Mechanics and Mechanical Engineering
Vol. 14, No. 2 (2010) 317–324

Dynamic Stability of Sandwich Rod and of Rotor in Bipolar Electric Drive Motor with Damping

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Received (10 June 2010)
Revised (18 July 2010)
Accepted (30 July 2010)

The work presents an analysis buckling of a sandwich bar and rotor in bipolar electric drive motor with damping. In order to determine the stability of the transverse motion, equation of its transverse vibration were formulated. From the equations of motion, differential equations interrelating the dynamic deflection with space and time were derived. Eventually, homogeneous, partial, differential equations have been obtained and solved by the Fourier's method. Then an ordinary differential equation (Hill's equation) describing the vibration have been solved. An analysis of the solution became the basis for determining the regions of bar and rotor motion instability. Finally, the critical damping coefficient values at which parametric resonance occurs have been determined.

Keywords: Sandwich bar, rotor, stability

1. Dynamic stability of sandwich bar

Sandwich constructions are characterized by light weight and high strength. Such features are highly valuable in aviation, building engineering and automotive applications. The primary aim of using sandwich constructions is to obtain properly strong and rigid structures with vibration damping capacity and good insulating properties. Figure 1 shows a scheme of a sandwich construction which is composed of two thin facing plates and a relatively thick core [9,10]. The core, made of plastic and metal sheet or foil, transfers transverse forces and maintains a constant distance between the plates. Sandwich constructions are classified into bars, plates and beams. A major problem in the design of sandwich constructions is the assessment of their stability under axial loads causing their buckling or folding. The existing methods of calculating such structures are limited to the assessment of their stability under loads constant in time [8,10].

There are no studies dealing with the analysis of parametric vibration and dynamic stability (dynamic buckling). This chapter presents a dynamic analysis of a sandwich bar compressed by a periodically variable force, assuming that the core is linearly viscoelastic. Differential equations describing the dynamic flexural buckling of bars are derived and regions of instability are identified. The dynamic analysis of sandwich constructions is of great importance for automotive vehicles and aeroplanes, since most of the loads which occur in them have the form of time-dependent forces.

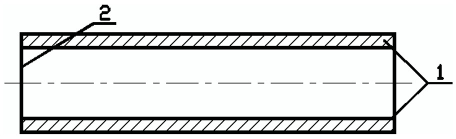

Figure 1 Scheme of sandwich construction 1 – plates, 2 – core

2. Dynamic buckling of a sandwich bar

A simply–supported sandwich bar compressed by time-dependent force F is shown in Fig. 2. Force F can be expressed as follows

$$F = F_1 + F_2 \cos pt \tag{1}$$

where

F_1 – constant component of the compressive force,
F_2 – amplitude of the variable component of the compressive force,
p – frequency of variable component F_2,
t – time.

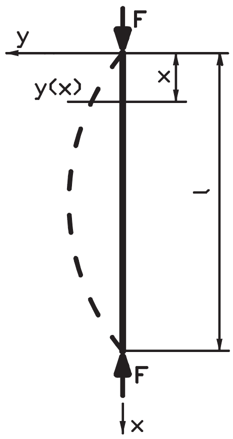

Figure 2 Sandwich bar compressed by force F

The cross section of the sandwich bar is shown in Fig. 3. The basis for describing the dynamic buckling of the sandwich bar is the differential equation of the sandwich beam centre line. The equation can be written as

$$B\frac{\partial^4 y}{\partial x^4} = q - k\frac{B}{S}\frac{\partial^2 q}{\partial x^2} \tag{2}$$

where:

B – flexural rigidity of the bar,
q – load intensity,
k – a coefficient representing the influence of the transverse force on the deflection of the bar,
S – transverse rigidity of the bar.

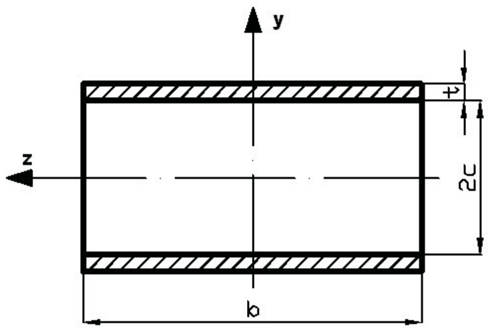

Figure 3 Cross section of sandwich bar

In sandwich constructions the core is merely sheared and does not transfer normal stresses whereby coefficient k is equal to one (k = 1).

$$S = 2bcG_c \tag{3}$$

where

b, c – dimensions of the core (Fig. 3),
G_c – modulus of rigidity of the core material.
Load intensity q can be written in the form:

$$q = q_1 + q_2 + q_3 \tag{4}$$

$$q_1 = -F\frac{\partial^2 y}{\partial x^2} \qquad q_2 = -\mu\frac{\partial^2 y}{\partial t^2} \qquad q_3 = -\eta_r\frac{\partial y}{\partial t} \tag{5}$$

where

μ – mass of the unit of length,
η_r – damping coefficient of the core material.

After substituting equations (5) into differential equation (2) the following differential equation is obtained:

$$B\left(1-\frac{F}{S}\right)\frac{\partial^4 y}{\partial x^4}+F\frac{\partial^2 y}{\partial x^2}-\frac{B}{S}\mu\frac{\partial^4 y}{\partial x^2\partial t^2}+\mu\frac{\partial^2 y}{\partial t^2}+\eta_r\frac{\partial y}{\partial t}-\frac{B}{S}\eta_r\frac{\partial^3 y}{\partial x^2\partial t}=0 \tag{6}$$

The above equation is a fourth–order homogeneous equation with time–dependent coefficients. It was solved by the method of separation of variables (Fourier's method). The solution can be presented in the form of an infinite series:

$$y=\sum_{n=1}^{\infty} X_n\left(x\right)T_n\left(t\right) \tag{7}$$

Eigenfunctions $X_n(x)$, satisfying the boundary conditions at the supports of the bar at its ends, have the following form:

$$X_n\left(x\right)=A_n\sin\left(\frac{\pi n x}{l}\right) \tag{8}$$

Having substituted equations (7) and (8) into the differential equation (6), one gets the following ordinary differential equation describing functions $T_n(t)$:

$$\ddot{T}_n+2h\dot{T}_n+\omega_{on}^2\left(1-2\psi_n\cos pt\right)T_n=0 \tag{9}$$

where

$$2h=\frac{\eta_r}{\mu} \qquad 2\psi_n=\frac{F_2\left(\frac{\pi n}{l}\right)^2}{\mu\omega_{on}^2} \tag{10}$$

The square of frequency ω_{on} can be expressed as follows:

$$\omega_{on}^2=\omega_o^2-\frac{F_1\left(\frac{\pi n}{l}\right)^2}{\mu} \tag{11}$$

where

ω_o – the natural frequency of vibration of the bar when $F_1=0$, $F_2=0$, $\eta_r=0$.

The square of frequency ω_o can be expressed as follows:

$$\omega_o^2=\frac{B\left(\frac{\pi n}{l}\right)^2}{\mu\left[1+\frac{B}{S}\left(\frac{\pi n}{l}\right)^2\right]} \tag{12}$$

Differential equation (9) is Hill's equation in the form [3, 5]:

$$\ddot{T}_n+2h\dot{T}_n+\Omega_n^2\left[1-f\left(t\right)\right]T_n=0 \tag{13}$$

If there is no damping in the core ($h=0$) and assuming $f(t)=2\psi_n\cos pt$, one gets the following classical Mathieu equation

$$\ddot{T}_n+\omega_{on}^2\left(1-2\psi_n\cos pt\right)T_n=0 \tag{14}$$

In order to solve equation (13), a change of variable was made and the solution was expressed in the form.

3. Dynamic stability of rotor

Among in electric machines, the two–pole asynchronous motors occupy a particular space. These motors have small value of the magnetic gap. For this reason, the basic problem encountered in the phase of construction of such machines is to estimate the stability of the rotors. The problem of stability of rotors is in relation to the problem of vibration. On certain values of some quantities, such a rotational speed, magnetic tension, rigidity, etc. the effect of unstability can take place. The assessment of the stability is of particular importance in the case of long rotors, for example rotors of motors of deep–well pumps.

Problem of estimation of stability of transverse motion of rotors without damping are presented in the works [4, 6, 7]. In this paper the influence of damping in rotors on the dynamic stability of its rotors in two-pole asynchronous motors have been determined.

The model of rotor accepted for calculations is shown in Fig.4.

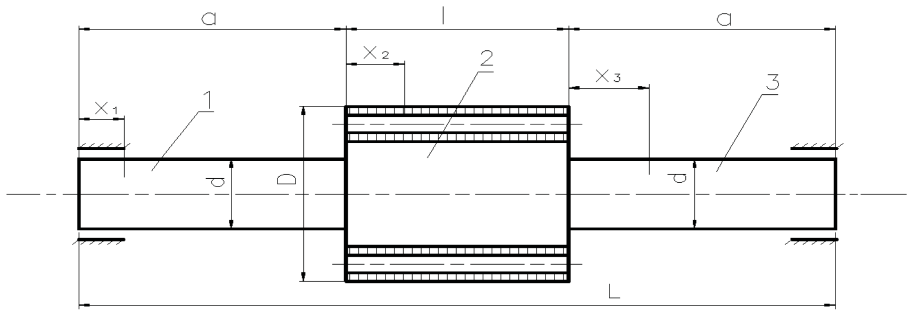

Figure 4 The model of rotor accepted for calculations

In order to simplify the considerations a vertical position of the rotor have been assumed. The basis for describing the dynamic stability of the rotor is the differential equation of the centre line of the beam. The equation can be written as:

$$S\frac{\partial^4 y}{\partial x^4} = -q_x \tag{15}$$

where:

S – flexural rigidity of the section 2

y – deflection of the rotor

q_x – load intensity.

The load intensity q_x can be introduced in the form:

$$q_x = q_{1x} + q_{2x} + q_{3x} \tag{16}$$

where:

q_{1x} – load intensity related to the influence of the forces of inertia,

q_{2x} – load intensity related to the influence of the magnetic tension,

q_{3x} – load intensity related to the influence of the damping.

The load intensity q_{1x} can be expressed as:

$$q_{1x} = -\mu \frac{\partial^2 y}{\partial t^2} \tag{17}$$

where:

μ – mass of the unit of length of the section 2,

t – time.

The load intensity q$_{2x}$ can be expressed as [1, 2]:

$$q_{2x} = (A_1 + A_2 \cos pt) y \tag{18}$$

where:

A_1, A_2, p – parameters of magnetic tension [2, 6, 7].

The load intensity q_{3x} can be expressed as:

$$q_{3x} = -\eta_r \frac{\partial y}{\partial t} \tag{19}$$

where:

η_r – damping coefficient on the rotor.

After substituting (16) in (15), the following differential equation in obtained:

$$\beta^2 \frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} + 2h \frac{\partial y}{\partial t} - (\gamma + \vartheta \cos pt) y = 0 \tag{20}$$

where:

$$\beta^2 = \frac{S}{\mu} \qquad 2h = \frac{\eta}{\mu} \qquad \gamma = \frac{A_1}{\mu} \qquad \vartheta = \frac{A_2}{\mu} \tag{21}$$

The above equation is a fourth – order homogeneous equation with time – dependent coefficients. It was solved by the Fourier's method. The solution can be presented in the form of an infinite series:

$$y = \sum_{n=1}^{\infty} X_n(x) T_n(t) \tag{22}$$

After a separation of variables and definion of parameter k$_n$ the following equations have been obtained:

$$\overset{IV}{X}_n(x) - k_n^4 \overset{II}{X}_n(x) = 0 \tag{23}$$

$$\ddot{T}_n + 2h\dot{T}_n + (\omega_n^2 - \vartheta \cos pt) T_n = 0 \tag{24}$$

where:

ω_n denotes the n–order frequency of free vibrations of rotor when $\vartheta = 0$, $\eta_r = 0$

The equation (24) can be expressed as follows:

$$\ddot{T}_n + 2h\dot{T}_n + \omega_n^2 (1 - 2\psi_n \cos pt) T_n = 0 \tag{25}$$

where:

$$2\psi_n = \frac{\vartheta}{\omega_n^2} \tag{26}$$

Differential equation (25) is Hill's equation in the form [3, 5]:

$$\ddot{T}_n + 2h\dot{T}_n + \Omega_n^2[1 - f(t)]T_n = 0 \tag{27}$$

If there is no damping in the rotor ($h = 0$) and assuming $f(t) = 2\psi_n \cos pt$, one gets the following classical Mathieu equation:

$$\ddot{T}_n + \omega_{on}^2(1 - 2\psi_n \cos pt)T_n = 0 \tag{28}$$

4. Solution of Hill's equation

In order to solve equation (27), a change of variable was made and the solution was expressed in the form:

$$T_n\,(t) = e^{-ht}\varphi_n\,(t) \tag{29}$$

In this way a new differential equation for function ϕ_n(t) was obtained:

$$\ddot{\varphi}_n + \omega_n^2\,[1 - f_1\,(t)]\,\varphi_n = 0 \tag{30}$$

where

$$\omega_n^2 = \Omega_n^2 - h^2 \tag{31}$$

$$f_1\,(t) = \frac{\Omega_n^2}{\omega_n^2} f\,(t) \tag{32}$$

Equation (30) is the Mathieu equation without damping. Therefore for the analysis of this equation one can use the solution of equation (28), substituting f_1(t) for f(t) and $\Omega_n^2 - h^2$ for ω_n^2.

Let us now analyze the stability of the solutions of the differential equation (30), limiting the analysis to the first (most important) region of instability.

By solving of equation (9) the boundary lines of the first region of instability has been obtained (Fig. 5).

In a similar way as in the case without damping the following relations for the boundary lines of the first region of instability are obtained:

$$\frac{p}{\Omega_n} < 2\sqrt{\frac{(1 - \xi_n)^2 - \psi_n^2}{1 - 3\xi_n - \sqrt{\psi_n^2 - 4\xi_n + 8\xi_n^2}}} \tag{33}$$

$$\frac{p}{\Omega_n} > 2\sqrt{\frac{(1 - \xi_n)^2 - \psi_n^2}{1 - 3\xi_n + \sqrt{\psi_n^2 - 4\xi_n + 8\xi_n^2}}} \tag{34}$$

where

$$\xi_n = \left(\frac{h}{\Omega_n}\right)^2 \tag{35}$$

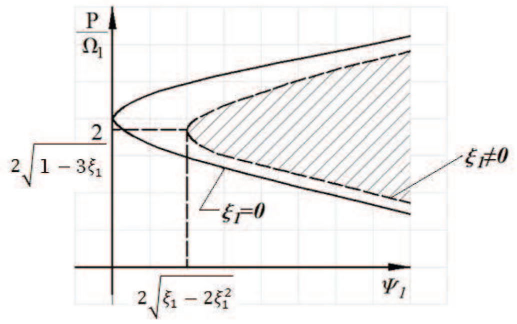

Figure 5 First region of instability ($\xi_1 = 0$, without damping, $\xi_1 \neq 0$, with damping)

Vertex of the first region instability has the coordinates:

$$\psi_{1gr} = 2\sqrt{\xi_1 - 2\xi_1^2}, \qquad \frac{P}{\Omega_1} = 2\sqrt{1 - 3\xi_1} \tag{36}$$

Relation (33) and (34) describe the upper and lower boundary line respectively. From formula (36) the boundary value of coefficient ψ_1 at which parametric resonance occurs has been obtained. If $\psi_1 < \psi_{1gr}$, no parametric resonance arises. It follows from the above that there exist compressive force components F_1 and F_2, and coefficient of damping at which the bar does not lose stability. Likewise exist coefficient A_1 and A_2 of magnetic tension and coefficient of damping at which the rotor does not lose stability.

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