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Variable Permeability Effect on Vortex Instability of Free Convection Flow Over Inclined Heated Surfaces in Porous Media

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A linear stability analysis is performed for the study of the onset of vortex instability in free convection flow over an inclined heated surface in a porous medium, where the wall temperature is a power function of the distance from the origin. The variation of permeability in the vicinity of the solid boundary is approximated by an exponential function. The variation rate itself depends slowly on the streamwise coordinate, such as to allow the problem to possess a set of solutions, invariant under a group of transformations. Velocity and temperature profiles as well as local Nusselt number for the base flow are presented for the uniform permeability UP and variable permeability VP cases. The resulting variable coefficient eigenvalue problem is solved numerically. The critical parameter $Ra_x^* \tan^2 \phi$ and the critical wave number k^* are computed for different prescribed wall temperature distribution of the inclined surface for both UP and VP cases. It is found that the larger the inclination angle with respect to the vertical, the more susceptible is the flow for the vortex mode of disturbances; and in the limit of zero inclination angle (i.e vertical heated plate) the flow is stable for this form of disturbances. Also, it is found that the variable permeability effect tends to increase the heat transfer rate and destabilize the flow to the vortex mode of disturbance.

Keywords: Free convection, vortex instability, inclined surface, porous media, variable permeability, variable porosity

1. Introduction

The problem of the vortex mode of instability in natural convection flow over a horizontal or an inclined heated plate in a saturated porous medium has recently received considerable attention. This is primarily due to a large number of technical applications, such as fluid flow in geothermal reservoirs, separation processes in chemical industries, storage of radioactive nuclear waste materials, transpiration cooling, transport processes in aquifers, etc. Hsu et al. [1] analyzed the vortex mode of instability of horizontal natural convection flows in a uniform porosity medium. Jang and Chang [2] studied the vortex instability of horizontal natural convection in a porous medium resulting from combined heat and mass buoyancy effects.

For an inclined surface, the buoyancy force causing motion has a component in both the tangential and normal directions. This causes a pressure gradient across the boundary layer and leads to a theoretical analysis more complicated than that for a vertical or a horizontal surface. Hsu and Cheng [3] studied the vortex instability in buoyancy–induced flow over inclined heated surfaces in a uniform porosity medium. Jang and Chang [4] re–examined the same problem for an inclined plate, where both the streamwise and normal components of the buoyancy force are retained in the momentum equations. The effects of a density extremum on the vortex instability of an inclined buoyant layer in porous media saturated with cold water were examined by Jang and Chang [5]. Rees and Basson [6] presented an account of the linear instability of Darcy–Boussineq convection in a uniform, unstably stratified porous layer at arbitrary inclinations from the horizontal.

All of the above mentioned papers considered with the Darcy formulation with uniform porosity. In some applications, such as fixed–bed catalytic reactors, packed bed heat exchangers and drying, the porosity is not uniform but has a maximum value at the wall and a minimum value away from the wall. This wall–channeling phenomenon has been reported by a number of investigators such as Vafai [7], Chandrasekhara et al. [8], Chandrasekhara [9] and Hong et al. [10] for forced, natural and mixed convection boundary layer flows adjacent to a horizontal and vertical surfaces. It is shown that the variable porosity effect increases the temperature gradient adjacent to the wall resulting in the enhancement of the surface heat flux. Chandrasekhara and Namboodiri [11] obtained the similarity solution for combined free and forced convection in the presence of inclined surface in saturated porous media with variable permeability. Ibrahim and Hassanien [12] reported non-similarity solutions for the variable permeability on combined convection along a non–isothermal wedge in a saturated porous medium. The effect variable porosity on vortex instability of a horizontal mixed and free convection flow in a saturated porous medium was studied by Jang and Cheng [13], Ibrahim et al. [14] and Ibrahim and Elaiw [15].

The purposed of this paper is to study the effect of variable permeability on vortex instability free convection boundary layer flow over an inclined heated plate in a saturated porous medium. The stability analysis is based on the linear theory. The disturbance quantities are assumed to be in the form of a stationary vortex roll that is periodic in the spanwise direction, with its amplitude function depending primarily on the normal coordinate and weakly on the streamwise coordinate. The resulting equations for the amplitude of the disturbances are solved based on the local similarity method. The resulting eigenvalue problem was solved numerically using finite difference scheme.

2. Analysis

2.1. *The main flow*

Consider an inclined impermeable surface embedded in a porous medium as shown in Fig. 1. The axial and normal coordinates are x and y, where $x$represents the distance along the plate from its leading edge, and y coordinate pointed toward the porous medium. The wall temperature is assumed to be a power function of x, i.e. $T_w = T_\infty + Ax^m$, where A is a constant and T_∞ is the free stream temperature. The angle of inclination ϕ is measured from the vertical.

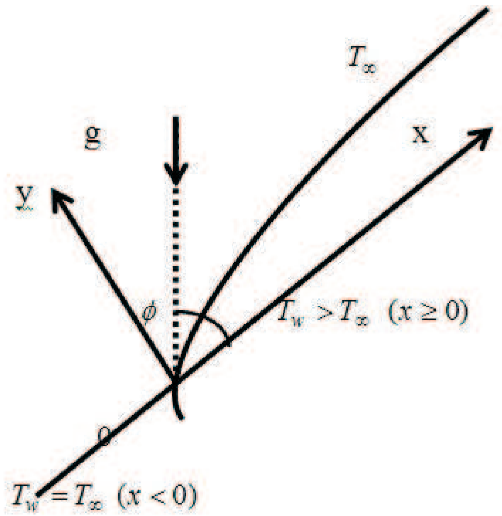

Figure 1 The physical model and coordinate system

In the formulation of the present problem the following common assumptions are made: the local thermal equilibrium exists between the fluid and solid phases; fluid properties are assumed to be constant except for density variations in the buoyancy force term. Under the Boussinesq and the boundary layer approximations, the governing equations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u = -\frac{K}{\mu}\left[\frac{\partial P}{\partial x} + \rho g \cos\phi\right] \tag{2}$$

$$v = -\frac{K}{\mu}\frac{\partial P}{\partial y} \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2} + \frac{\partial}{\partial y}\left(\alpha\frac{\partial T}{\partial y}\right) \tag{4}$$

$$\rho = \rho_\infty\left[1 - \beta(T - T_\infty)\right] \tag{5}$$

where u and v are the Darcian velocities in the x and y directions; P is the pressure; T is the temperature; ρ is the fluid density; μ is the dynamic viscosity; K is the permeability of the porous medium; β is the thermal expansion coefficient of the fluid and $\alpha = \lambda_m/(\rho_\infty c_p)_f$ is the effective thermal diffusivity of the porous medium,

λ_m denotes the effective thermal conductivity of the saturated porous medium and $(\rho_\infty c_p)_f$ denotes the product of density and specific heat of the fluid.

The pressure terms appearing in Eqs (2) and (2) can be eliminated through cross–differentiation. The boundary layer assumptions yields $\partial/\partial x << \partial/\partial y$ and $v << u$. With ψ being a stream function such that $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$, the equations (1)–(5) become

$$\frac{\partial^2\psi}{\partial y^2} = -K\frac{d}{dy}\left[\frac{1}{K}\right]\frac{\partial\psi}{\partial y} - \frac{K\rho_\infty g\beta\cos\phi}{\mu}\frac{\partial T}{\partial y} \tag{6}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{d\alpha}{dy}\frac{\partial T}{\partial y} \tag{7}$$

The boundary conditions of this problem are

$$\begin{array}{llll} \text{at} & y = 0 & v = -\frac{\partial\psi}{\partial x} = 0 & T_w = T_\infty + Ax^m \\ \text{as} & y \to \infty & u = \frac{\partial\psi}{\partial y} \to 0 & T \to T_\infty \end{array} \tag{8}$$

We define a set of solutions, which are invariant under group transformations as follows:

$$\eta(x,y) = Ra_x^{1/2}\frac{y}{x} \qquad f(\eta) = \frac{\psi(x,y)}{\alpha_\infty Ra_x^{1/2}} \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{9}$$

where

$$Ra_x = \frac{K_\infty\rho_\infty g\beta(T_w - T_\infty)x\cos\phi}{\mu\alpha_\infty}$$

is the modified local Rayleigh number. Here we consider that the porosity $\varepsilon$and permeability K vary exponentially from the wall (Chandrasekhara [9])

$$\varepsilon = \varepsilon_\infty(1 + de^{-y/\gamma}) \tag{10}$$

$$K = K_\infty(1 + d^* e^{-y/\gamma}) \tag{11}$$

where ε_∞ and K_∞ are the porosity and permeability at the edge of the boundary layer; d and d^* are constants whose values are taken as 1.5 and 3 respectively, (Chandrasekhara [9]). Further, $\alpha = \lambda_m/(\rho_\infty c_p)_f$ also varies since it is related to the effective thermal conductivity of the saturated porous medium λ_m, where λ_m can be computed according to the following semi–analytical expression given by Nayagam et al. [16]:

$$\lambda_m = \varepsilon\lambda_f + (1-\varepsilon)\lambda_s \tag{12}$$

where λ_f and λ_s are the thermal conductivities of the fluid and solid respectively. Hence the expression for the thermal diffusivity has the form

$$\alpha = \alpha_\infty\left[\varepsilon_\infty(1 + de^{-y/\gamma}) + \sigma\left\{1 - \varepsilon_\infty(1 + de^{-y/\gamma})\right\}\right] \tag{13}$$

where $\alpha_\infty = \lambda_f/(\rho_\infty c_p)_f$ and $\sigma = \lambda_s/\lambda_f$. Equations (6) and (7) become

$$f'' + \frac{d^* e^{-\eta}}{1 + d^* e^{-\eta}}f' = (1 + d^* e^{-\eta})\theta' \tag{14}$$

$$\frac{\alpha(\eta)}{\alpha_\infty}\theta'' + \frac{d}{d\eta}\left(\frac{\alpha(\eta)}{\alpha_\infty}\right)\theta' = mf'\theta - \frac{m+1}{2}f\theta' \tag{15}$$

with boundary conditions

$$\left.\begin{array}{ll} f(0) = 0 & \theta(0) = 1 \\ f'(\infty) = 0 & \theta(\infty) = 0 \end{array}\right\} \tag{16}$$

In the above equations, the primes denote the derivatives with respect to η and $\gamma = x/Ra_x^{1/2}$. In terms of new variables, it can be shown that the velocity components and the local Nusselt number are given by

$$u(x,y) = \frac{\alpha_\infty Ra_x}{x} f'(\eta) \tag{17}$$

$$v(x,y) = -\frac{\alpha_\infty Ra_x^{1/2}}{2x}\left[(m+1)f + (m-1)\eta f'\right] \tag{18}$$

$$Nu_x/Ra_x^{1/2} = -\theta'(0) \tag{19}$$

2.2. *The disturbance flow*

The standard method of linear stability theory yields the following:

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{20}$$

$$u_1 = -\frac{K}{\mu}\left[\frac{\partial P_1}{\partial x} - \rho_\infty g\beta T_1 \cos\phi\right] \tag{21}$$

$$v_1 = -\frac{K}{\mu}\left[\frac{\partial P_1}{\partial y} - \rho_\infty g\beta T_1 \sin\phi\right] \tag{22}$$

$$w_1 = -\frac{K}{\mu}\frac{\partial P_1}{\partial z} \tag{23}$$

$$u_0\frac{\partial T_1}{\partial x} + v_0\frac{\partial T_1}{\partial y} + u_1\frac{\partial T_0}{\partial x} + v_1\frac{\partial T_0}{\partial y} = \alpha\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial}{\partial y}\left(\alpha\frac{\partial T_1}{\partial y}\right) + \alpha\frac{\partial^2 T_1}{\partial z^2} \tag{24}$$

where the subscripts 0 and 1 signify the base flow and disturbance components respectively.

Following the method of order of magnitude analysis described in detail by Hsu and Cheng [3], the terms, $\partial u_1/\partial x$, $\partial P_1/\partial x$ and $\partial^2 T_1/\partial x^2$ in Eqs (20), (21) and (24) can be neglected. The omission of $\partial u_1/\partial x$ in Eq. (20) implies the existence of a disturbance stream function ψ_1, such as

$$w_1 = \frac{\partial \psi_1}{\partial y} \qquad v_1 = -\frac{\partial \psi_1}{\partial z} \tag{25}$$

Eliminating P_1 from Eqs (22) and (23) and applying Eq. (25), leads to

$$u_1 = \frac{K}{\mu}\rho_\infty g\beta T_1\cos\phi \tag{26}$$

$$\frac{\partial^2 \psi_1}{\partial y^2} + \frac{\partial^2 \psi_1}{\partial z^2} = \frac{1}{K}\frac{dK}{dy}\frac{\partial \psi_1}{\partial y} - \frac{\rho_\infty g\beta K\sin\phi}{\mu}\frac{\partial T_1}{\partial z} \tag{27}$$

$$u_0\frac{\partial T_1}{\partial x} + v_0\frac{\partial T_1}{\partial y} + u_1\frac{\partial T_0}{\partial x} - \frac{\partial \psi_1}{\partial z}\frac{\partial T_0}{\partial y} = \alpha\left(\frac{\partial^2 T_1}{\partial y^2} + \frac{\partial^2 T_1}{\partial z^2}\right) + \frac{d\alpha}{dy}\frac{\partial T_1}{\partial y} \tag{28}$$

As in Hsu et al. [1], we assume the three–dimensionaless disturbances for neutral stability are of the form

$$(\psi_1, u_1, T_1) = [\overline{\psi}(x,y), \overline{u}(x,y), \overline{T}(x,y)] \exp(iaz) \tag{29}$$

where a is the spanwise periodic wave number. Substituting Eq. (29) into Eqs (26)–(28) yields

$$\overline{u} = \frac{K}{\mu}\rho_\infty g\beta\overline{T}\cos\phi \tag{30}$$

$$\frac{\partial^2\overline{\psi}}{\partial y^2} - a^2\overline{\psi} = \frac{1}{K}\frac{dK}{dy}\frac{\partial\overline{\psi}}{\partial y} - ia\frac{\rho_\infty g\beta K\sin\phi}{\mu}\overline{T} \tag{31}$$

$$\alpha\left(\frac{\partial^2\overline{T}}{\partial y^2} - a^2\overline{T}\right) + \frac{d\alpha}{dy}\frac{\partial\overline{T}}{\partial y} = u_0\frac{\partial\overline{T}}{\partial x} + v_0\frac{\partial\overline{T}}{\partial y} + \overline{u}\frac{\partial T_0}{\partial x} - ia\overline{\psi}\frac{\partial T_0}{\partial y} \tag{32}$$

Eqs (30)–(32) are solved based on the local similarity approximation (Sparrow et al. [17]), wherein the disturbances are assumed to have weak dependence in the streamwise direction (i.e. $\partial/\partial x << \partial/\partial\eta$). Introducing the following dimensionless quantities (Hsu et. al. [1]),

$$k = \frac{ax}{Ra_x^{1/2}} \qquad F(\eta) = \frac{\overline{\psi}}{i\alpha_\infty Ra_x^{1/2}} \qquad \Theta(\eta) = \frac{\overline{T}}{Ax^m} \tag{33}$$

we obtain the following system of equations for the local similarity approximation:

$$F'' - k^2F + \frac{d^*e^{-\eta}}{1+d^*e^{-\eta}}F' = -k\tan\phi\,(1+d^*e^{-\eta})\Theta \tag{34}$$

$$\begin{aligned}&\frac{\alpha(\eta)}{\alpha_\infty}\left[\Theta'' - k^2\Theta\right] + \frac{d}{d\eta}\left[\frac{\alpha(\eta)}{\alpha_\infty}\right]\Theta' = mf'\Theta - \frac{m+1}{2}f\Theta' \\ &+kRa_x^{1/2}\theta'F + (1+d^*e^{-\eta})\left[m\theta + \frac{m-1}{2}\eta\theta'\right]\Theta\end{aligned} \tag{35}$$

Then the substitution of Θ from Eq. (34) into Eq. (35) yields

$$\begin{aligned}&G\frac{\alpha(\eta)}{\alpha_\infty}F'''' + \left[3G'\frac{\alpha(\eta)}{\alpha_\infty} + \left(\frac{d}{d\eta}\left(\frac{\alpha(\eta)}{\alpha_\infty}\right) + \frac{m+1}{2}f\right)G\right]F''' \\ &+\left[\frac{\alpha(\eta)}{\alpha_\infty}(3G'' - 2k^2G) + 2G'\left(\frac{d}{d\eta}\left(\frac{\alpha(\eta)}{\alpha_\infty}\right) + \frac{m+1}{2}f\right)\right. \\ &\left.-mGf' - \left(m\theta + \frac{m-1}{2}\eta\theta'\right)\right]F'' \\ &+\left[\frac{\alpha(\eta)}{\alpha_\infty}(G''' - 3k^2G') + \left(\frac{d}{d\eta}\left(\frac{\alpha(\eta)}{\alpha_\infty}\right) + \frac{m+1}{2}f\right)(G'' - k^2G)\right. \\ &\left.-mG'f' - \frac{G'}{G}\left(m\theta + \frac{m-1}{2}\eta\theta'\right)\right]F' \\ &+\left[\frac{\alpha(\eta)}{\alpha_\infty}(k^4G - k^2G'') - k^2G'\left(\frac{d}{d\eta}\left(\frac{\alpha(\eta)}{\alpha_\infty}\right) + \frac{m+1}{2}f\right) + mk^2Gf'\right. \\ &\left.+k^2\left(m\theta + \frac{m-1}{2}\eta\theta'\right)\right]F = -k^2Ra_x^{1/2}\tan\phi\,\theta'F\end{aligned} \tag{36}$$

with the boundary conditions

$$F(0) = F''(0) = F(\infty) = F''(\infty) = 0 \tag{37}$$

where

$$G = \frac{1}{1 + d^* e^{-\eta}}$$

Eq. (36) along with its boundary condition (37) constitutes a fourth–order system of linear ordinary differential equations for the disturbance amplitude distributions $F(\eta)$. For fixed values of d, d^*, ε_∞, σ, k and m, the solution F is an eigenfunction for the eigenvalue $Ra_x \tan^2 \phi$.

3. Numerical Scheme

In this section, we compute the approximate value of $Ra_x \tan^2 \phi$ for the Eq. (36) with the boundary conditions (37). An implicit finite difference method is used to solve first the base flow, system Eqs (14) and (15), and the results are stored for a fixed step size h, which is small enough to predict accurate linear interpolation between mesh points. The domain is $0 \le \eta \le \eta_\infty$, where η_∞ is the edge of the boundary layer of the basic flow. The problem is discretized with standard centered finite differences of order two, following Usmani [18]. Solving eigenvalue problem is achieved by using the subroutine GVLRG of the IMSL library, see [19].

4. Results and Discussion

Numerical results are obtained for the parameter m, for both uniform permeability (UP), i.e. $d = d^* = 0$ and variable permeability (VP), i.e. $d, d^* \neq 0$ cases. For the purpose of numerical integration we have assumed $d = 1.5$, $d^* = 3$, $\sigma = 0.2$ and $\varepsilon_\infty = 0.4$.

Figs 2–3 show the effect of the parameter m on the dimensionless tangential velocity and temperature profiles. It seen that, the velocity gradient at the wall increases and the momentum boundary layer thickness decreases as$m$ increases. Also, as m increase, the thermal boundary layer thickness decreases and the temperature gradient at the wall increases. This means a higher value of the heat transfer rate is associated with higher values m. Further, from these figures, variable permeability effect increases the velocity gradient and reduces the thermal boundary layer leading to an enhancement of heat transfer rate.

Numerical solutions of the local Nusselt number for selected values of m are shown in Fig. 4 for UP and VP cases. As expected, the local Nusselt number increases as m increases. This increment is more higher for VP case than UP one. Fig. 5 shows the neutral stability curves, for the present problem where the eigenvalues $Ra_x \tan^2 \phi$ as a function of dimensionless wave number k for selected values of m and for both uniform permeability UP and variable permeability VP. It is observed that as m increases, the neutral stability curves shift to higher parameter $Ra_x \tan^2 \phi$. In the case of the variable permeability, the neutral stability curves shift to lower $Ra_x \tan^2 \phi$ and higher wave number k, indicating a destabilization of the flow. At a given value of m, the minimum value of $Ra_x \tan^2 \phi$ as shown in Fig. 5, is the critical parameter for the onset of vortex instability in free convection flow about inclined surface in porous medium.

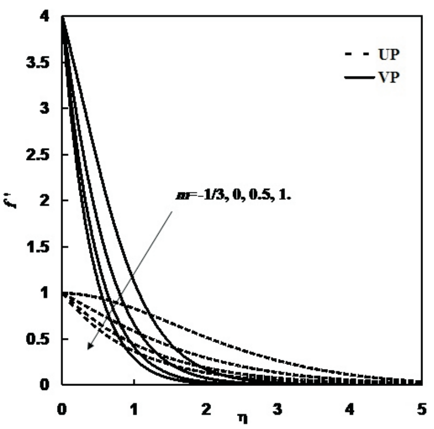

Figure 2 Velocity profile for selected values of m

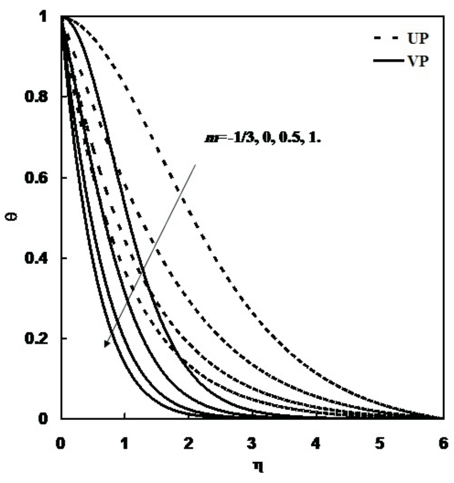

Figure 3 Temperature profile for selected values of m

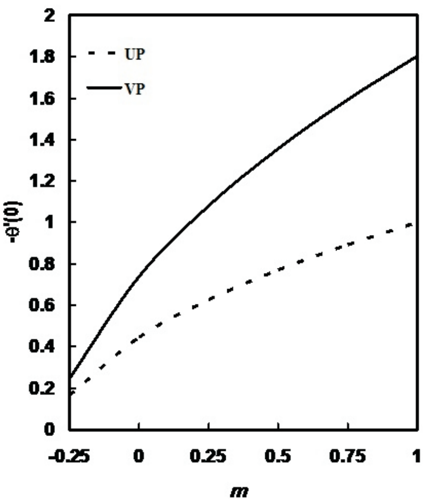

Figure 4 Local Nusselt number vs m

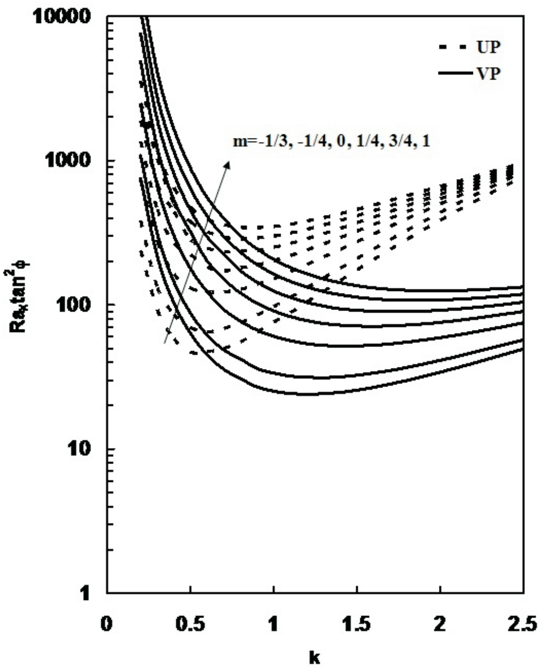

Figure 5 Eigenvalues as a function of dimensionless spanwise wave number

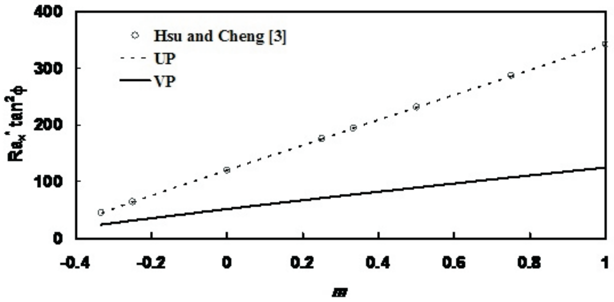

Figure 6 The critical eigenvalues as a function of m

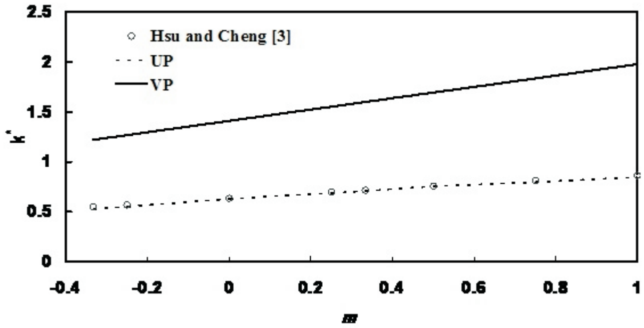

Figure 7 The critical wave numbers as a function of m

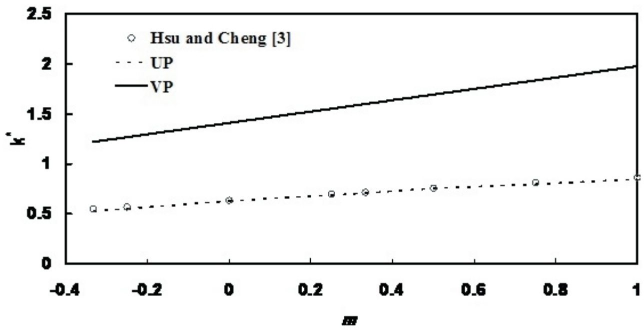

Figure 8 Values of $Ra_x^* \tan\phi$ for selected values of m

The critical parameter $Ra_x^* \tan^2 \phi$ and the associated wave number k^*as a function of m for both UP and VP cases are shown in Figs 6 and 7. In general the values of $Ra_x^* \tan^2 \phi$ and k^* increase as m increases for both UP and VP cases. Further, the results indicate that the variable permeability effect tends to destabilize the flow to the vortex mode of disturbance. Furthermore, the results of the critical values of the parameters $Ra_x^* \tan^2 \phi$ and k^* in UP case are in good agreement with Hsu and Cheng [3] as shown in Figs 6 and 7. It will be of interest to examine the special case of a vertical impermeable surface with $\phi = 0$. The finite value of $Ra_x^* \tan^2 \phi$ implies that the critical Raleigh number $Ra_x^* = K_\infty \rho_\infty g\beta(T_w - T_\infty)x/\mu\alpha_\infty$ for vortex instability in free convection flow about a vertical surface in a porous medium is infinite since both $\sin \phi$ and $\tan \phi$ approach to zero as $\phi \to 0$. It follows therefore that vortex mode of instability will not manifest in free convection flow about a vertical surface in a porous medium. Fig. 8 is a plot showing the effect of inclination angle ϕ from $\phi = 0$ (for a vertical surface) up to $\phi = 65^0$ for which the present analysis is valid. It is found that the larger the inclination angle with respect to the vertical, the more susceptible is the flow for the vortex mode of disturbances; and the limit of zero inclination angle (i.e. vertical heated plate) the flow is stable for this form of disturbances.

5. Conclusions

Vortex instability of free convection flow over an inclined impermeable surface in a porous medium incorporating the variation of permeability and thermal conductivity is studied, where the wall temperature is a power function of the distance from the origin. The permeability of the medium is assumed to vary exponentially with distance from the wall. Velocity and temperature profiles as well as local Nusselt number for the base flow are presented for the uniform and variable permeability cases. The numerical results demonstrate that variable permeability effect tends to increase the heat transfer rate and destabilize the flow to the vortex mode of disturbance. It is found that the larger the inclination angle with respect to the vertical, the more susceptible is the flow for the vortex mode of disturbances; and the limit of zero inclination angle (i.e vertical heated plate) the flow is stable for this form of disturbances.

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Nomenclature

A , m	real constants in equation (8)
d, d^*	constants defined in equations (10), (11)
f	dimensionless base state stream function
F	dimensionless disturbance stream function
g	gravitational acceleration
i	complex number
k	dimensionless wave number
a	spanwise wave number
P	Pressure
K	permeability of porous medium
Nu_x	local Nusselt number
Ra_x	local Rayliegh number
T	fluid temperature
u	Darcy's velocity in x-direction
v	Darcy's velocity in y-direction
w	Darcy's velocity in z-direction
x	coordinate in downstream direction
y	coordinate normal to bounding surface
z	coordinate tangent to bounding surface
Greek symbols	
α	thermal diffusivity
ϕ	inclination angle
β	volumetric coefficient of thermal expansion
ε	porosity of the saturated porous medium
η	pseudo-similarity variable
θ	dimensionless base state temperature
Θ	dimensionless disturbance temperature
λ_f	thermal conductivity of the fluid
λ_s	thermal conductivity of the solid
λ_m	effective thermal conductivity of the saturated porous medium
μ	dynamic viscosity of the fluid
ρ	fluid density
σ	ratio of thermal conductivity of the solid to the fluid
ψ	stream function
Subscripts	
w	conditions at the wall
∞	conditions at the free stream
0	basic undisturbed quantities
1	disturbed quantities
*	critical value
/	differentiation with respect to η