opened and closed cross–sections, are included. It is worth mentioned that there are a very few works which were published of dynamic behaviour of thin–walled plated columns. Similar consideration of dynamic buckling but of thin–walled shell structures is contained in Simitses's paper [16]. Królak et al. analyzed thin–walled members under static loading in postbuckling range but included a number of results of solutions in elastic–plastic range [7]. Patel et al. analyse dynamic instability of stiffened shell panels applying FEM tool to obtain the solution [11]. Mania and Kowal–Michalska [10] considered isotropic columns of flat panel walls under axial pulse compression, especially the influence of cross–section shape (square versus rectangle) and pulse shape (rectangle, triangle and sine) on dynamic critical load value. In [8] Mania presents some results of investigations of orthotropic thin walled columns under dynamic pulse compression. The materials properties considered in mentioned above and other not cited known papers and publications are obtained from static tests. The loading rate sensitivity of many structural materials is well known [5], [13]. However this dynamic material properties have not been employed in dynamic buckling analysis. As to the author's knowledge there is no published solutions of buckling analysis of thin–walled columns made of strain rate sensitive material. Thereby this paper deals with thin–walled isotropic column made of material with a viscoplastic characteristic, axially loaded by compression impulse of finite duration.

2. Solution of the problem

The subject of the study is the short thin–walled column described above and shown in Fig. 1. It is assumed that the loaded edges of the column are simply supported and remain straight and parallel during loading. The shape of walls' initial imperfections fulfill the boundary conditions along all edges of component plates and corresponds directly to the first static buckling mode. The ratio of imperfection amplitude to the thickness of column walls (all walls are of equal thickness) was in the range (0.01, 0.1). The maximal in–plane dimension of wall (length or width) to its thickness fulfils the inequality

$$max\{l,\, b_1,\, b_2\}/\{h_1 = h_2\} \geq 20$$

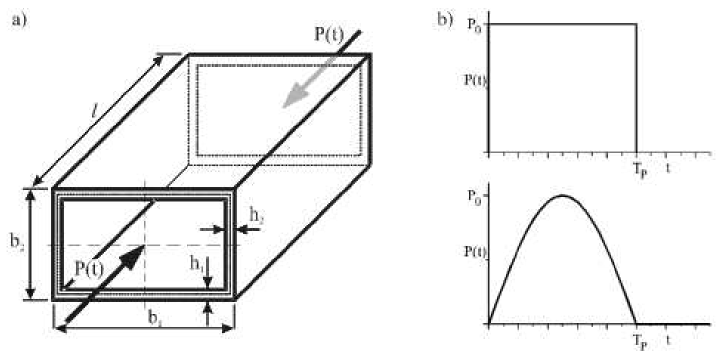

Figure 1 Geometric dimensions (a) and shapes of considered pulses (b)

The transient analysis was performed for pulse of rectangular and sinusoidal shapes with time duration T_p equal to or close to the period of fundamental natural flexural vibrations T. For each considered structure this period was obtained from modal analysis (eigenvalue problem). Zero initial conditions were assumed for velocity and initial imperfection with chosen amplitude for deflections.

The presented solution was obtained on the basis of the first shear deformation theory ($FSDT$). The displacement field was assumed as follows:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\varphi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\varphi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

where u_0, v_0, w_0 are the displacement components along the coordinate directions of a point on the midplane ($z = 0$) and φ_x, φ_y denote rotations about the y and x axes, respectively. In order to determine both out–of–plane and in–plane geometric plate behaviour under dynamic loading, the full strain tensor (Green–Lagrange strain tensor) for in–plane deformation was employed:

$$\{\varepsilon\} = \left\{\varepsilon^{(0)}\right\} + z\left\{\varepsilon^{(1)}\right\} \tag{2}$$

where $\left\{\varepsilon^{(0)}\right\}$ are the membrane strains:

$$\left\{\varepsilon^{(0)}\right\} = \left\{\begin{array}{c} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{yz}^{(0)} \\ \gamma_{xz}^{(0)} \\ \gamma_{xy}^{(0)} \end{array}\right\} = \left\{\begin{array}{l} u_0,_x + \frac{1}{2}\left(u_0^2,_x + v_0^2,_x + w_0^2,_x\right) \\ v_0,_y + \frac{1}{2}\left(u_0^2,_y + v_0^2,_y + w_0^2,_y\right) \\ w_0,_y + u_0,_y\, u_0,_z + v_0,_y\, v_0,_z + w_0,_y\, w_0,_z + \varphi_y \\ w_0,_x + u_0,_x\, u_0,_z + v_0,_x\, v_0,_z + w_0,_x\, w_0,_z + \varphi_x \\ u_0,_y + v_0,_x + u_0,_x\, u_0,_y + v_0,_x\, v_0,_y + w_0,_x\, w_0,_y \end{array}\right\} \tag{3}$$

and $\left\{\varepsilon^{(1)}\right\}$ are the flexural strains:

$$\left\{\varepsilon^{(1)}\right\} = \left\{\begin{array}{c} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{yz}^{(1)} \\ \gamma_{xz}^{(1)} \\ \gamma_{xy}^{(1)} \end{array}\right\} = \left\{\begin{array}{l} \varphi_x,_x \\ \varphi_y,_y \\ 0 \\ 0 \\ \varphi_x,_y + \varphi_y,_x \end{array}\right\} \tag{4}$$

Recalling the Hamilton's principle for a single column wall, the Euler-Lagrange equations could be obtained in the following form:

$$\begin{array}{rcl} -N_{xx},_x - N_{xy},_y - \left(N_{xx}u_0,_x\right),_x - \left(N_{yy}u_0,_y\right),_y & & \\ -\left(N_{xy}u_0,_x\right),_y - \left(N_{xy}u_0,_y\right),_x + I_0u_0,_{tt} + I_1\varphi_x,_{tt} & = & 0 \\ -N_{xy},_x - N_{yy},_y - \left(N_{xx}v_0,_x\right),_x - \left(N_{yy}v_0,_y\right),_y & & \\ -\left(N_{xy}v_0,_x\right),_y - \left(N_{xy}v_0,_y\right),_x + I_0v_0,_{tt} + I_1\varphi_y,_{tt} & = & 0 \\ -Q_x,_x - Q_y,_y - \left(N_{xx}w_0,_x\right),_x - \left(N_{yy}w_0,_y\right),_y & & \\ -\left(N_{xy}w_0,_x\right),_y - \left(N_{xy}w_0,_y\right),_x + q + I_0w_0,_{tt} & = & 0 \\ -M_{xx},_x - M_{xy},_y + Q_x + I_1u_0,_{tt} + I_2\varphi_x,_{tt} & = & 0 \\ -M_{xy},_x - M_{yy},_y + Q_y + I_1v_0,_{tt} + I_2\varphi_y,_{tt} & = & 0 \end{array} \tag{5}$$

In (5) N_{xx}, N_{yy}, N_{xy} denote resultants of membrane force, M_{xx}, M_{yy}, M_{xy} are moment resultants, Q_x, Q_y denote the transverse force resultants [6]. I_0, I_1, I_2 are the mass moments of inertia defined by integrals (6).

$$\left\{\begin{array}{c} I_0 \\ I_1 \\ I_2 \end{array}\right\} = \int\limits_{-h/2}^{h/2} \left\{\begin{array}{c} 1 \\ z \\ z^2 \end{array}\right\} \rho\, dz \tag{6}$$

Generally, for complex geometry, arbitrary boundary conditions and/or nonlinearities, the exact (analytical or variational) solution to the equations (5) cannot be developed. Therefore a finite element method was chosen to solve the problem. After some transformations of presented above set of equations (5) (integrating by parts, rearranging etc.) the weak form of equations of motion, associated with the

$FSDT$ was obtained:

$$\begin{aligned}
&\oint_\Gamma \{[N_{xx} + N_{xx}u_{0,x} + N_{xy}u_{0,y}]\ n_x \\
&+ [N_{xy} + N_{xy}u_{0,y} + N_{yy}u_{0,y}]\ n_y\}\delta u_0 ds = 0 \\
&\oint_\Gamma \{[N_{xy} + N_{xx}v_{0,x} + N_{xy}v_{0,y}]\ n_x \\
&+ [N_{yy} + N_{xy}v_{0,x} + N_{yy}v_{0,y}]\ n_y\}\delta v_0 ds = 0 \\
&\oint_\Gamma \{[Q_x + N_{xx}w_{0,x} + N_{xy}w_{0,y}]\ n_x \\
&+ [Q_y + N_{xy}w_{0,x} + N_{yy}w_{0,y}]\ n_y\}\delta w_0 ds = 0 \\
&\oint_\Gamma (M_{xx}n_x + M_{xy}n_y)\delta\varphi_x ds = 0 \\
&\oint_\Gamma (M_{xy}n_x + M_{yy}n_y)\delta\varphi_y ds = 0
\end{aligned} \tag{7}$$

Finally after approximation of the primary variables u_0, v_0, w_0, ϕ_x, ϕ_y with the shape functions F_i of the form:

$$\begin{aligned}
F_i \quad = \quad & \frac{1}{4}\{u_I(1-s)(1-t) + u_J(1+s)(1-t) \\
& + u_K(1+s)(1+t) + u_L(1-st)(1+t)\}
\end{aligned} \tag{8}$$

the FEM model of analyzed thin–walled column was obtained. In equation (8) s and t are normalized element local coordinates and $\underline{u}_I$ is the motion of node I of a four node quadrilateral shell element applied in the meshing. To perform the calculations for some chosen cases the ANSYS package software was employed [17]. The column of a shape of four wall cubic structure was meshed with a SHELL181 element, which is four nodes isoparametric nonlinear element. This element has six degrees of freedom: three displacements along the axes of local coordinate system and three rotations around these axes respectively. However the in–plane rotation around the axis normal to the element surface is controlled in Allman's sense [2].

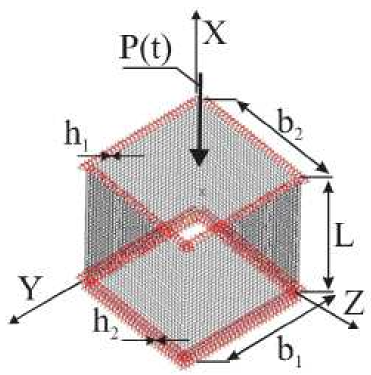

Figure 2 Meshed column model in reference coordinate system

The uniformly distributed mesh was applied with up to 10000 quadrilateral elements [8] what allowed to analyze even the high frequency modes in dynamic response of the column, giving acceptable time of computations (Fig 2).

The simply supported boundary conditions [8], [10] were chosen for the loaded edges of the column and the uniform compression of impulse type of finite duration, parallel to the walls, dynamically loaded the structure. The walls of the column are composed of isotropic plates. The material of column is assumed a structural steel. The fact that mild steel is strain rate sensitive is well known and in literature widely documented [13], [15]. It is reported that mild steel under dynamic loading increases the yield limit and the hardening part of strain–stress curve lies over static characteristic. Consequently the dynamic ultimate strength is also higher than in static case. To define this phenomenon by Perzyna viscoplastic constitutive equation [12] the following approach is performed.

According to Pradtl–Reuss associated flow rule idealization, the strain increment in the solid in elastoplastic range can be decomposed into elastic and plastic part [3]:

$$d\varepsilon_{ij} = \frac{1+\nu}{E} d\sigma_{ij} - \frac{\nu}{E} d\sigma_0 \delta_{ij} + \frac{3}{2}\frac{d\varepsilon_i^p}{\sigma_i}\left(\sigma_{ij} - \delta_{ij}\sigma_0\right) \tag{9}$$

where in (9) the first two terms simply obey generalized Hooke's law – elastic strain part and the last term – is plastic strain component. It relates equivalent plastic strain increment $d\varepsilon_{ij}^p$, effective stress σ_i and deviatoric stress $s_{ij} = \sigma_{ij} - \delta_{ij}\sigma_0$. Including the time dependence into elastoplastic equation (9) its viscoplastic form in rate terms is as follows:

$$d\dot{\varepsilon}_{ij} = \frac{1+\nu}{E} d\dot{\sigma}_{ij} - \frac{\nu}{E} d\dot{\sigma}_0 \delta_{ij} + \frac{3}{2}\frac{d\dot{\varepsilon}_i^p}{\sigma_i}\dot{s}_{ij} \tag{10}$$

Next introducing the effect of strain rate in deformation described by equation (10), Perzyna proposed the constitutive equation for general state of stress in the form of [12]:

$$d\dot{\varepsilon}_{ij} = \frac{1+\nu}{E} d\dot{\sigma}_{ij} - \frac{\nu}{E} d\dot{\sigma}_0 \delta_{ij} + \gamma \left\langle \Phi\left(F\right)\right\rangle \frac{\partial f}{\partial \sigma_{ij}} \tag{11}$$

where γ is material viscosity constant and f is the viscoplastic potential. The function $\Phi(F)$ should be determined from set of experiments. The notation $\langle\ \rangle$ is for McCauley's bracket. For steel, the relation between stress and strain is defined by Perzyna as:

$$\sigma = \sigma_0 \left[1 + \Phi^{-1}\left(\frac{\dot{\varepsilon}^p}{\gamma^*}\right)\right] \tag{12}$$

or simply:

$$\sigma = \sigma_0 \left[1 + \left(\frac{\dot{\varepsilon}^p}{\gamma^*}\right)^m\right] \tag{13}$$

In equation (13) $\dot{\varepsilon}^p$ means strain rate, m and γ^* are material constants and δ_0 is current static yield stress. Jones [5] suggests for mild steel $m = 0.2$ and $\gamma^* = 40.4$. This values were used in discussed later results of numerical calculations. Static material characteristic was modeled as bilinear one.

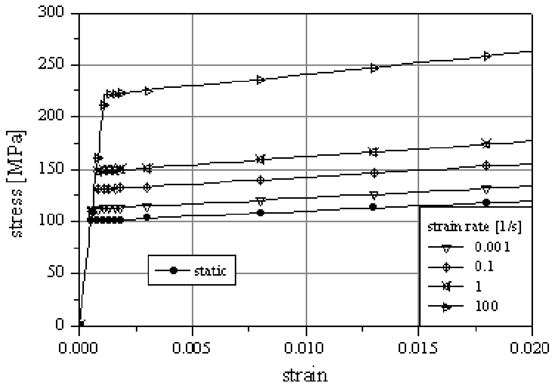

Figure 3 Dynamic material characteristics obtained with Perzyna constitutive equation (13)

The effect of strain rate on dynamic material characteristics is presented in Fig. 3. The real material static tensile test curve was approximated with bilinear characteristic. From this curve with Perzyna constitutive equation, the dynamic characteristic can be obtained for various strain rates. The stress – strain curves are nearly equidistant in the flow stress region. It is clearly seen that even for middle strain rate $\dot{\varepsilon}^p = 1\,[1/\mathrm{s}]$ the yield limit raises in 50 %. For impact rates ($\dot{\varepsilon}^p > 10^2\,[1/\mathrm{s}]$) this effect is more evident.

The critical conditions for dynamic buckling of analyzed columns were determined on the basis of one of criteria mentioned in the introduction: Budiansky-Hutchinson [3], Ari–Gur and Simoneta [1] and Petry&Fahlbusch [14]. The dynamic critical loads obtained from these criteria were compared as well.

3. Numerical results

Some chosen results of numerical calculations are presented in diagrams. The static material properties of considered material are collected in Table 1.

Table 1

Property	Unit	Isotropic value
E	GPa	200
G	GPa	77
ν		0.30
ρ	kg/m^3	7850

The dynamic load factor – DLF was introduced as a quotient of pulse load amplitude to the static buckling load for perfect structure. The static buckling load was determined in the first step of performed investigation, it is in linear eigenbuckling analysis.

In the performed dynamic buckling analysis the first maximal deflection of column walls was registered. Usually it took place during the acting pulse load or immediately after it was released. The differences of this effect are connected with the time duration of pulse load and pulse amplitude [6].

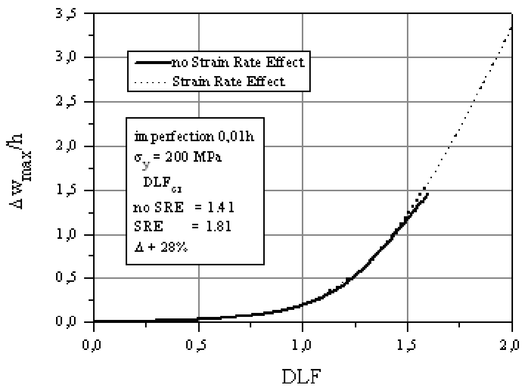

Figure 4 Maximal isotropic column deflections versus *DLF* value (rectangular pulse)

It a column of cubic shape (Fig. 2) with walls of equal thickness was considered thus the geometry relations were $b/l = 1$ and $b/h = 62$. The static buckling stress for this column was located under assumed yield limit $\sigma_y = 200$ [MPa]. The column was dynamically compressed with rectangular pulse load which lasted in time equal to the period of fundamental natural vibration of analyzed column. The graphs in figure 4 present the maximal column walls deflections as a function of pulse amplitude (DLF). The full line relates to transient analysis of material no sensitive to strain rate. The short dotted line was obtained for the case when the strain rate effect was included in analyzed material. To determine the dynamic critical load value the Budiansky–Hutchinson criterion was employed [4]. The critical ranges according to this criterion are difficult to distinguish for both cases because both curves almost coincide. Thus the Budiansky–Hutchinson criterion in its basic form is insufficient. Although from its approach it is obvious that in the critical range there is a inflection point of $\Delta w_{max}/h = f\ DLF$ curve. It can be derived with the application of finite difference method [9]. Following this procedure it was obtained that the critical DLF_{cr} for strain rate sensitive solution is greater in 28% comparing with DLF_{cr} for material where this effect was not considered (1.81 versus 1.41 respectively). From the column dynamic response history the maximal registered strain rate value is equal to 462 [1/s] for the dynamic pulse amplitude $DLF = 1.8$.

Analyzing column of similar geometry as the previous one but assuming lower yield limit - it is $\sigma_y = 100$ [MPa] with other material constants unchanged, leads to consideration of the problem of dynamic column buckling in elastic–plastic range even for low DLF values. Results of this analysis are presented in Fig. 5. Also in this case the critical values of DLF were determined by deriving the root of second determinant of scatter function $\Delta w_{max}/h = f\ DLF$. In this case the critical dynamic buckling load according to Budiansky–Hutchinson criterion is greater at 22% for rate sensitive material than for strain rate independent behavior. For both cases (results shown in Fig. 4 and in Fig. 5) the analysis of column made of material without strain rate dependence was limited to dynamic amplitudes (DLF) not greater than 1.6. The greater dynamic impulse amplitude were connected with large deformations and the plastic solution process was not converged.

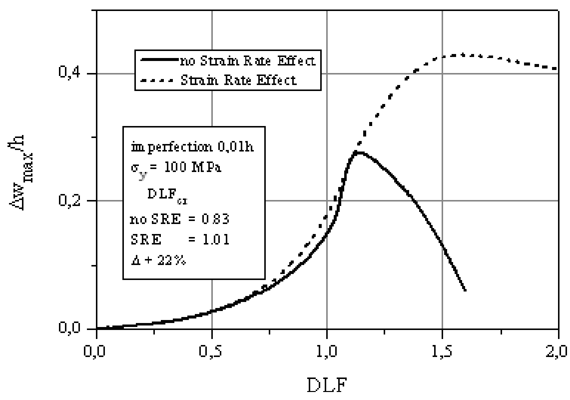

Figure 5 Maximal isotropic column deflections versus DLF value (rectangular pulse)

These cases are also a good example for comparison of application of different dynamic buckling criteria. As it is shown in Fig. 5 the DLF critical values are 0.83 and 1.01 for not rate sensitive and rate dependent material respectively. These values were obtained on the basis of the Budiansky-Hutchinson criterion. Both curves in Fig. 5 display rapid drop – solid one for $DLF = 1.2$ and dashed for $DLF = 1.55$. This is a result of change in buckling mode shape. For dynamic amplitudes lower than those denoted above, one half–wave deflection occurs on the column walls. For higher amplitudes more half–waves overlap causing lower deflection amplitude. For this situation one of Ari–Gur and Simoneta criteria can be used. It defines the critical condition for dynamic load as such for which the shape change in dynamic response occurs. According to this condition the dynamic critical loads are 1.2 and 1.55 - for respective material properties. Additionally they are greater in 50% comparing with values determined with application of Budiansky–Hutchinson criterion. Considering all these results the relations between the determined dynamic buckling load and applied criterion can be seen. This is one of basic differences between dynamic and static buckling analysis where for the last the bifurcation buckling load exists.

For comparative study the plated walls column of the same geometry as in the preceding consideration was loaded with sinusoidal compression impulse, acting in time equal to the period of fundamental natural vibration. Results of this analysis are shown in Fig. 6 together with results repeated after Fig. 5 for rectangular dynamic pulse loading. The sinusoidal pulse amplitude was 1.57 times greater than rectangle pulse amplitude to achieve the mechanical equality of both pulses. The maximal deflections of column walls for sinusoidal dynamic compression are about four times greater than those for rectangular pulse.

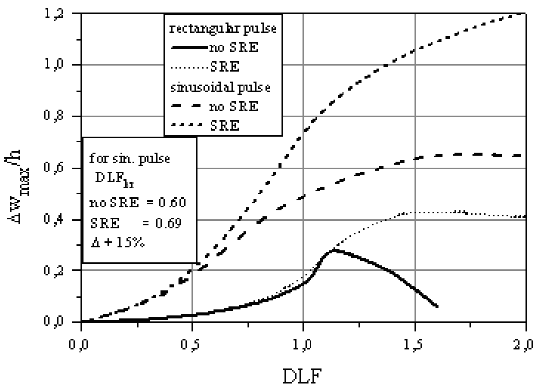

Figure 6 Maximal isotropic column deflections versus *DLF* value – pulse shape influence comparison

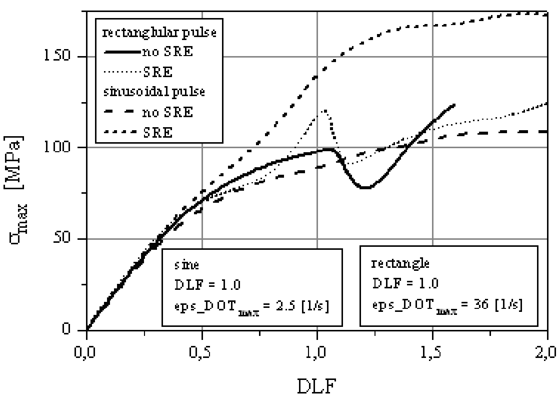

Figure 7 Maximal equivalent stresses (HMH) versus *DLF* value for sinusoidal and rectangular pulses

It can be explained by the fact that during sinusoidal loading the material has more time for plastic deformations when for rectangular pulse material achieves the full load from the very first moment. For sinusoidal pulse the plasticity undergoes with lower rate. This is confirmed by the stress curves for both examined loadings depicted in Fig. 7. The curves for sinusoidal impulse increase gradually when for rectangular pulse, curves local peaks are present. The maximal registered strain rate is grater for rectangular pulse load than for sinusoidal load and occurs at the beginning of acting impulse. However the critical *DLF* values for sinusoidal dynamic pulse are lower than for rectangular dynamic loading. This agrees with conclusions presented in [6] and [10].

4. Conclusions

In the paper the representative results of dynamic buckling analysis of thin-walled isotropic columns under impulse compression were presented. The plated walls of columns were made of isotropic strain rate sensitive material. For comparative study two different yield limits were assumed for this material. The special emphasis has been placed on the consideration of the effect of strain rate material dependence. The numerical calculations were obtained with Finite Element Method and ANSYS software package applications. The results of carried out analysis confirm the sensitiveness of the column dynamic response on the pulse shape and pulse duration (not presented in the paper) on the dynamic buckling load. It was proved that consideration of dynamic material properties in dynamic buckling analysis results in higher values of critical dynamic buckling loads. Due to lack of bifurcation buckling load in the dynamic analysis there is the necessity of dynamic buckling criterion assumption. The applied criterion influences the drawn conclusions and *DLF* critical value. The study of column with orthotropic walls demonstrates that similar strain rate sensitivity can be seen in dynamic response and these results will be presented soon.

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