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Optimal Grneralized Coplanar Bi–elliptic Transfer Orbits Part II

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In this part II, we extend our analysis to include all of the four feasible configurations. We have four generalized bi–elliptic configurations for the transfer problem, for a central gravitational field. We apply three impulses as usual for the bi–elliptic case, at the points A, C, B. x, z are our independent variables and are equal to the ratio between values of the velocities after and before the application of the impulses at points of pericenter and apocenter. Similarly y is defined as the corresponding parameter for the point C. We utilize the optimum condition of ordinary infinitesimal calculus for algebraic functions to evaluate the minimum values of x, z, y. In this part II we expand the domain of application of the numerical results.

Keywords: Rocket dynamics, orbital mechanics, bi–elliptic transfer, optimization

1. Introduction

Most generally, the change of kinematic conditions represented by t_i, r_i, v_i $\rightarrow$ t_f, r_f, v_f is the definition of a "transfer", where t is the physical time, r the radius, and v the velocity. Deterministic aspects of optimization of rendez–vous orbital transfer is an essential application. Among all three impulse transfers, applying the gradient method, the simple bi–elliptic transfer is the most economic or equivalently the optimal transfer. If $11.94 < R < 15.58$ and midcourse impulse location r_i ($r_i > r_2$) is sufficiently large, then the bi–elliptic transfer is more economic than the Hohmann transfer [1]. L. Ting demonstrated that for optimality the terminal and transfer trajectories should be coplanar [2]. Billik and Roth discussed, in quite a general manner, the two dimensional simple bi–elliptic transfer, with or without parking in one of the two transfer ellipses. They concluded that the bi–elliptic transfer is and alternative of the Hohmann transfer where $r_f/r_i \approx 1$ [3]. Moreover, four or more impulse transfers are never optimal. The bi–elliptic and the three impulse transfers connect pericenters. The intermediate impulse is always at the outer limit of the annulus [4].

2. Methods and results

In a previous research paper [5], we wrote down the calculations for the first configuration Fig. 1. Herein, we cite the computations for the second, third and fourth configurations.

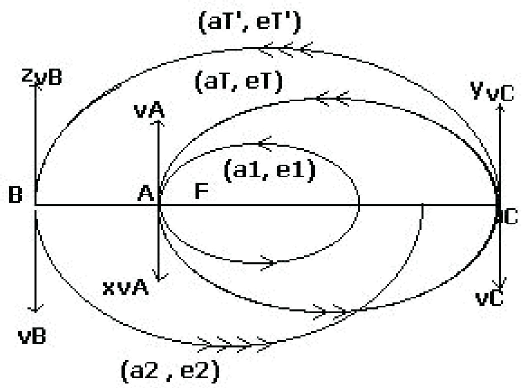

Figure 1

2.1. *Calculations for the second configuration*

For Fig. 2, we have

$$\begin{aligned}
v_A &= \sqrt{\frac{\mu\,(1+e_1)}{a_1\,(1-e_1)}} & xv_A &= \sqrt{\frac{\mu\,(1+e_T)}{a_T\,(1-e_T)}} \\
v_C &= \sqrt{\frac{\mu\,(1-e_T)}{a_T\,(1+e_T)}} & yv_C &= \sqrt{\frac{\mu\,(1+e_{T'})}{a_{T'}\,(1-e_{T'})}} \\
v_B &= \sqrt{\frac{\mu\,(1-e_{T'})}{a_{T'}\,(1+e_{T'})}} & zv_B &= \sqrt{\frac{\mu\,(1-e_2)}{a_2\,(1+e_2)}}
\end{aligned} \tag{1}$$

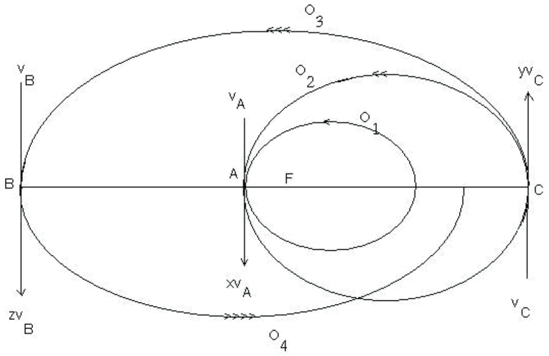

Figure 2

Accordingly

$$x = \frac{xv_A}{v_A} = \sqrt{\frac{1+e_T}{1+e_1}}$$
$$y = \frac{yv_C}{v_C} = \sqrt{\frac{1+e_{T'}}{1-e_T}} \tag{2}$$
$$z = \frac{zv_B}{v_B} = \sqrt{\frac{1-e_2}{1-e_{T'}}}$$

Where

$$a_1 (1 - e_1) = a_T (1 - e_T) \tag{3}$$
$$a_T (1 + e_T) = a_{T^{\cdot}} (1 - e_{T^{\cdot}}) \tag{4}$$
$$a_{T^{\cdot}} (1 + e_{T^{\cdot}}) = a_2 (1 + e_2) \tag{5}$$

and

$$b_1 = a_1 (1 - e_1) \qquad b_2 = a_1 (1 + e_1) \tag{6}$$
$$b_3 = a_2 (1 - e_2) \qquad b_4 = a_2 (1 + e_2)$$

whence

$$1 - e_T = 2 - x^2 (1 + e_1) \qquad 1 + e_T = x^2 (1 + e_1) \tag{7}$$
$$1 - e_{T'} = \frac{1 - e_2}{z^2} \qquad 1 + e_{T'} = 2 - \frac{1 - e_2}{z^2}$$

and

$$y = \frac{\sqrt{2z^2 - 1 + e_2}}{z\sqrt{2 - x^2 (1 + e_1)}} \tag{8}$$

By differentiation of Eq. (8) w.r.t. x, we get

$$\frac{\partial y}{\partial x} = \frac{x (1 + e_1) \sqrt{2z^2 - (1 - e_2)}}{z \left\{2 - x^2 (1 + e_1)\right\}^{3/2}} \tag{9}$$

And differentiating Eq. (8) w.r.t. z, then

$$\frac{\partial y}{\partial z} = \frac{1-e_2}{z^2\sqrt{\{2-x^2(1+e_1)\}\{2z^2-(1-e_2)\}}} \tag{10}$$

$$\Delta v_T = \Delta v_A + \Delta v_C + \Delta v_B \tag{11}$$

$$\Delta v_T = v_A(x-1) + v_C(y-1) + v_B(z-1) \tag{12}$$

We may write also,

For optimum condition

$$\frac{\partial \Delta v_T}{\partial x} = 0; \frac{\partial \Delta v_T}{\partial y} = 0; \frac{\partial \Delta v_T}{\partial z} = 0 \tag{13}$$

Consequently

$$\frac{\partial \Delta v_T}{\partial x} = v_A + \frac{\partial v_C}{\partial x}(y-1) + v_C\frac{\partial y}{\partial x} = 0 \tag{14}$$

$$v_C = \frac{2-x^2(1+e_1)}{x}\sqrt{\frac{\mu}{b_1(1+e_1)}} \tag{15}$$

But

$$\frac{\partial v_C}{\partial x} = -\frac{2+x^2(1+e_1)}{x^2}\sqrt{\frac{\mu}{b_1(1+e_1)}} \tag{16}$$

whence Eq. (14) can be written as :

$$\sqrt{\frac{\mu(1+e_1)}{b_1}} - \frac{\{2+x^2(1+e_1)\}}{x^2}\sqrt{\frac{\mu}{b_1(1+e_1)}}\left\{\frac{1}{z}\sqrt{\frac{2z^2-1+e_2}{2-x^2(1+e_1)}} - 1\right\}$$
$$+\frac{1}{z}\sqrt{\frac{\mu}{b_1(1+e_1)}}\frac{\{2-x^2(1+e_1)\}}{x}\frac{x(1+e_1)\sqrt{2z^2-1+e_2}}{\{2-x^2(1+e_1)\}^{3/2}} = 0 \tag{17}$$

Let

$$c = 1+e_1 \qquad c_1 = e_2 - 1$$

After some algebraic reductions and rearrangements

$$z^2\left\{3x^2c - x^6c^3\right\} - c_1 = 0 \tag{18}$$

We may write

$$\frac{\partial \Delta v_T}{\partial z} = v_C\frac{\partial y}{\partial z} + (z-1)\frac{\partial v_B}{\partial z} + v_B = 0 \tag{19}$$

where

$$\frac{\partial v_B}{\partial z} = -\frac{1}{z^2}\sqrt{\frac{\mu(1-e_2)}{b_4}} \tag{20}$$

$$v_B = \sqrt{\frac{\mu\frac{1-e_2}{z^2}}{b_4}} = \sqrt{\frac{\mu(1-e_2)}{b_4 z^2}} \tag{21}$$

From Eqs (10), (19) and after some reductions

$$z^2 = \frac{-2c_1 b_4 + x^2 c c_1 (b_4 - b_1)}{2x^2 b_1 c} \tag{22}$$

Let

$$-2c_1 b_4 = c_2; c c_1 (b_4 - b_1) = c_3; 2b_1 c = c_4 \tag{23}$$

i.e.

$$z^2 = \frac{c_2 + x^2 c_3}{x^2 c_4} \tag{24}$$

From the above equations and after some rearrangements and reductions, we get a sixth order algebraic equation, written as follows

$$x^6 + c_6 x^4 + c_7 x^2 + c_8 = 0 \tag{25}$$

Where

$$c_5 = 3c_2 c - c_1 c_4; c_6 = \frac{c_2}{c_3}; c_7 = \frac{-3}{c^2}; c_8 = \frac{c_5}{-c_3 c^3} \tag{26}$$

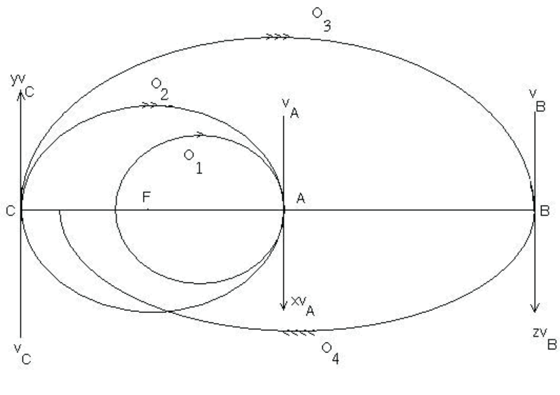

Figure 3

2.2. *Calculations for the third configuration*

For Fig. 3, we have

$$\begin{aligned} v_A &= \sqrt{\frac{\mu(1-e_1)}{a_1(1+e_1)}} & xv_A &= \sqrt{\frac{\mu(1-e_T)}{a_T(1+e_T)}} \\ v_C &= \sqrt{\frac{\mu(1+e_T)}{a_T(1-e_T)}} & yv_C &= \sqrt{\frac{\mu(1+e_{T'})}{a_{T'}(1-e_{T'})}} \\ v_B &= \sqrt{\frac{\mu(1-e_{T'})}{a_{T'}(1+e_{T'})}} & zv_B &= \sqrt{\frac{\mu(1-e_2)}{a_2(1+e_2)}} \end{aligned} \tag{27}$$

Accordingly

$$
\begin{aligned}
x &= \frac{xv_A}{v_A} = \sqrt{\frac{1-e_T}{1-e_1}} \\
y &= \frac{yv_C}{v_C} = \sqrt{\frac{1+e_{T'}}{1+e_T}} \\
z &= \frac{zv_B}{v_B} = \sqrt{\frac{1-e_2}{1-e_{T'}}}
\end{aligned} \tag{28}
$$

Where

$$a_1\left(1+e_1\right) = a_T\left(1+e_T\right) \tag{29}$$

$$a_T\left(1-e_T\right) = a_{T'}\left(1-e_{T'}\right) \tag{30}$$

$$a_{T'}\left(1+e_{T'}\right) = a_2\left(1+e_2\right) \tag{31}$$

whence

$$
\begin{aligned}
1+e_T &= 2 - x^2\left(1-e_1\right) \qquad & 1-e_T &= x^2\left(1-e_1\right) \\
1-e_{T'} &= \frac{1-e_2}{z^2} \qquad & 1+e_{T'} &= 2 - \frac{1-e_2}{z^2}
\end{aligned} \tag{32}
$$

and

$$y = \frac{\sqrt{2z^2 - 1 + e_2}}{z\sqrt{2 - x^2\left(1-e_1\right)}} \tag{33}$$

Let

$$c = 1 - e_1 \qquad c_1 = e_2 - 1$$

By differentiation of Eq. (33) w.r.t. x, we get

$$\frac{\partial y}{\partial x} = \frac{xc\sqrt{2z^2 + c_1}}{z\left\{2 - x^2 c\right\}^{3/2}} \tag{34}$$

And differentiating Eq. (33) w.r.t. z, then

$$\frac{\partial y}{\partial z} = \frac{-c_1}{z^2\sqrt{\left\{2 - x^2 c\right\}\left\{2z^2 + c_1\right\}}} \tag{35}$$

We may write also,

$$\Delta v_T = \Delta v_A + \Delta v_C + \Delta v_B \tag{36}$$

$$\Delta v_T = v_A\left(x-1\right) + v_C\left(y-1\right) + v_B\left(z-1\right) \tag{37}$$

For optimum condition

$$\frac{\partial \Delta v_T}{\partial x} = 0 \qquad \frac{\partial \Delta v_T}{\partial y} = 0 \qquad \frac{\partial \Delta v_T}{\partial z} = 0 \tag{38}$$

Consequently

$$\frac{\partial \Delta v_T}{\partial x} = v_A + \frac{\partial v_C}{\partial x}(y-1) + v_C \frac{\partial y}{\partial x} = 0 \tag{39}$$

$$v_C = \frac{2 - x^2 c}{x}\sqrt{\frac{\mu}{b_2 c}} \tag{40}$$

But

$$\frac{\partial v_C}{\partial x} = -\frac{2 + x^2 c}{x^2}\sqrt{\frac{\mu}{b_2 c}} \tag{41}$$

whence Eq. (39) can be written as :

$$\sqrt{\frac{\mu c}{b_2}} - \frac{\left\{2 + x^2 c\right\}}{x^2}\sqrt{\frac{\mu}{b_2 c}}\left\{\frac{1}{z}\sqrt{\frac{2z^2 + c_1}{2 - x^2 c}} - 1\right\}$$
$$+\frac{1}{z}\sqrt{\frac{\mu}{b_2 c}}\frac{\left\{2 - x^2 c\right\}}{x}\frac{xc\sqrt{2z^2 + c_1}}{\left\{2 - x^2 c\right\}^{3/2}} = 0 \tag{42}$$

After some algebraic reductions and rearrangements

$$z^2\left\{3x^2 c - x^6 c^3\right\} - c_1 = 0 \tag{43}$$

We may write

$$\frac{\partial \Delta v_T}{\partial z} = v_C \frac{\partial y}{\partial z} + (z-1)\frac{\partial v_B}{\partial z} + v_B = 0 \tag{44}$$

$$v_B = \sqrt{\frac{\mu \frac{1-e_2}{z^2}}{b_4}} = \frac{1}{z}\sqrt{\frac{-\mu c_1}{b_4}} \tag{45}$$

where

$$\frac{\partial v_B}{\partial z} = -\frac{1}{z^2}\sqrt{\frac{-\mu c_1}{b_4}} \tag{46}$$

From Eqs (35), (44) and after some reductions

$$z^2 = \frac{2c_1 b_4 + x^2 c c_1 (b_2 - b_4)}{-2x^2 b_2 c} \tag{47}$$

Let

$$-\frac{c_1 (b_2 - b_4)}{2b_2} = c_2 \qquad -\frac{c_1 b_4}{b_2 c} = c_3 \tag{48}$$

i.e.

$$z^2 = c_2 + \frac{c_3}{x^2} \tag{49}$$

From the above equations and after some rearrangements and reductions, we get a sixth order algebraic equation, written as follows

$$x^6 + c_4 x^4 + c_5 x^2 + c_6 = 0 \tag{50}$$

Where

$$c_4 = \frac{c_3}{c_2} \qquad c_5 = \frac{-3}{c^2} \qquad c_6 = \frac{-3c_3}{c_2 c^2} + \frac{c_1}{c_2 c^3} \tag{51}$$

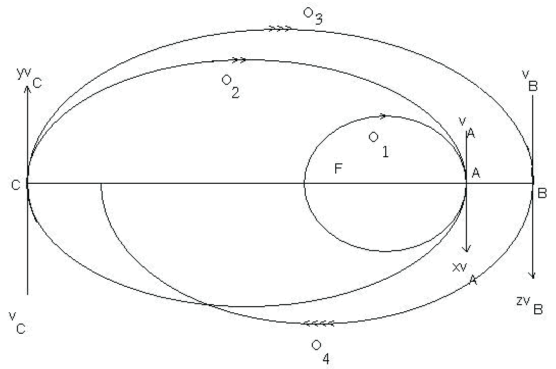

Figure 4

2.3. *Calculations for the fourth configuration*

For Fig. 4., we have

$$
\begin{aligned}
v_A &= \sqrt{\frac{\mu\,(1-e_1)}{a_1\,(1+e_1)}} \qquad & xv_A &= \sqrt{\frac{\mu\,(1+e_T)}{a_T\,(1-e_T)}} \\
v_C &= \sqrt{\frac{\mu\,(1-e_T)}{a_T\,(1+e_T)}} \qquad & yv_C &= \sqrt{\frac{\mu\,(1-e_{T'})}{a_{T'}\,(1+e_{T'})}} \\
v_B &= \sqrt{\frac{\mu\,(1+e_{T'})}{a_{T'}\,(1-e_{T'})}} \qquad & zv_B &= \sqrt{\frac{\mu\,(1+e_2)}{a_2\,(1-e_2)}}
\end{aligned} \tag{52}
$$

Accordingly

$$
\begin{aligned}
x &= \frac{xv_A}{v_A} = \sqrt{\frac{1+e_T}{1-e_1}} \\
y &= \sqrt{\frac{1-e_{T'}}{1-e_T}} \\
z &= \sqrt{\frac{1+e_2}{1+e_{T'}}}
\end{aligned} \tag{53}
$$

Where

$$a_1\,(1+e_1) = a_T\,(1-e_T) \tag{54}$$

$$a_T\,(1+e_T) = a_{T'}\,(1+e_{T'}) \tag{55}$$

$$a_{T'}\,(1-e_{T'}) = a_2\,(1-e_2) \tag{56}$$

whence

$$
\begin{aligned}
1+e_T &= x^2\,(1-e_1) \qquad & 1-e_T &= 2-x^2\,(1-e_1) \\
1+e_{T'} &= \frac{1+e_2}{z^2} \qquad & 1-e_{T'} &= 2-\frac{1+e_2}{z^2}
\end{aligned} \tag{57}
$$

and

$$y=\frac{\sqrt{2z^2-1-e_2}}{z\sqrt{2-x^2\left(1-e_1\right)}} \tag{58}$$

Let

$$c=1-e_1 \qquad c_1=-(e_2+1)$$

By differentiation of Eq. (58) w.r.t. x,we get

$$\frac{\partial y}{\partial x}=\frac{xc\sqrt{2z^2+c_1}}{z\left\{2-x^2c\right\}^{3/2}} \tag{59}$$

And differentiating Eq. (58) w.r.t. z, then

$$\frac{\partial y}{\partial z}=\frac{-c_1}{z^2\sqrt{\left\{2-x^2c\right\}\left\{2z^2+c_1\right\}}} \tag{60}$$

We may write also,

$$\Delta v_T=\Delta v_A+\Delta v_C+\Delta v_B \tag{61}$$

$$\Delta v_T=v_A\left(x-1\right)+v_C\left(y-1\right)+v_B\left(z-1\right) \tag{62}$$

For optimum condition

$$\frac{\partial\Delta v_T}{\partial x}=0 \qquad \frac{\partial\Delta v_T}{\partial y}=0 \qquad \frac{\partial\Delta v_T}{\partial z}=0 \tag{63}$$

$$\frac{\partial\Delta v_T}{\partial x}=v_A+\frac{\partial v_C}{\partial x}\left(y-1\right)+v_C\frac{\partial y}{\partial x}=0 \tag{64}$$

Consequently

$$v_C=\frac{2-x^2c}{x}\sqrt{\frac{\mu}{b_2c}} \tag{65}$$

But

$$\frac{\partial v_C}{\partial x}=-\frac{2+x^2c}{x^2}\sqrt{\frac{\mu}{b_2c}} \tag{66}$$

whence Eq. (64) can be written as :

$$\sqrt{\frac{\mu c}{b_2}}-\frac{\left\{2+x^2c\right\}}{x^2}\sqrt{\frac{\mu}{b_2c}}\left\{\frac{1}{z}\sqrt{\frac{2z^2+c_1}{2-x^2c}}-1\right\}$$
$$+\frac{1}{z}\sqrt{\frac{\mu}{b_2c}}\frac{\left\{2-x^2c\right\}}{x}\frac{xc\sqrt{2z^2+c_1}}{\left\{2-x^2c\right\}^{3/2}}=0 \tag{67}$$

After some algebraic reductions and rearrangements

$$z^2\left\{3x^2c-x^6c^3\right\}-c_1=0 \tag{68}$$

We may write

$$\frac{\partial\Delta v_T}{\partial z}=v_C\frac{\partial y}{\partial z}+\left(z-1\right)\frac{\partial v_B}{\partial z}+v_B=0 \tag{69}$$

where

$$v_B = \frac{1}{z}\sqrt{\frac{-\mu c_1}{b_3}} \tag{70}$$

$$\frac{\partial v_B}{\partial z} = -\frac{1}{z^2}\sqrt{\frac{-\mu c_1}{b_3}} \tag{71}$$

From Eqs. (60), (69) and after some reductions

$$z^2 = \frac{2c_1 b_3 + x^2 c c_1 (b_2 - b_3)}{-2x^2 b_2 c} \tag{72}$$

Let

$$-\frac{c_1 (b_2 - b_3)}{2b_2} = c_2 \qquad -\frac{c_1 b_3}{b_2 c} = c_3 \tag{73}$$

i.e.

$$z^2 = c_2 + \frac{c_3}{x^2} \tag{74}$$

From the above equations and after some rearrangements and reductions, we get a sixth order algebraic equation, written as follows

$$x^6 + c_4 x^4 + c_5 x^2 + c_6 = 0 \tag{75}$$

Where

$$c_4 = \frac{c_3}{c_2} \qquad c_5 = \frac{-3}{c^2} \qquad c_6 = \frac{-3c_3}{c_2 c^2} + \frac{c_1}{c_2 c^3} \tag{76}$$

3. Numerical results

For Fig. (2), we consider two cases

- **Case 1**: Earth – Mars:

  For Earth – Mars , we have[6]

  $a_1 = a_E = 1 \qquad a_2 = a_M = 1.5237$
  $e_1 = e_E = 0.0167 \qquad e_2 = e_M = 0.0934$

- **Case 2**: Earth – Uranus

  For Earth – Uranus , we have[6]

  $a_1 = a_E = 1 \qquad a_2 = a_U = 19.1913$
  $e_1 = e_E = 0.0167 \qquad e_2 = e_U = 0.0472$

By solving Eq. (25) for the above two cases numerically (we put μ =1), we get $(x)_{Min}$, then from Eq.(24), we get $(z)_{Min}$ and from Eq.(8), we get $(y)_{Min}$, finally from Eq.(12), we get $(\Delta v_T)_{Min}$, as:

Case	$(x)_{Min}$	$(z)_{Min}$	$(y)_{Min}$	$(\Delta v_T)_{Min}$
1	1.4026	0.6732	1.3017	0.0513
2	1.3463	1.1433	2.8437	0.5947

4. Appendix: Angular momentum concept

For Fig.(2), let h_2 be the angular momentum w.r.t. orbit O_2, the first transfer orbit. Let

$$r_A = a_1(1-e_1) \qquad r_C = a_T(1+e_T) \qquad r_B = a_{T'}(1+e_{T'})$$

whence

$$h_2 = \sqrt{2\mu}\sqrt{\frac{r_A r_C}{r_A + r_C}}$$

$$xv_A = \frac{h_2}{r_A} = \sqrt{2\mu}\sqrt{\frac{r_A r_C}{r_A + r_C}}\frac{1}{r_A}$$

$$x = \frac{xv_A}{v_A} = \sqrt{\frac{1+e_T}{1+e_1}} \tag{77}$$

$$1+e_T = x^2(1+e_1) \qquad 1-e_T = 2-x^2(1+e_1) \tag{78}$$

Let h_3 be the angular momentum w.r.t. orbit O_3, the second transfer orbit.

$$h_3 = \sqrt{2\mu}\sqrt{\frac{r_C r_B}{r_C + r_B}}$$

$$v_B = \frac{h_3}{r_B} = \sqrt{2\mu}\sqrt{\frac{r_C r_B}{r_C + r_B}}\frac{1}{r_B}$$

$$z = \frac{zv_B}{v_B} = \sqrt{\frac{1-e_2}{1-e_{T'}}} \tag{79}$$

$$1-e_{T'} = \frac{1-e_2}{z^2}; 1+e_{T'} = 2-\frac{1-e_2}{z^2} \tag{80}$$

$$yv_C = \frac{h_3}{r_C} = \sqrt{2\mu}\sqrt{\frac{r_C.r_B}{r_C + r_B}}\frac{1}{r_C} \tag{81}$$

$$y = \frac{yv_C}{v_C} = \sqrt{\frac{1+e_{T'}}{1-e_T}}$$

$$y = \frac{1}{z}\sqrt{\frac{2z^2-1+e_2}{2-x^2(1+e_1)}} \tag{82}$$

Eqs (77–78), are the same as in the case of change of energy concept.

5. Concluding remarks

It is possible to reduce the impulsive optimal transfer problem to a parametric optimization one with constraints. A numerical solution, or even analytical one in some simple cases could be acquired. In addition we may have the semi–analytical resolution as shown in this article [7]. For the second configuration we assigned the values of $(x,\, z,\, y,\, \Delta v_T)_{Min}$ for the generalized Earth – Mars and Earth – Uranus bi – elliptic transfer. Our procedure is elementary and straightforward, using only the properties of the elliptic conic section, and the minimum – partial ordinary calculus – conditions. Our choice of the independent parameters x, z proved simplicity and efficiency of this analysis when compared with other sophisticated approaches. The parameter $x$is determined from a numerical solution of a sixth degree polynomial equation. The numerical results may be repeatedly acquired for subsystems with exterior member as one of the outer planets Jupiter, Saturn, and Neptune, or even more the inner planets Venus and Mercury.

References

[1] **Prussing J. and Conway B.**: Orbital Mechanics, *Oxford University Press*, **1993**.

[2] **Ting L.**: *ARS J.*, pp. 1013–1018, **1960**.

[3] **Billik B.H. and Roth H.L.**: *Astronautica Acta*, Vol. 13, pp. 23–36, **1966**.

[4] **Gobetz F.W. and Doll J.R.**: *AIAA Journal*, Vol. 7, No 5, pp 801–834, **1969**.

[5] **Kamel O.M. and Soliman A.S.**: *Mechanics and Mechanical Engineering Journal*, Tech. University of Lodz, Poland, Vol. 13, No. 1, 73–78, **2009**.

[6] **Murray C.D. and Dermott S.F.**: Solar System Dynamics, *Cambridge University Press*, **1999**.

[7] **Marec J.P.**: Space – Vehicle Trajectories : Optimization, Reprinted from Systems & Control Encyclopedia, Theory, Technology, Applications, *Pergamon Press*, **1988**.