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Some Aspects of Application of the Hydrostatic Bearings in Machine Tools

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The possibility of increasing the stiffness of machine tools spindle units thanks to taking advantage of a poorly known property of the hydrostatic journal bearings is described in the article. This is the ability to reach displacements of shaft consistent with the load sign, opposite to it or equal to zero, for one of the acting force directions. The bearings are equipped with inflow restrictors which have a fixed geometry. The results of investigations of these bearings are presented herein, illustrating the mentioned phenomenon. Examples of spindle units of different machine tools are discussed where advantages of the described phenomenon could be used.

Keywords: Hydrostatic bearing, machine tool, spindle

1. Introduction

The following advantages of hydrostatic bearings are generally emphasized in publications: great stiffness and bearing capacity, high accuracy of movement, practically unlimited durability, perfect ability of vibration dumping, less power losses than in hydrodynamic bearings, lower requirements regarding the machining accuracy than in case of other bearings, assuring the fluid friction in the whole range of loads and rotational speed changes – including also the motionless spindle [5, 8, 9]. The possibility of static balancing the grinding wheel directly on the grinder spindle is mentioned sometimes. The outlook of using hydrostatic bearings for measurement of cutting forces is often pointed out, that proves to be useful in the process of adaptive control.

It is worth to pay attention to the specific characteristics of the journal hydrostatic bearing that may be significant in case of machine tools spindles. It is possible to reach displacements of the shaft consistent with the load sign, opposite to it or equal to zero for one of the acting force directions. This arises in case of a bearing equipped with fixed geometry inflow restrictors, and it does not introduce the danger of stability loss. In case of bearings with pressure-sensing valves such possibility was described in literature, but negative stiffness of such bearings may lead to static instability [5].

2. Displacements of shaft in hydrostatic journal bearing

Displacements of machine tool spindles are considered in two mutually perpendicular directions Oz and Oy that correspond with the directions of cutting forces – the main and the thrust. The displacements of the shaft in the assumed coordinate system (Fig. 1) are described [2, 3, 4] by the relations:

$$z = \frac{K_p W_z + K_\omega W_y}{K_p^2 + K_\omega^2} \tag{1}$$

$$y = \frac{K_p W_y - K_\omega W_z}{K_p^2 + K_\omega^2} \tag{2}$$

where W_z and W_y – components of bearing load.

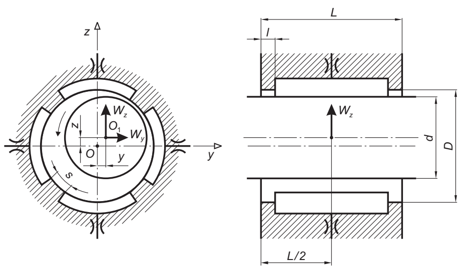

Figure 1 Loads and displacements of the shaft in the hydrostatic journal bearing

The pressure component of stiffness K_p and the motional component of stiffness K_ω, present in the above mentioned formula are expressed according to [2, 3, 4], as follows:

$$K_p = U \frac{D_r L_r \beta}{h_0} p_s \tag{3}$$

$$K_\omega = U \frac{D_r L_r^2 l \eta_b}{h_0^3} \omega \tag{4}$$

where

- reduced diameter of bearing

$$D_r = \left(1 - \frac{ks}{2\pi D} \sin^2 \frac{\pi}{k}\right) D \tag{5}$$

- radial clearance in unloaded bearing

$$h_0 = \frac{1}{2}(D - d) \tag{6}$$

- reduced length of bearing

$$L_r = L - l \tag{7}$$

- pressure ratio in unloaded bearing

$$\beta = \frac{p_0}{p_s} \tag{8}$$

- dimensionless coefficient

$$U = \frac{3k^2}{4p\left[\frac{1-(1-w)\beta}{2(1-\beta)\sin^2\frac{\pi}{k}} + \delta\right]} \tag{9}$$

- circumferential flow factor in unloaded bearing

$$\delta = \frac{kL_r l}{\pi D s} \tag{10}$$

w – coefficient dependent on the pattern of flow through the restrictor:
$w = 1$ – for laminar flow,
$w = 0.5$ – for turbulent flow,
$0.5 < w < 1$ – for intermediate flow, e.g. for the set of restrictors connected in series or in parallel; when the flow in some restrictors is laminar and in others is turbulent.

Variables, very similar to K_p and K_ω, were named by Rowe [5] as hydrostatic and hydrodynamic stiffness.

The above mentioned relations are valid for the bearing with any number of recesses $k \geq 3$ under conditions expressed in details in the works [1, 3, 5, 6, 7]. It is worth to notice, that forces and displacements are positive when their senses are consistent with the assumed coordinate system Oyz (Fig. 1). The trigonometric direction was assumed as positive for the angular velocity ω.

The shaft displacements z or y will be equal to zero, when the numerators in formulae (1) or (2) become zero. Consequently, it will turn out that $z = 0$ when

$$p_s = [(p_s)_{lim}]_z = -\frac{L_r l \eta_b}{\beta h_0^2}\omega\lambda_b \tag{11}$$

Similarly, it will be $y = 0$, when

$$p_s = [(p_s)_{lim}]_y = \frac{L_r l \eta_b}{\beta h_0^2}\frac{\omega}{\lambda_b} \tag{12}$$

The bearing load ratio

$$\lambda_b = \frac{W_y}{W_z} \tag{13}$$

was assumed in the above formulae.

The analysis of the relations (11) and (12) shows that the simultaneous fulfilment of both these conditions is not possible. Only one of them can have physical sense. For instance, if the signs of λ_b and ω are equal, then only the pressure $[(p_s)_{lim}]_y$ calculated from the formula (12) may be positive. The $[(p_s)_{lim}]_z$ value will be less than zero – and that is nonsense because in the hydrostatic bearing p_s must be positive ($p_s > 0$).

3. Spindle units of some machine tools

One of the aims in machine tools is to minimize the deformations in direction of the thrust force acting, i.e. Oy. Just these deformations have direct influence on the accuracy of machining. In hydrostatic bearing y displacements can be reduced to zero, when the condition (12) has the physical sense. This happens, when the signs of the angular velocity ω and the force ratio λ_b are equal.

In general, the analysis of spindle behaviour is complex, as the forces from the cutting and from the drive have to be taken into account, as well as their mutual location in space. In the present paper the subject is limited to the spindles relieved from the forces coming from the drive (Fig. 2). Such spindles are encountered in many types of precise machine tools.

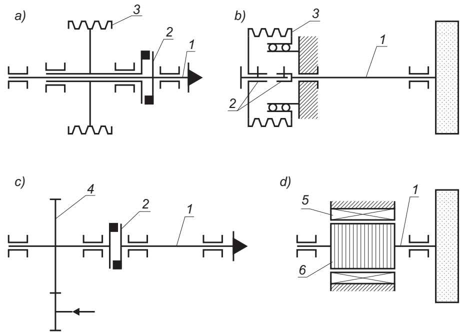

Figure 2 Spindles relieved from the forces coming from the drive: 1 – spindle, 2 – coupling, 3 – pulley, 4 – gear, 5 – stator, 6 – rotor

In this case the spindle is loaded with the torque, and with the components of cutting forces applied to the spindle nose. It is attained by separate bearing assemblies of the spindle and the pulleys (Fig. 2a, b), the gear wheel (Fig. 2c) or by placing the electric motor rotor directly onto the spindle (Fig. 2d).

Prior to cutting the bearing is affected by the gravity forces of the spindle and of the elements mounted on it. When the external load occurs, with the components F_y and F_z (Fig. 3), the load of the bearings changes by the values: ΔW_{Ay} and ΔW_{Az} in the front bearing, and ΔW_{By} and ΔW_{Bz} — in the rear one. For the spindles relieved from the drive forces the following relation will be met:

$$\frac{\Delta W_{Ay}}{\Delta W_{Az}} = \frac{\Delta W_{By}}{\Delta W_{Bz}} = \frac{F_y}{F_z} = \lambda_c \tag{14}$$

In steady–state machining conditions it may be admitted that the cutting force ratio λ_c is constant and is equal to:
$\lambda_c = 0.4 - 0.6$ – for longitudinal turning of steel,
$\lambda_c = 0.3 - 0.6$ – for longitudinal turning of cast iron,
$\lambda_c = 1 - 3$ – for longitudinal grinding,
$\lambda_c \approx 0.4$ – for milling.

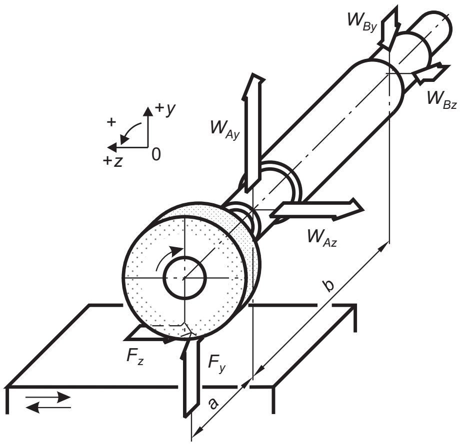

Figure 3 The surface grinder spindle relieved from the forces coming from the drive

The changes of bearing loads will cause the displacement of the shaft in relation to the initial position. These displacements can be calculated from the formulae:

$$\Delta z = \frac{K_p \Delta W_z + K_\omega \Delta W_y}{K_p^2 + K_\omega^2} \tag{15}$$

$$\Delta y = \frac{K_p \Delta W_y - K_\omega \Delta W_z}{K_p^2 + K_\omega^2} \tag{16}$$

The similarity of the formulae (15) and (16) to the relations (1) and (2) results from the linear dependence of displacements on the loads.

Similarly, as mentioned before, the change of the shaft position Δy in the direction normal to the work surface will be equal to zero, if

$$p_s = [(p_s)_{lim}]_y = \frac{L_r l \eta_b}{\beta h_0^2} \cdot \frac{\omega}{\lambda_c} \tag{17}$$

In case of the spindles performing the primary motion (Fig. 3) the sense of F_z force changes together with the direction of the angular velocity ω. Therefore the signs of the λ_c coefficient and the speed ω will always be here identical, as shown in Fig. 4 with reference to turning. This also refers to the grinding wheel spindles of surface grinders (Fig. 3), cylindrical grinders and internal grinders, and spindles of milling machines, too.

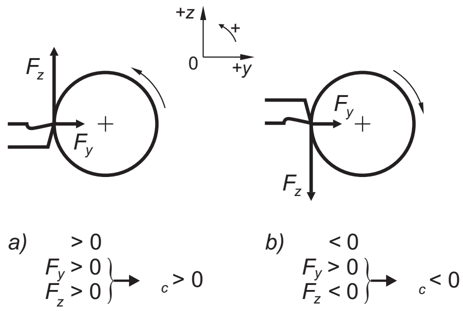

Figure 4 The signs of the λ_c coefficient and the angular velocity ω at turning

Therefore, in case of spindles performing the primary motion, the adequate selection of supply pressure will enable to achieve displacements of shaft in Oy direction consistent with the sign of the thrust force F_y, opposite to its sign or equal to zero. The displacements in Oz direction will always be consistent with the sign of F_z force.

Another situation may occur in case of spindles that perform the rotary feed motion. It is possible to minimize the displacements in Oy direction for the workpiece spindle of the cylindrical grinders (Fig. 5) by selecting suitable supply pressure, because it occurs that $\omega > 0$ and $\lambda_c > 0$. It is the beneficial situation, resulting from the chosen of the up–grinding method.

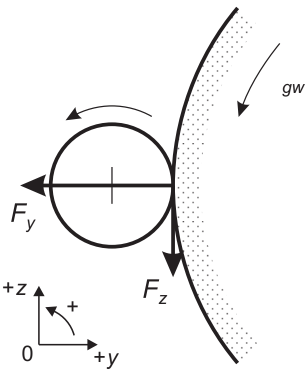

Figure 5 The loads acting on a workpiece at cylindrical grinding: ω – angular velocity of the workpiece, ω_{gw} – angular velocity of the grinding wheel

The same up–grinding method applied in internal grinding (Fig. 6) is unfavourable in the aspect of the presented characteristic of the hydrostatic bearings, as this is where the sign of Δydisplacement is always consistent with the sign of F_y force because $\omega > 0$ and $\lambda_c < 0$.

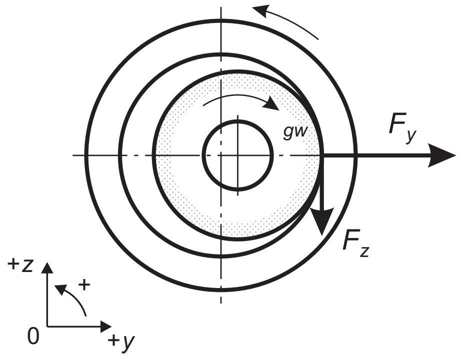

Figure 6 The loads acting on a workpiece at internal grinding: ω – angular velocity of the workpiece, ω_{gw} – angular velocity of the grinding wheel

This situation could be altered by reversing the direction of the workpiece rotation, if the drive system would enable that without the grinding quality deterioration.

4. Results of experimental investigations

The results of examinations of hydrostatic journal bearing with fixed geometry inflow restrictors are presented in Fig. 7-9. Data for this bearing are presented in Table 1. The bearing was designed for the spindle of a lathe.

Table 1 Data of the tested bearing

No.	Meaning of symbol	Symbol	Unit	Value
1	Number of recesses	k	-	4
2	Bearing diameter	D	mm	80.0
3	Bearing length	L	mm	112
4	Leakage flow land width	l	mm	4
5	Inter-recess land width	s	mm	12.5
6	Radial clearance in unloaded bearing	h_0	μm	39

Fig. 7 illustrates the relation of the shaft displacements: Δz to the force increment ΔW_z, and Δy to the force increment ΔW_y. The ratio of these increments was constant, and for the described example it was equal to $\lambda_c = 0.364$. That means that two components W_y and W_z acted simultaneously on the shaft. When component W_y increased by e.g. $\Delta W_y = 166$ [N], the component W_z rose by $\Delta W_z = 455$ [N] Then the shaft was loaded with forces enlarged by $\Delta W_y = 248$ [N] and $\Delta W_z = 682$ [N], etc. The shaft rotated with the rotational speed $n = 1240$ rpm. The supply pressure was $p_s = 2.29$ [MPa], and it was less than the limit pressure $[(p_s)_{lim}]_y = 3.14$ [MPa]. In consequence the displacement values Δy turned out to be negative, i.e. their sense was opposite to the sense of the force increments ΔW_y. This refers to both theoretical and experimental relations. The relative differences of the theoretical and experimental values of Δy were in this case quite substantial, but the absolute differences did not exceed 1 [μm]. Much better conformity between the theory and the experience was found for the displacement Δz.

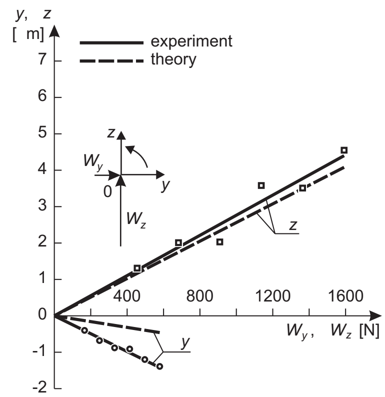

Figure 7 The relation of the shaft displacements: Δz to the force increment ΔW_z, and Δy to the force increment ΔW_y for $\lambda_c = 0.364$, $p_s = 2.29$ [MPa], $n = 1240$ rpm, $\beta = 0.484$, $\eta_b = 0.0150$ [Pa·s]

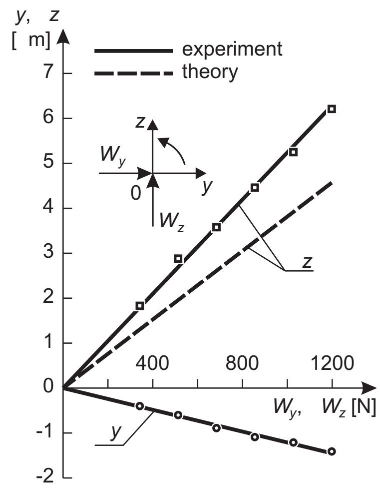

Figure 8 The relation of the shaft displacements: Δz to the force increment ΔW_z, and Δy to the force increment ΔW_y for $\lambda_c = 1.00$, $p_s = 1.00$ [MPa], $n = 2400$ rpm, $\beta = 0.494$, $\eta_b = 0.0136$ [Pa·s]

Similar relations for the shaft loaded with identical force increments ΔW_y and ΔW_z are presented in Fig. 8, i.e. for the ratio $\lambda_c = 1.00$. The supply pressure $p_s = 1.00$ [MPa] was less than the limit pressure, that for the rotational speed $n = 2400$ rpm was equal to $[(p_s)_{lim}]_y = 1.96$ [MPa]. Similarly, as aforementioned, the displacements Δy were negative. However it may be stated that in the range of the graph accuracy full conformity of theory and experiment was achieved. The divergence between theory and experience for displacements Δz was equal to approx. 37%.

The best conformity between theoretical and experimental characteristics occurred for the ratio $\lambda_c = 2.75$ (Fig. 9). At the rotational speed $n = 2400$ rpm the limit pressure was equal to $[(p_s)_{lim}]_y = 0.65$ [MPa]. The investigations were carried out with the supply pressure $p_s = 0.50$ [MPa], which caused that again the displacements Δy turned out to be less then zero.

In the three presented examples the displacements Δz were positive, i.e. the senses of displacements Δz and the force increments ΔW_z remained consistent.

The dependence of the displacement y on the load W_y, as well as the dependence of the displacement z on $W_z$can be regarded as linear in the range of investigated values. The values of the pressure ratio β and oil viscosity η_b are quoted below the discussed figures.

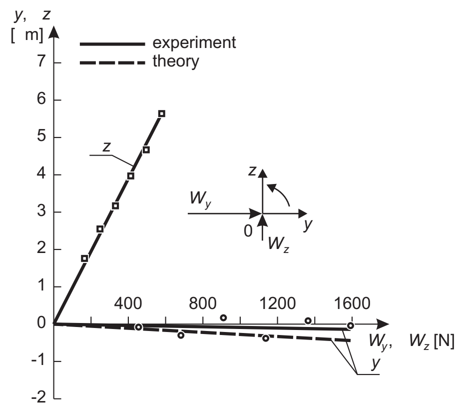

Figure 9 The relation of the shaft displacements: Δz to the force increment ΔW_z, and Δy to the force increment ΔW_y for $\lambda_c = 2.75$, $p_s = 0.50$ [MPa], n=2400 rpm, $\beta = 0.504$, $\eta_b = 0.0127$ [Pa·s]

It may be acknowledged that the theoretical consideration results were confirmed with rather sufficient accuracy in experimental investigations. During the experiments the bearing operation was reliable and no disturbances were noticed. The shaft kept the stable position towards the bearing shell in the whole range of loads.

The necessary and sufficient condition for the stability of the hydrostatic journal bearing is:

$$p_s > \frac{mh_0\omega^2}{4UD_rL_r\beta} \tag{18}$$

where m — mass reduced to the centre of the bearing.

Pressure values, for the mass $m = 20$ [kg] ensuring the stable bearing operation, in cases from Fig. 7-9, were duly equal to: $5.23 \cdot 10^{-4}$ [MPa], $1.97 \cdot 10^{-3}$ [MPa] and $1.98 \cdot 10^{-3}$ [MPa]. These values are few orders lower than recommended $[(p_s)_{lim}]_y$ limit.

5. Conclusions

The presented results of the theoretical analysis and experimental investigations confirm the thesis that the radial hydrostatic bearing with inflow restrictors having fixed geometry, supplied with oil at constant pressure, enable to reach the shaft displacements with the sign opposite to the component of the cutting thrust force. The displacements in just this direction have the supreme influence on the machining accuracy. The described phenomenon enables to compensate partially the very spindle deformations, and to decrease the total displacements of the spindle nose, resulting from the flexibility of the spindle and bearings. The mentioned effect may be achieved by the adequate selection of the supply pressure, according to the relation (17). However, caution should be observed, that at low angular spindle velocities, the recommended pressure value may appear to be too low for securing the proper stiffness of the bearings in the direction of the acting main cutting force component.

It should not be allowed that the total spindle nose displacements become negative (opposite to the sign of the thrust component), as that will cause the unstable operation of the spindle unit. The increase of load in such case would cause the increase of the cutting depth, and that could bring about the increase of forces, etc., till the failure of the unit.

It should be emphasized that there is no fear of the bearing stability loss in the described operating conditions.

The mentioned effect of the shaft negative displacements can always be achieved in the direction normal to the work surface, for spindles performing the primary motion. The displacements in the tangential direction are then consistent with the sign of the main cutting force component.

In case of spindles performing the rotational feed motion (i.e. workpiece spindles in cylindrical grinders and internal grinders) this effect will occur in one or in the other considered directions – depending on the senses of the forces and the angular velocity.

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Nomenclature

d	diameter of journal
D	internal diameter of bearing bush (bearing diameter)
D_r	reduced diameter of bearing
h_0	radial clearance in unloaded bearing
k	number of recesses
l	leakage flow land width
L	bearing length
L_r	reduced bearing length
p_0	recess pressure in unloaded bearing
p_s	constant pressure at supply source
s	inter-recess land width
β	pressure ratio in unloaded bearing
δ	circumferential flow factor in unloaded bearing
η_b	dynamic viscosity in bearing film
λ_b	bearing load ratio
λ_c	cutting force ratio
ω	angular velocity of shaft