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The Effect of Yield Strength on Inelastic Buckling of Reinforcing Bars

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The paper presents the results of numerical analyses of inelastic buckling of reinforcing bars of various geometrical parameters, made of steel of various values of yield strength. The results of the calculations demonstrate that the yield strength of the steel the bars are made of influences considerably the equilibrium path of the compressed bars within the range of postyielding deformations

Comparative diagrams of structural behaviour (loading paths) of thin–walled sections under investigation for different strain rates are presented. Some conclusions and remarks concerning the strain rate influence are derived.

Keywords: Reinforcing bars, inelastic buckling, yield strength, tensil strength

1. Introduction

The impact of some exceptional loads, e.g. seismic loads, on a structure may result in the occurrence of post–critical states. Therefore the National Standards regulations for designing reinforced structures on seismically active areas e.g. EC8 [15] require the ductility of a structure to be examined on a cross–sectional level, and additionally, the structures should demonstrate a suitable level of global ductility. The results of the examinations of members of reinforced concrete structures show that inelastic buckling of longitudinal reinforcement bars occurs in the state of post–critical deformations, [1, 2, 4, 7], and in some cases it occurs yet within the range of elastic deformations [8]. Therefore, in order to evaluate the ductility of a reinforced concrete structure properly, the assumed relations $\sigma - \varepsilon$ for longitudinal reinforcement should consider their possible inelastic buckling. The results of the experimental studies of reinforcing bars [1, 2, 5, 11, 12, 13] as well as the results of numerical analyses [4, 6, 10] demonstrate distinctly that inelastic buckling of bars is not only influenced by their slenderness and the mechanical properties of steel but also the effect of the level of yield strength is noticeable. This is essential in terms of new grades of reinforcing steel with increasing values of yield strength launched onto the market.

The paper presents the evaluation of the effect of yield strength on inelastic buckling of reinforcing bars. Numerous numerical analyses were purposefully carried out with the use of Cosmos/M system. The selected results of the analyses carried out for reinforcement steel of various mechanical properties are presented in this study.

2. Bar model and description of calculations

Steel grades which are used for reinforcing bars for reinforced concrete structures are characterized by considerable variation of yield strength and mechanical properties after yielding. The differences refer to the length of the plastic plateau ε_h, reinforcement index f_u/f_y and the branch of the strengthening curve E_h (Fig. 1a).

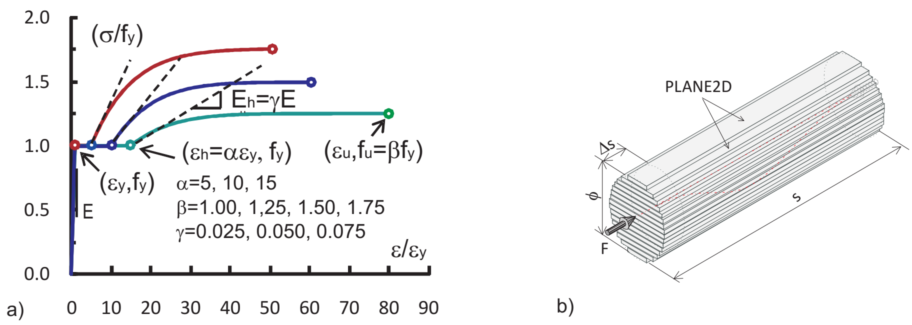

Figure 1 a) Parameters of mechanical properties of steel, b) Numerical model of a bar

The numerical analyses were carried out for various material curves $\sigma - \varepsilon$ characterizing various reinforcement steels. Fig.1a illustrates the assumed material curves, with four changing parameters α (length of plastic plateau), β (reinforcement index) and γ (initial rigidity of reinforcement) as well as yield strength f_y = 200, 400, 600, 800 and 1000 MPa. Young's modulus equaled E = 200000 MPa. The material curves were the same for both compressed and tensioned fibers. Mander's model, assumed for the description of the strengthening curve and presented in [1], takes the following form (1):

$$\sigma = f_u + (f_y - f_u)\left(\frac{\varepsilon_u - \varepsilon}{\varepsilon_u - \varepsilon_h}\right)^p \qquad (1)$$

The longitudinal reinforcing bars are most frequently subjected to buckling which is preceded by loosening of coating between two neighboring ties. Therefore the calculations were carried out for the scheme of a bar clamped on both sides. The assumed diameter of a bar is 16 mm. The ratios of the distances between the support points s, to the bar diameters equaled: $s/\o$ = 5, 6, 7, 8, 9, 10, 12, 15, which, for the assumed method of bar supporting, corresponds to the classical definition of slenderness $\lambda = 10, 12, 14, 16, 18, 20, 24, 30$.

System COSMOS/M [14] was applied in the numerical analysis of large elasto–plastic deformations of compressed reinforcing bars. A four–node finite element PLANE2D was used in the numerical model. The bar consisted of sixteen layers 1 mm high with varied thicknesses which ensure that the surface area of the bar cross–section $A = 201$ mm^2 (Fig. 1b). In all the analyzed cases, the bar was divided into 320 elements, which led to a discrete system of 698 degrees of freedom. Elastic analyses were carried out with Huber–Mises–Hencki's elasticity condition, the associated flow rule and isotropic strengthening. A nonlinear material characteristic $\sigma - \varepsilon$ with option PLASTIC was introduced to the analysis. Various material curves, of changing parameters α, β, γ and various values of yield strength were used in the analysis. A displacement control was consistently applied in the analysis, selecting a longitudinal displacement of the loaded end of the bar (Δs) as a leading parameter, as well as the Newton–Raphson's method for solving a nonlinear system of equations in subsequent incremental steps. An option of the automatic selection of a step was applied, modifying slightly the parameters of the option for particular cases. The aim of the calculations was to obtain a full loaddisplacement characteristic corresponding to the experimental compression.

3. Results and analysis of calculations

The numerical analyses were carried out for two models of reinforcement i.e. for an elasto–plastic model and an elasto–perfectly plastic model with a nonlinear strengthening according to Mander's [1] recommendations. The material curves differed one from another with yield strength f_y and with characteristic after yielding, expressed by parameters α, β, γ. As a result, fifty material curves were obtained. The calculations were carried out for each material curve and eight values of bar slenderness. The calculations resulted in relations between force F transmitted by the bar, and the displacement of the end of the bar Δs. The paper presents only a part of the results of the calculations.

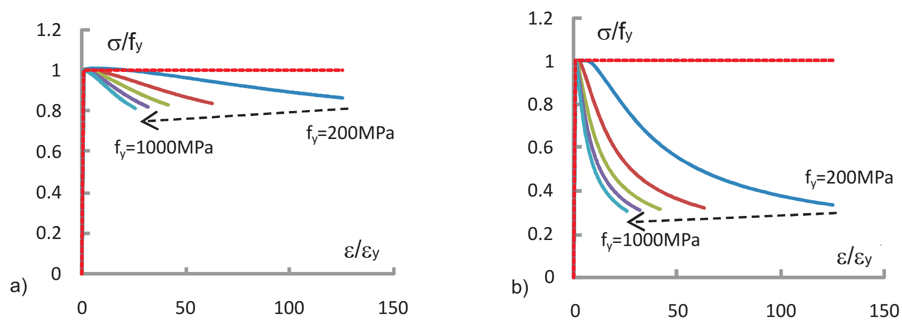

Figure 2 The impact of yield strength on the equilibrium paths of bars of elasto–perfectly plastic characteristic: a) bar slenderness $s/\o$=5, b) bar slenderness $s/\o = 10$

3.1. *Elasto–perfectly plactic model*

The paths of static equilibrium of bars made of steel of elasto–perfectly plastic characteristic, for various values of yield strength and selected bar slenderness are presented in Fig. 2. The figure demonstrates relations between average stress $\sigma = F/A$ standardized with regard to the yield strength and the average axial strain of bars $\varepsilon = \Delta s/s$ standardized with regard to strain in the moment of yielding ε_y.

Observing the curves presented in Fig. 2, it can be stated that for all the cases, the critical buckling stresses (defined as maximum average stress in a bar $\sigma_{max} = F_{max}/A$) are equal to the yield strength, and the intensity of decrease in stresses in bars after yielding depends on their slenderness and the yield strength of steel. The more slender the bars are, the more intense decrease in stresses; the gradient of the declining part of the curve rises. The yield strength also influences considerably the behavior of bars. The behavior of bars, made of steel of lower values of yield strength, after exceeding the yield strength is more stable than the behavior of bars made of steel of higher values of yield strength. It can also be observed that after yielding the behavior of a bar, made of steel of yield strength f_y = 1000 MPa and slenderness $s/ø$=5, is very similar to the behavior of a bar made of steel of yield strength f_y = 200 MPa and slenderness $s/ø$=10; the changes in stresses after yielding are of a similar character.

3.2. *Elasto–plastic model with strengthening*

Fig. 3 presents the equilibrium paths for bars made of steel of various strengthening characteristics, with bar slenderness $s/ø$ = 5 and 10. Buckling stresses (σ_{max}/f_y) for slender bars with $s/ø$ = 5 are higher than the yield strength in all cases and depend on the value of the yield stress and the characteristic of steel after yielding i.e. the length of the yield plateau α, the ratio of the tensile strength to the yield strength β, as well as the rigidity of the strengthening γ. If yield strength f_y = 200 MPa, buckling stresses for slender bars with $s/ø$ = 5 are equal to the tensile strength (f_u/f_y) for all the steel characteristics. Buckling stresses decrease with the increase in the yield strength. What is more, it can be noticed that the shorter the plastic plateau and the higher rigidity of strengthening are, the higher are buckling stresses.

The presented results demonstrate that the current opinion, presented among others by Mau [9,10], Monti and Nuti [11], according to which the response of the compressed slender bars with $s/ø \leq 5$ overlaps the material curve i.e. a compressed bar has the same characteristic as a tensioned bar may only refer to bars made of steel of a low yield strength. Only bars made of steel with yield strength f_y = 200 MPa revealed buckling stresses equal to the tensile strength f_u, irrespectively to other strength parameters of steel. The curves of behavior of compressed bars made of steel of higher values of yield strength overlap the material curves only within a certain range of strains which depends on the characteristic of steel after yielding.

Buckling stresses for slender bars with $s/ø$ = 10 are significantly smaller than for bars with $s/ø$ = 5 and also depend on mechanical properties of steel after yielding and the yield strength.

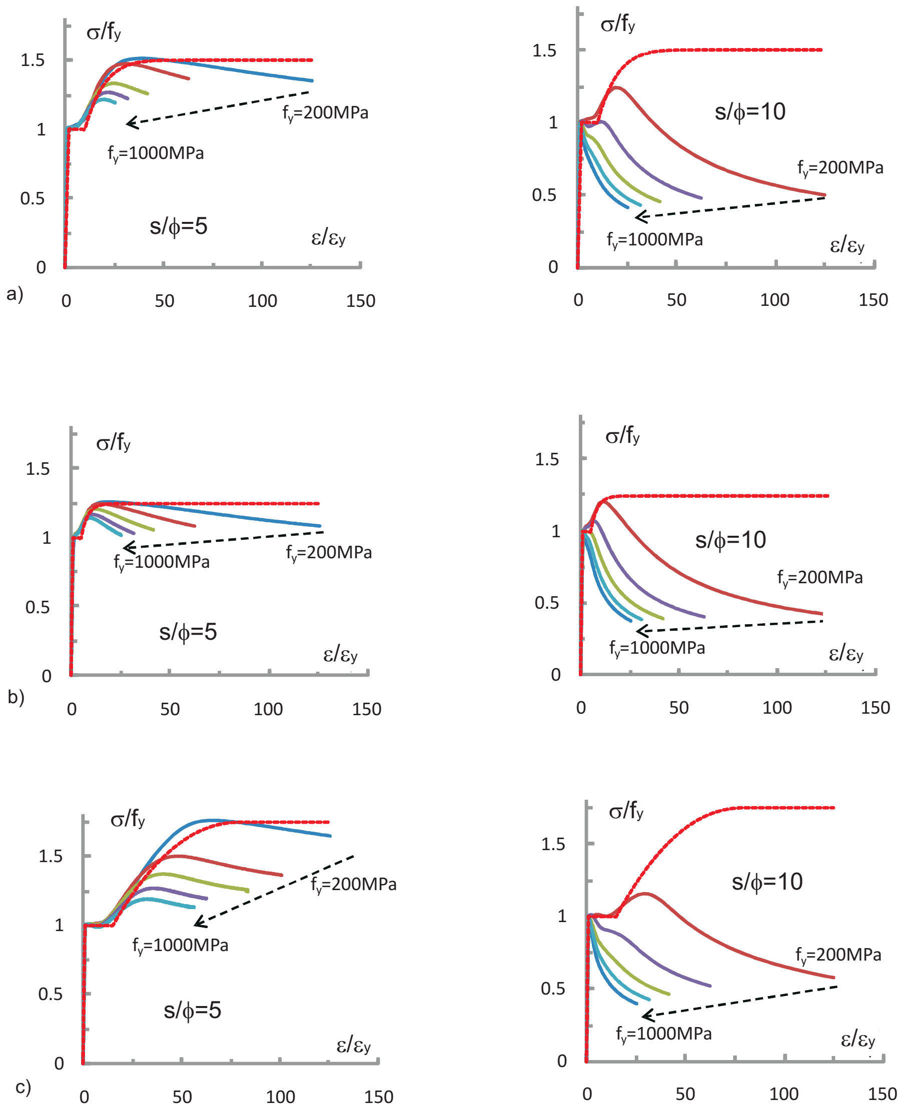

Figure 3 Impact of yield strength on equilibrium paths of bars of elasto–plastic characteristic with strengthening: a) steel bars with steel parameters $\alpha = 10$, $\beta = 1.50$, $\gamma = 0.050$, b) steel bars with steel parameters $\alpha = 5$, $\beta = 1.25$, $\gamma = 0.075$, c) steel bars with steel parameters $\alpha = 15$, $\beta = 1.75$, $\gamma = 0.025$

In this case, buckling stresses are slightly higher than the yield strength for bars made of steel with yield strength $f_y = 400$ MPa and are close to the strength for bars made of steel with the yield strength $f_y = 200$ MPa (Fig. 3b, $s/ø = 10$).In the remaining cases, the buckling stresses are equal to yield strength. After reaching yield strength, bars made of steel with yield strength $f_y \leq 400$ remain in the static equilibrium state up to a certain level of strains, whereas the force transmitted by bars made of steel with higher yield strength $f_y > 400$ MPa lowers just after reaching the yield strength, while with in–crease in the yield strength, the decrease in the load becomes more sudden. A similar regularity has been stated in Mau and El–Mabsout's research [10], where the response of the bars made of steel of higher yield strength was less static than for bars made of steel of lower yield strength.

The known theories of inelastic buckling such as theory of a substitutive modulus (Engesser–Karaman's) and the theory of a tangent modulus (Engesser–Stanley's) refer to the state equilibrium of a bar in a rectilinear form and thus they cannot consider the impact of the yield strength on the inelastic buckling. In the performed numerical analyses, the horizontal displacements of the centre of the bar were also observed [6, 9]. In each case, the loss of the rectilinear form of the bar occurred just after the stresses in the bar reached the yield strength. The results of the research [6, 9], as well as the graphs presented in fig.3 show that in case of a inelastic buckling, a leaning form of the stability equilibrium is also possible. Therefore, to the author's understanding, the inelastic buckling should be treated as buckling in a physical sense, but the theoretical grounds for the impact of yield strength on static equilibrium paths could be searched in the Ayrton's–Perry's theory referring to the depletion of load capacity of bars with an initial curvature, cited in [16].

4. Summary of the calculation results

Fig. 4 presents graphs illustrating the behavior of bars of the same strength characteristic for various values of yield strength fy and various bar slenderness $s/ø$.

As can be noticed, for the same steel parameters α, β, γ, the bars of slenderness $s/ø = 10$ and yield strength $f_y = 200$ as well as the bars of slenderness $s/ø = 5$ and yield strength $f_y = 800$ behave very similar. The same similarity of bars' behavior has been noticed in case of bars of slenderness $s/ø$=12 and of yield strength $f_y = 200$ MPa as well as for bars of slenderness $s/ø$=6 and yield strength $f_y = 800$ MPa. The regularity is noticeable for both elasto–plastic steel models and elasto–plastic steel models with strengthening. The similarity of behavior of bars of various values of yield strength and slenderness was also observed in numerical analyses carried out by Dhakal and Maekaw [4] for bars of slenderness values $s/ø = 5$ and $s/ø = 10$ and yield strength $f_y = 1600$ MPa and $f_y = 400$ MPa respectively as well as for yield stress $f_y = 400$ MPa and $f_y = 100$ MPa respectively.

Analyzing the above mentioned values of yield strength and slenderness, it can be stated that a fourfold difference in the yield strength and a double difference in the bar slenderness occurs in all the cases. What is more, the mentioned pairs of bars, despite their different values of yield strength fy and slenderness $s/ø$, had the same values of the expression $f_y(s/ø)^2$. And thus $800 \cdot 52 = 20000$, $200 \cdot 102 = 20000$, $800 \cdot 62 = 28800$, $200 \cdot 122 = 28800$. The expression $f_y(s/ø)^2$ can be treated as a certain materialgeometric constant characterizing a particular way of inelastic buckling of compressed bars. The following features (among others) can be attributed to this expression: a maximum load transmitted by the bar (critical load), or maximum average stress in a bar (critical stress), or the curtailment of a bar at maximum load (critical strain). For example, the buckling stress equal $1.21 f_y$ (comp. Fig. 4) will be observed in case of bars of slenderness and yield stress satisfying condition $f_y(s/ø)^2 = 800 \cdot 52 = 20000$.

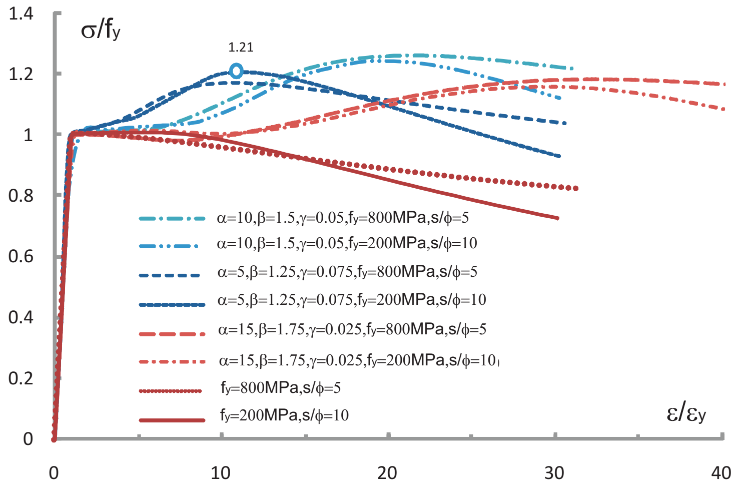

Figure 4 The impact of yield strength and bar slenderness on equilibrium paths

The graphs of the expression $f_y(s/ø)^2$ in the function of bar slenderness $s/ø$ for various values of yield strength are presented in Fig. 5. The expression can constitute a parameter describing the same behavior of bars of various values of yield strength and slenderness. For example, if the equilibrium path for a bar of determined slenderness made of steel of a determined yield strength is known, it is possible to determine parameters (yield strength and slenderness) of other bars behaving in the same way on the basis of graph in Fig.5. The expression $f_y(s/ø)^2$ may be very useful in determining the models of behavior of compressed bars of various mechanical properties.

5. Final remarks

On the basis of the carried out calculations it can be stated that inelastic buckling is influenced not only by the their geometrical parameters and mechanical properties of steel after yielding they are made of, but also the yielding stress of steel.

For a determined bar slenderness and a determined material curve, the critical force and thus the average buckling stresses decrease with the increase in the yield strength.

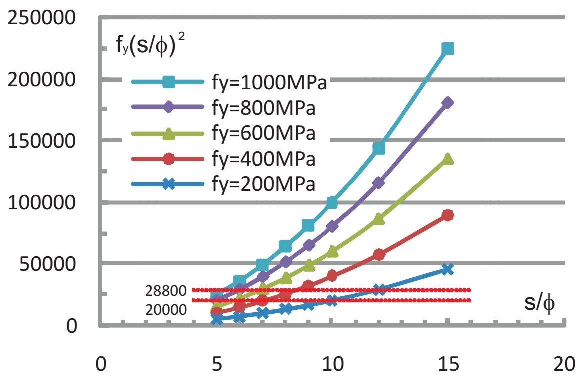

Figure 5 Graphs of relations f_y $(s/ø)^2$ for various yield strengthes

The static equilibrium paths for compressed bars with slenderness $s/ø \leq 5$ overlap the material curve only for steel of yield strength $f_y < 400$ MPa. The behavior of compressed bars in the state of post–critical deformation made of steel of the same strength characteristic actually depends on a single parameter described with expression $f_y(s/ø)^2$.

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