Mechanics and Mechanical Engineering
Vol. 13, No. 2 (2009) 69–79

Circular Pipe Flow of a Dusty Casson Fluid Considering the Hall Effect

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Received (10 October 2008)
Revised (5 March 2009)
Accepted (8 June 2009)

In this paper, the transient flow of a dusty viscous incompressible electrically conducting non–Newtonian Casson fluid through a circular pipe is studied taking the Hall effect into consideration. A constant pressure gradient in the axial direction and an uniform magnetic field directed perpendicular to the flow direction are applied. The particle–phase is assumed to behave as a viscous fluid. A numerical solution is obtained for the governing nonlinear equations using finite differences.

Keywords: Fluid mechanics, magneto–fluid mechanics, computational fluid, flow in channels, circular pipe flow, non–Newtonian fluid, Casson fluid

1. Introduction

The flow of a dusty and electrically conducting fluid through a circular pipe in the presence of a transverse magnetic field has important applications such as magnetohydrodynamic (MHD) generators, pumps, accelerators, and flowmeters. The performance and efficiency of these devices are influenced by the presence of suspended solid particles in the form of ash or soot as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. When the particle concentration becomes high, mutual particle interaction leads to higher particle–phase viscous stresses and can be accounted for by endowing the particle phase by the so–called particle–phase viscosity. There have been many articles dealing with theoretical modelling and experimental measurements of the particle–phase viscosity in a dusty fluid (Soo [1], Gidaspow et al. [2], Grace [3], and Sinclair et al. [4]).

The flow of a conducting fluid in a circular pipe has been investigated by many authors (Gadiraju et al. [5], Dube et al. [6], Ritter et al. [7], and Chamkha [8]). Gadiraju et al. [5] investigated steady two–phase vertical flow in a pipe. Dube et al. [6] and Ritter et al. [7] reported solutions for unsteady dusty-gas flow in a circular pipe in the absence of a magnetic field and particle–phase viscous stresses. Chamkha [8] obtained exact solutions which generalize the results reported in Dube et al. [6] and Ritter et al. [7] by the inclusion of the magnetic and particle–phase viscous effects. It should be noted that in the above studies the Hall effect is ignored.

A number of industrially important fluids such as molten plastics, polymers, pulps and foods exhibit non–Newtonian fluid behavior (Nakayama et al. [10]). Due to the growing use of these non–Newtonian materials, in various manufacturing and processing industries, considerable efforts have been directed towards understanding their flow characteristics. Many of the inelastic non–Newtonian fluids, encountered in chemical engineering processes, are known to follow the so–called "power–law model" in which the shear stress varies according to a power function of the strain rate (Metzner et al. [11]). It is of interest in this paper to study the influence of the magnetic field as well as the non–Newtonian fluid characteristics on the dusty fluid flow properties in situations where the particle–phase is considered dense enough to include the particulate viscous stresses.

In the present study, the unsteady flow of a dusty non–Newtonian Casson fluid through a circular pipe is investigated considering the Hall effect. The carrier fluid is assumed viscous, incompressible and electrically conducting. The particle phase is assumed to be incompressible pressureless and electrically non–conducting. The flow in the pipe starts from rest through the application of a constant axial pressure gradient. The governing nonlinear momentum equations for both the fluid and particle–phases are solved numerically using the finite difference approximations. The effect of the Hall current, the non–Newtonian fluid characteristics and the particle–phase viscosity on the velocity of the fluid and particle–phases are reported.

2. Governing equations

Consider the unsteady, laminar, and axisymmetric horizontal flow of a dusty conducting non–Newtonian Casson fluid through an infinitely long pipe of radius "d" driven by a constant pressure gradient. A uniform magnetic field is applied perpendicular to the flow direction. The Hall current is taken into consideration and the magnetic Reynolds number is assumed to be very small, consequently the induced magnetic field is neglected (Sutton et al. [12]). We assume that both phases behave as viscous fluids and that the volume fraction of suspended particles is finite and constant (Chamkha [8]). Taking into account these and the previously mentioned assumptions, the governing momentum equations can be written as

$$\rho \frac{\partial V}{\partial t} = -\frac{\partial P}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(\mu r \frac{\partial V}{\partial r}\right) + \frac{\rho_p \phi}{1-\phi} N(V_p - V) - \frac{\sigma B_o^2 V}{1+m^2} \tag{1}$$

$$\rho_p \frac{\partial V_p}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(\mu_p r \frac{\partial V_p}{\partial r}\right) + \rho_p N(V - V_p) \tag{2}$$

where t is the time, r is the distance in the radial direction, V is the fluid–phase velocity, V_p is the particle–phase velocity, ρ is the fluid–phase density, ρ_p is the particle–phase density, $\partial P/\partial z$ is the fluid pressure gradient, ϕ is the particle–phase volume fraction, N is a momentum transfer coefficient (the reciprocal of the relaxation time, the time needed for the relative velocity between the phases to reduce e^{-1}

of its original value (Chamkha [8]), σ is the fluid electrical conductivity, $m = \sigma\beta B_o$ is the Hall parameter, β is the Hall factor (Sutton et al. [12]), B_o is the magnetic induction, μ_p is the particle–phase viscosity which is assumed constant, and μ is the apparent viscosity of the fluid which is given by,

$$\mu = \left(K_c + \sqrt{\frac{\tau_o}{\left|\frac{\partial V}{\partial r}\right|}} \right)^2$$

where K_c is the coefficient of viscosity of a Casson fluid, τ_o is the yield stress, and $|\partial V/\partial r|$ is the magnitude of the velocity gradient which is always positive regardless of the sign of $\partial V/\partial r$. In this work, ρ, ρ_p, μ_p, ϕ, and B_o are all constant. It should be pointed out that the particle–phase pressure is assumed negligible and that the particles are being dragged along with the fluid–phase.

The initial and boundary conditions of the problem are given as

$$V(r,0) = 0, V_p(r,0) = 0, \tag{3}$$

$$\frac{\partial V(0,t)}{\partial r} = 0, \frac{\partial V_p(0,t)}{\partial r} = 0, V(d,t) = 0, V_p(d,t) = 0 \tag{4}$$

where d is the pipe radius.

Eqs (1)–(4) constitute a nonlinear initial–value problem which can be made dimensionless by introducing the following dimensionless variables and parameters

$$\bar{r} = \frac{r}{d}$$

$$\bar{t} = \frac{tK_c}{\rho d^2}$$

$$G_o = -\frac{\partial P}{\partial z}$$

$$k = \frac{\rho_p \phi}{\rho(1-\phi)}$$

$$\bar{\mu} = \frac{\mu}{K_c}$$

$$\bar{V}(r,t) = \frac{K_c V(r,t)}{G_o d^2}$$

$$\bar{V}_p(r,t) = \frac{K_c V_p(r,t)}{G_o d^2}$$

$\alpha = Nd^2\rho/K_c$ is the inverse Stoke's number

$\beta = \mu_p/K_c$ is the viscosity ratio

$\tau_D = \tau_o/G_o d$ is the Casson number (dimensionless yield stress)

$H_a = B_o d\sqrt{\sigma/K_c}$ is the Hartmann number (Sutton et al. [12])

By introducing the above dimensionless variables and parameters as well as the expression of the fluid viscosity defined above, Eqs (1)–(4) can be written as (the bars are dropped),

$$\frac{\partial V}{\partial t} = 1 + \sqrt{\mu}\frac{\partial^2 V}{\partial r^2} + \frac{\mu}{r}\frac{\partial V}{\partial r} + k\alpha(V_p - V) - \frac{H_a^2 V}{1+m^2} \tag{5}$$

$$\frac{\partial V_p}{\partial t} = \beta\left(\frac{\partial^2 V_p}{\partial r^2} + \frac{1}{r}\frac{\partial V_p}{\partial r}\right) + \alpha(V - V_p) \tag{6}$$

$$\mu = \left(1 + \sqrt{\frac{\tau_D}{\left|\frac{\partial V}{\partial r}\right|}}\right)^2$$

$$V(r,0) = 0 \quad V_p(r,0) = 0, \tag{7}$$

$$\frac{\partial V(0,t)}{\partial r} = 0 \quad \frac{\partial V_p(0,t)}{\partial r} = 0 \quad V(1,t) = 0 \quad V_p(1,t) = 0 \tag{8}$$

The volumetric flow rates and skin–friction coefficients for both the fluid and particle phases are defined, respectively, as (Chamkha [8])

$$\begin{aligned} Q &= 2\pi\int_0^1 rV(r,t)dr \\ Q_p &= 2\pi\int_0^1 rV_p(r,t)dr \\ C &= -\frac{\partial V(1,t)}{\partial r} \\ C_p &= -\beta k\frac{\partial V_p(1,t)}{\partial r} \end{aligned} \tag{9}$$

3. Results and Discussion

Eqs (5) and (6) represent a coupled system of nonlinear partial differential equations which are solved numerically under the initial and boundary conditions (7) and (8), using the finite difference approximations. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank–Nicolson implicit method (Mitchell et al. [13] and Evans et al. [14]) is used at two successive time levels. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tridiagonal system is solved using the generalized Thomas algorithm (Mitchell et al. [13] and Evans et al. [14]). Computations have been made for $\alpha = 1$ and $k = 10$. Grid–independence studies show that the computational domain $0 < t < \infty$ and $0 < r < 1$ can be divided into intervals with step sizes $\Delta t = 0.0001$ and $\Delta r = 0.005$ for time and space respectively. It should be mentioned that the results obtained herein reduce to those reported by Dube et al. [6] and Chamkha [8] for the cases of non–magnetic, inviscid particle–phase (β=0), and Newtonian fluid. These comparisons lend confidence in the accuracy and correctness of the solutions.

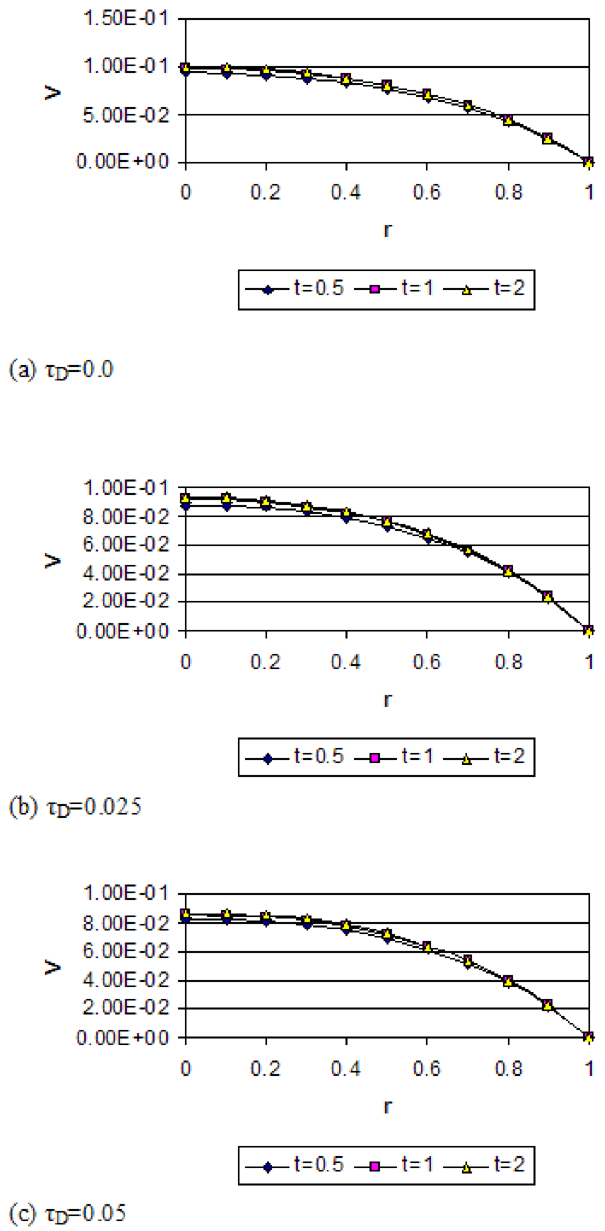
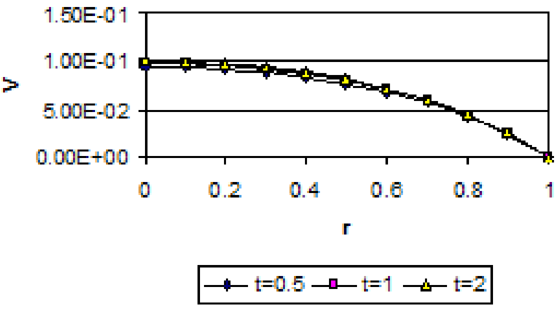
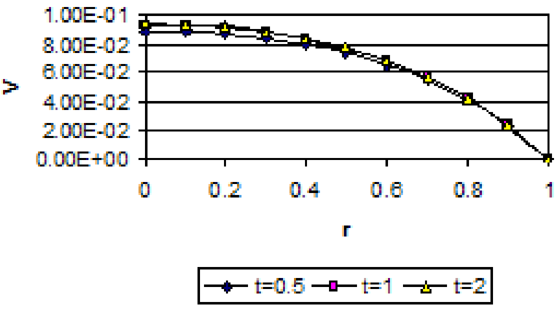


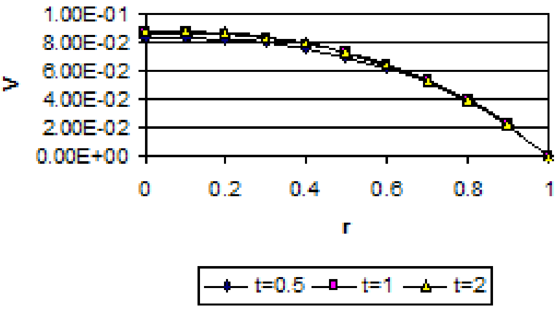
Figure 1 Time development of the profile of V for various values of τ_D ($m = 0,\ H_a = 0.5,\ \beta = 0.5$)



(a) τ_D=0.0

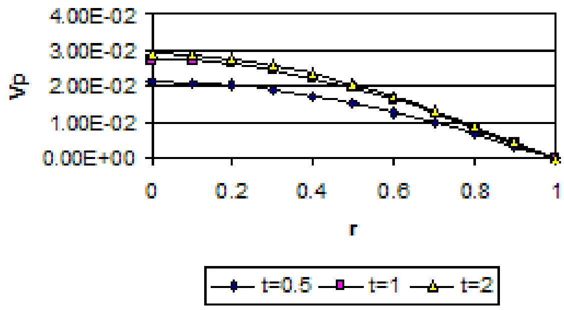


(b) τ_D=0.025

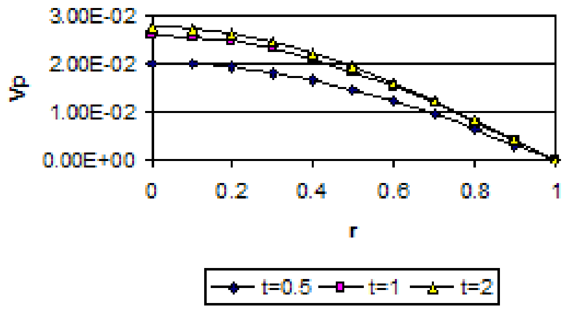


(c) τ_D=0.05

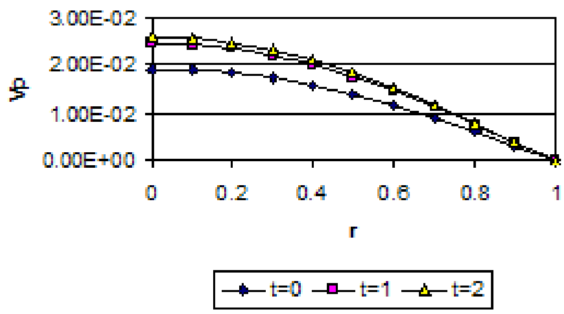
Figure 2 Time development of the profile of V for various values of τ_D ($m = 1$, $H_a = 0.5$, $\beta = 0.5$)



(a) τ_D=0.0



(b) τ_D=0.025



(c) τ_D=0.05

Figure 3 Time development of the profile of V_p for various values of τ_D ($m = 0,\ H_a = 0.5,\ \beta = 0.5$)

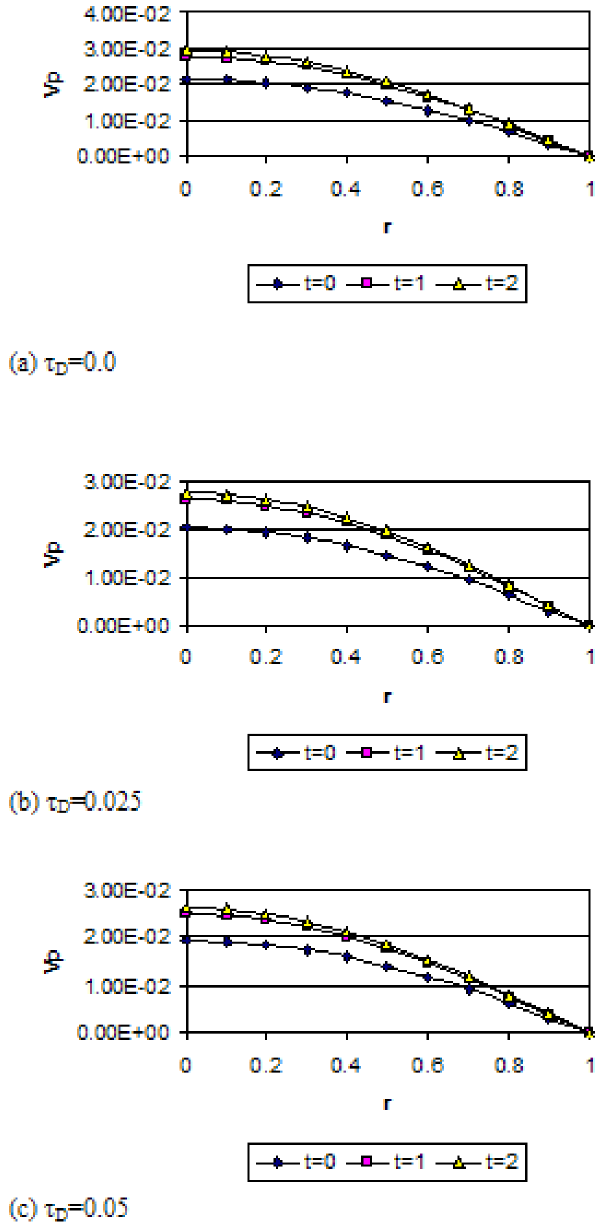


Figure 4 Time development of the profile of V_p for various values of τ_D ($m = 1,\ H_a = 0.5,\ \beta = 0.5$)

Figs 1–4 present the time evolution of the profiles of the velocity of the fluid V and dust particles V_p, respectively for various values of the Bingham number τ_D and the Hall parameter m and for H_a=0.5 and β=0.5. Both V and V_p increase with time and V reaches the steady–state faster than V_p for all values of τ_D. It is clear from the figures that increasing τ_D increases both V and V_p while its effect on their steady–state times can be neglected. It is indicated in the figures that increasing m increases V and, in turn, V_p due to the decrease in the effective conductivity $(\sigma/(1+m^2))$ which reduces the damping magnetic force on V. It is shown that the influence of the Hall parameter m on V is more apparent for higher values of τ_D.

Table 1 The steady state values of Q, Q_p, C, C_p for various values of m and τ_D

τ_D=0	$m = 0$	$m = 1$	$m = 2$
Q	0.1763	0.1779	0.1789
Q_p	0.0425	0.0429	0.0432
C	0.2817	0.2833	0.2843
C_p	0.2106	0.2125	0.2136
τ_D=0.025	$m = 0$	$m = 1$	$m = 2$
Q	0.1669	0.1684	0.1693
Q_p	0.0403	0.0406	0.0408
C	0.2671	0.2686	0.2695
C_p	0.1995	0.2012	0.2022
τ_D=0.05	$m = 0$	$m = 1$	$m = 2$
Q	0.1576	0.1589	0.1597
Q_p	0.0379	0.0383	0.0385
C	0.2523	0.2537	0.2546
C_p	0.1883	0.1899	0.1908

Table 1 presents the steady state values of the fluid–phase volumetric flow rate Q, the particle–phase volumetric flow rate Q_p, the fluid–phase skin friction coefficient C, and the particle–phase skin friction coefficient C_p for various values of the parameters τ_D and m and for H_a=0.5 and β=0.5. It is clear that increasing the parameter m increases Q, Q_p, C, and C_p for all values of τ_D. It is also shown that increasing τ_D increases Q, Q_p, C, and C_p for all values of m. Table 2 presents the steady state values of the fluid–phase volumetric flow rate Q, the particle–phase volumetric flow rate Q_p, the fluid–phase skin friction coefficient C, and the particle–phase skin friction coefficient C_p for various values of the parameters m and β and for H_a=0.5 and τ_D=0. It is clear that, increasing m increases Q, Q_p, C, and C_p for all values of β and its effect becomes more pronounced for smaller values of β. Increasing the parameter β decreases the quantities Q, Q_p, and C, but increases C_p for all values of m.

4. Conclusion

The transient MHD flow of a particulate suspension in an electrically conducting non–Newtonian Casson fluid in a circular pipe is studied considering the Hall effect.

Table 2 The steady state values of Q, Q_p, C, C_p for various values of m and β

$\beta = 0$	$m = 0$	$m = 1$	$m = 2$
Q	0.2653	0.2686	0.2706
Q_p	0.1975	0.1997	0.2009
C	0.3759	0.3791	0.3811
C_p	0	0	0
$\beta = 0.5$	$m = 0$	$m = 1$	$m = 2$
Q	0.1763	0.1779	0.1789
Q_p	0.0425	0.0429	0.0432
C	0.2817	0.2833	0.2843
C_p	0.2106	0.2125	0.2136
$\beta = 1$	$m = 0$	$m = 1$	$m = 2$
Q	0.1640	0.1654	0.1662
Q_p	0.0226	0.0228	0.0229
C	0.2702	0.2716	0.2724
C_p	0.2231	0.2249	0.2260

The governing nonlinear partial differential equations are solved numerically using finite differences. The effect of the magnetic field parameter H_a, the Hall parameter, the non–Newtonian fluid characteristics (Bingham number τ_D), and the particle–phase viscosity β on the transient behavior of the velocity, volumetric flow rates, and skin friction coefficients of both fluid and particle–phases is studied. It is shown that increasing the magnetic field decreases the fluid and particle velocities, while increasing the Hall parameter increases both velocities. It is found that increasing the parameter m increases Q, Q_p, C, and C_p for all values of τ_D. The effect of the Hall parameter on the quantities Q, Q_p, C, and C_p becomes more pronounced for smaller values of β.

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