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Dynamic Stability of Micro–Periodic Cylindrical Shells

Barbara Tomczyk
Department of Structural Mechanics
Technical University of Lodz
al. Politechniki 6, 90-924 Łódź, PL

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The object of considerations are thin linear–elastic Kirchhoff–Love–type circular cylindrical shells having a micro–periodic structure along one direction tangent to the shell midsurface. Shells of this kind are called uniperiodic. The aim of this paper is twofold. First, we formulate an averaged non–asymptotic model for the analysis of dynamical stability of periodic shells under consideration, which has constant coefficients and takes into account the effect of a cell size on the overall shell behavior. This model is derived employing *the tolerance modeling procedure*. Second, we apply the obtained model to derivation of frequency equations being a starting point in the analysis of dynamical shell stability. *The effect of the microstructure length on these frequency equations is discussed*. The system of two the second–order ordinary differential frequency equations being a certain generalization of the known Mathieu equation is obtained. This system reduces to the Mathieu equation provided that the length–scale effect is neglected. Moreover, in the framework of the tolerance model proposed here the new additional higher–order free vibration frequencies and the new additional higher–order critical forces are derived. These frequencies and critical forces cannot be obtained from the asymptotic models commonly used for investigations of the shell stability.

Keywords: Micro–periodic cylindrical shells, dynamical stability, mathematical modelling, length–scale effect

1. Introduction

Thin linear–elastic Kirchhoff–Love–type cylindrical shells with a periodically inhomogeneous structure along one direction tangent to the shell midsurface are analyzed. By periodic inhomogeneity we shall mean periodically variable shell thickness and/or periodically variable inertial and elastic properties of the shell material. Shells of this kind are termed *uniperiodic*. As an example we can mention cylindrical shells with periodically spaced families of stiffeners as shown in Fig. 1. The period of inhomogeneity is assumed to be very large compared with the maximum shell thickness and very small as compared to the midsurface curvature radius as well as the smallest characteristic length dimension of the shell midsurface.

Because properties of such shells are described by highly oscillating and non-continuous periodic functions, the exact equations of the shell theory are too complicated to apply to investigations of engineering problems. That is why a lot of different approximate modeling methods for shells of this kind have been proposed. Periodic cylindrical shells (plates) are usually described using *homogenized models* derived by means of *asymptotic methods*, cf. [2, 4, 9]. Unfortunately, in models of this kind *the effect of a cell size* (called *the length–scale effect*) on the overall shell behavior is neglected.

The periodically densely stiffened shells are also modeled as homogeneous orthotropic structures, cf. [1, 5, 6]. The orthotropic model equations with coefficients independent of the period length cannot be used to the analysis of phenomena related to the existence of microstructure length–scale effect (e.g. the dispersion of waves, the occurrence of additional higher–order free vibration frequencies and higher-order critical forces).

In order to analyze the length–scale effect in dynamics or/and stability problems, the new averaged non–asymptotic models of thin cylindrical shells with a periodic micro–heterogeneity either along two directions tangent to the shell midsurface (*biperiodic structure*) or along one direction (*uniperiodic structure*) have been proposed by Tomczyk in a series of papers, e.g. [14, 15, 16, 17, 18, 19, 23], and also in the books [20, 21, 22]. These, co called, *the tolerance models* have been obtained by applying *the non–asymptotic tolerance averaging technique*, proposed and discussed in the monographs [24, 25, 26], to the known governing equations of Kirchhoff–Love theory of thin elastic shells (partial differential equations with functional highly oscillating non–continuous periodic coefficients). Contrary to starting equations, the governing equations of the tolerance models have coefficients which are constant or slowly–varying and depend on the period length of inhomogeneity. Hence, these models make it possible to investigate the effect of a cell size on the global shell dynamics and stability. This effect is described by means of certain extra unknowns called *fluctuation amplitudes* and by known *fluctuation shape functions* which represent oscillations inside the periodicity cell. Moreover, the tolerance models describe selected problems of the shell micro–dynamics, cf. [22, 23]. It means that contrary to equations derived by using the asymptotic homogenized methods, the tolerance model equations make it possible to investigate the micro–dynamics of periodic shells independently of their macro–dynamics. In the papers and books, mentioned above, the applications of the proposed models to analysis of special problems dealing with dynamics and stationary stability of *uniperiodic shells* as well as dynamics and dynamical stability of *biperiodically densely stiffened cylindrical shells* have been presented. It was shown that the length–scale effect plays an important role in these problems and cannot be neglected.

In this paper the influence of a cell size on the dynamic stability of *uniperiodically densely stiffened cylindrical shells* will be analyzed. It has to be emphasized that the non–asymptotic tolerance models of shells with uni– and biperiodic structure have to be led out independently, because they are based on different modeling assumptions. The governing equations for uniperiodic shells are more complicated. It means that contrary to the asymptotic approach, *the uniperiodic shell is not a special case of biperiodic shell.*

The application of the tolerance averaging technique to the investigations of selected dynamical and/or stability problems for periodic plates can be found in many papers, e.g. in [8] and [3], where stability of Kirchhoff-type plates and of Hencky–Bolle–type plates is analyzed, respectively, in [7] and [12], where dynamics of Kirchhoff–type plates and of wavy–type plates is investigated, respectively. For review of application of the tolerance averaging technique to the modeling of different periodic and also non–periodic structures the reader is referred to [24, 25, 26]. The aim of this contribution is three–fold:

- First, to formulate a new mathematical non–asymptotic model for the analysis of parametric vibrations and dynamical stability of uniperiodically stiffened cylindrical shells. This model will be derived by applying *a new approach to the tolerance modeling of micro–heterogeneous media* proposed by Woźniak in [26].
- Second, to formulate an asymptotic model in which the length–scale effect is neglected. This model will be derived by applying *the consistent asymptotic modeling* proposed by Woźniak in [26].
- Third, to apply the derived models to investigate the effect of a microstructure size on the frequency equation being a starting point in the analysis of parametric vibrations and dynamical stability of periodic shells under consideration

It is well known that stability problems of shells being homogeneous or weakly heterogeneous have to be investigated by using the geometrically nonlinear shell theory, cf. [5, 13]. However, in the case of the highly heterogeneous structures considered here (i.e. densely stiffened shells), which are described by using continuum models, we are often interested in the upper state of critical forces and hence we can use the geometrically linear stability theory for thin linear–elastic cylindrical Kirchhoff–Love–type shells.

The periodic cylindrical shells, being object of considerations in this paper, are widely applied in civil engineering, most often as roof girders and bridge girders. They are also applied as housings of reactors and tanks. The periodic cylindrical shells having small length dimensions are widely used as elements of air–planes, ships and machines.

In the subsequent section the basic denotations, preliminary concepts and starting equations will be presented.

2. Preliminaries

In this paper we investigate linear–elastic thin circular cylindrical shells. The shells are reinforced by families of longitudinal ribs, which are periodically and densely distributed in circumferential direction. Shells of this kind are termed *uniperiodic*. At the same time, these shells have constant structure in an axial direction. Example of such shell is shown in Fig. 1.

In order to describe the shell geometry define $\tilde{\Omega} = (0, L_1) \times (0, L_2)$ as a set of points (x^1, x^2) in R^2; x^1, x^2 being the Cartesian orthogonal coordinates parametrizing region $\tilde{\Omega} \subset R^2$. Let $O\,\bar{x}^1\bar{x}^2\bar{x}^3$ stand for a Cartesian orthogonal coordinate system in the physical space E^3. Points of E^3 will be denoted by $\bar{\mathbf{x}} \equiv (\bar{x}^1, \bar{x}^2, \bar{x}^3)$. A cylindrical shell midsurface $M$is given by its parametric representation

$$M \equiv \left\{ \bar{\mathbf{x}} \in E^3 : \bar{\mathbf{x}} = \bar{\mathbf{r}}\left(x^1, x^2\right),\ \left(x^1, x^2\right) \in \tilde{\Omega} \right\}$$

where $\bar{\mathbf{r}}(\cdot)$ is the smooth function such that

$$\partial\, \bar{\mathbf{r}}/\partial x^1 \cdot \partial\, \bar{\mathbf{r}}/\partial x^2 = 0 \qquad \partial\, \bar{\mathbf{r}}/\partial x^1 \cdot \partial\, \bar{\mathbf{r}}/\partial x^1 = 1 \qquad \partial\, \bar{\mathbf{r}}/\partial x^2 \cdot \partial\, \bar{\mathbf{r}}/\partial x^2 = 1$$

It means that on M we have introduced the orthonormal parametrization and hence L_1, L_2 are length dimensions of M. It is assumed that x^1 and x^2 are coordinates parametrizing the shell midsurface along the lines of its principal curvature and along its generatrix, respectively, cf. Fig. 1.

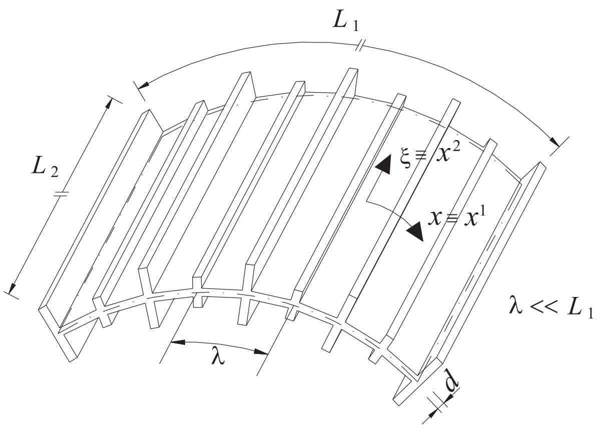

Figure 1 A fragment of periodically stiffened cylindrical shell

Subsequently, sub– and superscripts $\alpha, \beta, \ldots$ run over sequence 1, 2 and are related to midsurface parameters x^1, x^2; summation convention holds. The partial differentiation related to x^α is represented by ∂_α. Moreover, it is denoted $\partial_{\alpha\ldots\delta} \equiv \partial_\alpha \ldots \partial_\delta$. Differentiation with respect to time coordinate $t \in [t_0, t_1]$ is represented by the overdot. Denote by $a_{\alpha\beta}$ and $a^{\alpha\beta}$ the covariant and contravariant midsurface first metric tensors; respectively. For the introduced parametrization $a_{\alpha\beta} = a^{\alpha\beta} = \delta^{\alpha\beta}$ are the unit tensors.

We define a bounded domain $\Omega \times \Xi$ by means of $\Omega = (0, L_1)$ and $\Xi = (0, L_2) \times [t_0, t_1]$ as well as we shall denote $x \equiv x^1 \in (0, L_1)$ and $\xi \equiv x^2 \in (0, L_2)$.

Let $d(x)$ and r stand for the shell thickness and the constant midsurface curvature radius, respectively. We define λ as a period of the stiffened shell structure in $x \equiv x^1$–direction, which represents here the distance between axes of two neighbouring stiffeners belonging to the same family, cf. Fig. 1. The period λ satisfies conditions: $\lambda/d_{\max} >> 1$, $\lambda/r << 1$ and $\lambda/L_1 << 1$. We also assume that $L_2 > L_1$ and hence $\lambda/L_2 << 1$. *The basic cell* Δ and *the cell distribution* (Ω, Δ) assigned to $\Omega = (0, L_1)$ are defined by $\Delta \equiv [-\lambda/2,\ \lambda/2]$, $(\Omega, \Delta) \equiv \{\Delta(x) \equiv x + \Delta,\ x \in \bar{\Omega}\}$. Setting $z \equiv z^1 \in [-\lambda/2,\ \lambda/2]$, we assume that the cell Δ has a symmetry axis for $z = 0$. It means that inside the cell, the geometrical, elastic and inertial properties of the stiffened shell are described by symmetric (i.e. even) functions of argument z. At the same time, these functions are independent of argument $\xi \equiv x^2$.

Denote by $u_\alpha = u_\alpha(x, \xi, t)$, $w = w(x, \xi, t)$, $x \in \Omega$, $(\xi, t) \in \Xi$, the midsurface shell displacements in directions tangent and normal to M , respectively. Elastic properties of the shell are described by shell stiffness tensors $D^{\alpha\beta\gamma\delta}(x)$, $B^{\alpha\beta\gamma\delta}(x)$. Let $\mu(x)$ stand for a shell mass density per midsurface unit area. We denote by $\bar{N}^{\alpha\beta}(t)$ the time–dependent compressive membrane forces. In the problem considered here the external forces will be neglected.

It is assumed that the behavior of the stiffened shell under consideration is described by the action functional

$$A(u_\alpha, w) = \int_0^{L_1}\int_0^{L_2}\int_{t_0}^{t_1} L(x, \partial_\beta u_\alpha, \dot{u}_\alpha, \partial_{\alpha\beta} w, \partial_\alpha w, w, \dot{w}) dt d\xi dx \tag{1}$$

where lagrangian $L(x, \partial_\beta u_\alpha, \dot{u}_\alpha, \partial_{\alpha\beta} w, \partial_\alpha w, w, \dot{w})$ is highly oscillating function with respect to x and has the well-known form, cf. [6]

$$\begin{aligned} L = \frac{1}{2}(&D^{\alpha\beta\gamma\delta}\partial_\beta u_\alpha \partial_\delta u_\gamma + \frac{2}{r} D^{\alpha\beta 11} w \partial_\beta u_\alpha + \frac{1}{r^2} D^{1111} ww \\ &+ B^{\alpha\beta\gamma\delta}\partial_{\alpha\beta} w \partial_{\gamma\delta} w + \bar{N}^{\alpha\beta}(t)\partial_\alpha w \partial_\beta w - \mu a^{\alpha\alpha}(\dot{u}_\alpha)^2 - \mu \dot{w}^2) \end{aligned} \tag{2}$$

The principle of stationary action applied to A leads to the following system of Euler–Lagrange equations

$$\begin{aligned} &\partial_\beta \frac{\partial L}{\partial(\partial_\beta u_\alpha)} + \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{u}_\alpha} = 0 \\ &-\partial_{\alpha\beta}\frac{\partial L}{\partial(\partial_{\alpha\beta} w)} + \partial_\alpha \frac{\partial L\lambda}{\partial(\partial_\alpha w)} - \frac{\partial L}{\partial w} + \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{w}} = 0 \end{aligned} \tag{3}$$

After combining (3) with (2) the above system can be written in the form

$$\begin{aligned} &\partial_\beta(D^{\alpha\beta\gamma\delta}\partial_\delta u_\gamma) + r^{-1}\partial_\beta(D^{\alpha\beta 11} w) = \mu a^{\alpha\alpha}\ddot{u}_\alpha \\ &r^{-1}D^{\alpha\beta 11}\partial_\beta u_\alpha + \partial_{\alpha\beta}(B^{\alpha\beta\gamma\delta}\partial_{\gamma\delta} w) + r^{-2}D^{1111}w - \bar{N}^{\alpha\beta}\partial_{\alpha\beta} w = -\mu\ddot{w} \end{aligned} \tag{4}$$

It can be observed that equations (4) coincide with the well–known governing equations of simplified Kirchhoff–Love second–order theory of thin elastic shells, cf. [5]. In the above equations the displacements $u_\alpha(x, \xi, t)$, $w(x, \xi, t)$ are the basic unknowns. For periodic shells coefficients of lagrangian L and hence also of equations (4) are highly oscillating non–continuous functions depending on x with a period λ. That is why equations (4) cannot be directly applied to investigations of engineering problems. Our aim is to "replace" these equations by equations with constant coefficients depending on the microstructure size. To this end *the tolerance modelling technique* given by Woniak in [26] will be applied. To make the analysis more clear, in the next section we shall outline the basic concepts and the main assumptions of *the tolerance averaging approach and of the consistent asymptotic modeling*, following the book [26].

3. Modeling Concepts and Assumptions

Following the monograph [26], we outline below the basic concepts and assumptions which will be used in the course of modeling procedure.

3.1. *Basic concepts*

The fundamental concepts of the tolerance modeling are those of *tolerance* (determined by *tolerance parameter* δ), *cell distribution*, *tolerance periodic function* and its two special cases: *slowly–varying* and *highly–oscillating functions.* The tolerance approach is based on the notion of *the averaging of tolerance periodic function.*

- The main statement of the modelling procedure is that every measurement as well as numerical calculation can be realized in practice only within a certain accuracy defined by *tolerance parameter* δ being a positive constant.

- For the shells discussed here, the concept of *cell distribution* (Ω, Δ) assigned to $\Omega = (0, L_1)$ has been introduced in the previous Section.

- A bounded integrable function $f(x)$ defined on $\bar{\Omega} = [0, L_1]$ (which can also depend on $\xi \in [0, L_2]$ and t as parameters) is called *tolerance periodic* with respect to cell Δ and tolerance parameter δ, if roughly speaking, its values in an arbitrary cell $\Delta(x)$ can be approximated, with sufficient accuracy, by the corresponding values of a certain Δ–periodic function $\tilde{f}(x, z)$, $z \in \Delta(x)$, $x \in \bar{\Omega}$. Function $\tilde{f}$ is a Δ–*periodic approximation* of f on $\Delta(x)$. This condition has to be fulfilled by all derivatives of f up to the R–th order; i.e. by all its derivatives which occur in the problem under consideration. In this case we shall write $f \in TP^R_\delta(\Omega, \Delta)$.

- A continuous bounded diferentiable function $v(x)$ defined on $\bar{\Omega} = [0, L_1]$ (which can also depend on $\xi \in [0, L_2]$ and t as parameters) is called *slowly-varying* with respect to cell Δ and tolerance parameter δ, if $v \in TP^R_\delta(\Omega, \Delta)$ and its periodic approximation $\tilde{v}(x, z)$, $z \in \Delta(x)$, $x \in \bar{\Omega}$ in $\Delta(x)$ together with periodic approximations of its derivatives up to the R–th order (i.e. all its derivatives which occur in the problem under consideration) are constant functions in arbitrary periodicity cell $\Delta(x)$; we shall write $v \in SV^R_\delta(\Omega, \Delta)$.

- A λ–periodic function $h(\cdot)$ defined on $\bar{\Omega}$, which is continuous together with its derivatives up to the $(R-1)$ order and has either continuous or a piecewise continuous bounded derivative of the R–th order, is called *the highly–oscillating function* with respect to cell Δ and tolerance parameter δ,

  $h \in HO^R_\delta(\Omega, \Delta) \subset TP^R_\delta(\Omega, \Delta)$

  if it depends on λ and satisfies conditions:

(a) $(\forall v(x) \in SV_\delta^R(\Omega,\Delta))\ (f = hv \in TP_\delta^R(\Omega,\Delta))$,
(b) $\partial_z^k \tilde{f}(x,z) = \partial_z^k h(z)\ v(x)$
for every $z \in \Delta(x),\ x \in \Omega,\ k = 0,1,..,R,$
(c) $\partial_z^k h \in O(\lambda^{R-k}), \quad k = 0,1,..,R,$
(d) $\frac{1}{\lambda} \int\limits_{-\lambda/2}^{\lambda/2} \mu(z)h(z)dz = 0, \quad z \in \Delta(x),$
for μ being a certain positive valued λ–periodic function defined on $\bar{\Omega}$,
(e) $\frac{1}{\lambda} \int\limits_{-\lambda/2}^{\lambda/2} \partial_z^k h(z)dz = 0, \quad z \in \Delta(x),\ x \in \Omega, \quad k = 1,..,R,$

where we have introduced denotations
$\partial_z^k \tilde{f}(x,z) \equiv \partial^k \tilde{f}(x,z)\ /\partial z^k,\ k = 0,1,..,R,\ \partial_z^0 \tilde{f}(x,z) \equiv f(x,z)$
and
$\partial_z^k h(z) \equiv \partial^k h(z)\ /\partial z^k,\ k = 0,1,..,R,\ \partial_z^0 h(z) \equiv h(z).$
By *the averaging of tolerance periodic function*
$f(x) \in TP_\delta^0(\Omega,\Delta),\ x \in \bar{\Omega} = [0, L_1],$
which can also depend on $\xi \in [0, L_2]$ and t as parameters, we shall mean function

$$< f > (x) \equiv \frac{1}{\lambda} \int\limits_{x-\lambda/2}^{x+\lambda/2} \tilde{f}(x,z)dz \qquad z \in \Delta(x), \qquad x \in \bar{\Omega} \tag{5}$$

where $\tilde{f}(x,z)$ is a periodic approximation of $f(x)$ in $\Delta(x) = [x - \lambda/2,\ x + \lambda/2]$. For function $f(x) \in TP_\delta^R(\Omega,\Delta)$, the above formula also holds for derivatives of f up to the R–th order, i.e. for all its derivatives which occur in the problem under consideration. It can be seen that if f is a periodic function then $\tilde{f}$ is independent of x and $< f >$ is constant.

It follows that if $h(z) \in HO_\delta^0(\Omega,\Delta)$ and $v(x) \in SV_\delta^0(\Omega,\Delta)$ then

$$< v > (x) \equiv \frac{1}{\lambda} \int\limits_{x-\lambda/2}^{x+\lambda/2} v(x)dz = v(x), \qquad z \in \Delta(x),\ x \in \bar{\Omega} \tag{6}$$

$$< vh > (x) \equiv \frac{1}{\lambda} \int\limits_{x-\lambda/2}^{x+\lambda/2} h(z)dz\ v(x), \qquad z \in \Delta(x),\ x \in \bar{\Omega} \tag{7}$$

For functions $v(x) \in SV_\delta^R(\Omega,\Delta)$ and $f = vh \in TR_\delta^R(\Omega,\Delta)$, where $h(z) \in HO_\delta^R(\Omega,W)$, the above formulae also hold for derivatives of v and f up to the R-th order.

On passing from *tolerance averaging* to *the consistent asymptotic averaging* we retain only the concept of *highly–oscillating function.* In the asymptotic approach we deal with *mean (constant) value* $< f >$ *of* Δ*–periodic function* $f(\cdot)$ defined by

$$< f >\equiv \frac{1}{\lambda} \int\limits_{x-\lambda/2}^{x+\lambda/2} f(z)dz\ , \quad z \in \Delta(x),\ \ x \in \bar{\Omega} \tag{8}$$

3.2. *Modeling assumptions*

The fundamental assumption imposed on the lagrangian under consideration in the framework of *the tolerance averaging approach* is called *the micro–macro decomposition.* It states that the displacement fields occurring in this lagrangian have to be *the tolerance periodic functions* in x. Hence, they can be decomposed into *unknown averaged displacements* being *slowly–varying functions* in x and *fluctuations* represented by *known highly–oscillating functions* called *fluctuation shape functions* and by *unknown fluctuation amplitudes* being *slowly–varying* in x.

The fundamental assumption imposed on the lagrangian under consideration in the framework of *the consistent asymptotic averaging approach* is called *the consistent asymptotic decomposition.* It states that the displacement fields occurring in this lagrangian have to be replaced by families of fields depending on small parameter $\varepsilon = 1/m,\ m = 1, 2, ...$ and defined in an arbitrary cell. These families of displacements are decomposed into averaged part described by unknown functions being continuously bounded in $\bar{\Omega}$ and highly–oscillating part depending on ε. This highly–oscillating part is represented by known *fluctuation shape functions* and by unknown functions being continuously bounded $\bar{\Omega}$.

For details the reader is referred to [26] and also to [24, 25].

4. Tolerance Model

The tolerance modeling procedure for Euler–Lagrange equations (3) is realized in two steps.

The first step is *the tolerance averaging of action functional* (1). To this end let us introduce two systems of linear independent *highly–oscillating functions*, called *the fluctuation shape functions*, being λ–periodic in x:

$$h^a(x) \in HO^1_\delta(\Omega, \Delta) \qquad a = 1, .., n$$

and

$$g^A(x) \in HO^2_\delta(\Omega, \Delta) \qquad A = 1, .., N$$

These functions are assumed to be known in every problem under consideration. They have to satisfy conditions:

$$\begin{array}{l} h^a \in O(\lambda) \quad \lambda\partial_1 h^a \in O(\lambda) \quad g^A \in O(\lambda^2) \quad \lambda\partial_1 g^A \in O(\lambda^2) \\ \lambda^2\partial_{11} g^A \in O(\lambda^2) \quad <\mu\, h^a> = <\mu\, g^A> = 0 \end{array}$$

and

$$<\mu\, h^a h^b> = <\mu\, g^A g^B> = 0 \quad \text{for} \quad a \neq b,\ A \neq B$$

where $\mu(\cdot)$ is the shell mass density being a λ–periodic function with respect to x. In dynamic problems, functions $h^a(x), g^A(x)$ represent either the principal modes of free periodic vibrations of the cell $\Delta(x)$ or physically reasonable approximation of these modes. Hence, they can be obtained as solutions to certain periodic eigenvalue problems describing free periodic vibrations of the cell, cf. [21]. Because of these functions are periodic in x with a period λ we can restrict their domain $(0, L_1)$ to arbitrary cell $\Delta(x) = [x - \lambda/2,\ x + \lambda/2]$ with center at x. Bearing in mind the symmetry of the cell geometry and symmetric distribution of the material properties inside the cell we assume that $h^a(z)$ and $g^A(z)$, $z \in [-\lambda/2,\ \lambda/2]$, are respectively *odd* and *even* functions of z.

Now, we have to introduce *the micro–macro decomposition* of displacements $u_\alpha(x,\xi,t) \in TP^1_\delta(\Omega,\Delta)$, $w(x,\xi,t) \in TP^2_\delta(\Omega,\Delta)$, $x \in \Omega$, $(\xi,t) \in \Xi$, which in the problem under consideration is assumed in the form

$$\begin{aligned} u_\alpha(x,\xi,t) = u_{h\alpha}(x,\xi,t) = u^0_\alpha(x,\xi,t) + h^a(x)U^a_\alpha(x,\xi,t)\ , &\quad a = 1,...,n \\ w(x,\xi,t) = w_g(x,\xi,t) = w^0(x,\xi,t) + g^A(x)W^A(x,\xi,t)\ , &\quad A = 1,...,N \end{aligned} \tag{9}$$

where

$$\begin{aligned} &u^0_\alpha(x,\xi,t),\ U^a_\alpha(x,\xi,t) \in SV^1_\delta(\Omega,\Delta) \subset TP^1_\delta(\Omega,\Delta) \\ &w^0(x,\xi,t),\ W^A(x,\xi,t) \in SV^2_\delta(\Omega,\Delta) \subset TP^2_\delta(\Omega,\Delta) \end{aligned} \tag{10}$$

and where summation convention over a and A holds. Functions u^0_α, w^0, called *averaged variables*, and functions U^a_α, W^A, called *fluctuation amplitudes*, are *the new unknowns* being slowly–varying in x.

Due to the fact that $u_{h\alpha}(\cdot,\xi,t) \in TP^1_\delta(\Omega,\Delta)$ and $w_g(\cdot,\xi,t) \in TP^2_\delta(\Omega,\Delta)$ there exist periodic approximations of these functions and of their pertinent derivatives in every $\Delta(x)$.

Bearing in mind properties of the slowly–varying and highly–oscillating functions, cf. Section 3 of this paper or a book [26], the periodic approximations of $u_{h\alpha}(\cdot,\xi,t)$, $\partial_\beta u_{h\alpha}(\cdot,\xi,t)$ and $\dot{u}_{h\alpha}(\cdot,\xi,t)$ in $\Delta(x)$, $x \in \bar{\Omega}$, have the form

$$\begin{aligned} &\tilde{u}_{h\alpha}(x,z,\xi,t) = u^0_\alpha(x,\xi,t) + h^a(z)U^a_\alpha(x,\xi,t) \\ &\partial_1\tilde{u}_{h\alpha}(x,z,\xi,t) = \partial_1 u^0_\alpha(x,\xi,t) + \partial_1 h^a(z)U^a_\alpha(x,\xi,t) \\ &\partial_2\tilde{u}_{h\alpha}(x,z,\xi,t) = \partial_2 u^0_\alpha(x,\xi,t) + h^a(z)\partial_2 U^a_\alpha(x,\xi,t) \\ &\dot{\tilde{u}}_{h\alpha}(x,z,\xi,t) = \dot{u}^0_\alpha(x,\xi,t) + h^a(z)\dot{U}^a_\alpha(x,\xi,t) \end{aligned} \tag{11}$$

for every $x \in \bar{\Omega}$, almost every $z \in \Delta(x)$ and every $(\xi,t) \in \Xi$.

The periodic approximations of $w_g(\cdot,\xi,t)$, $\partial_\alpha w_g(\cdot,\xi,t)$, $\partial_{\alpha\beta} w_g(\cdot,\xi,t)$ and $\dot{w}_g(\cdot,\xi,t)$ in $\Delta(x)$, $x \in \bar{\Omega}$, have the form

$$\begin{aligned} &\tilde{w}_g(x,z,\xi,t) = w^0(x,\xi,t) + g^A(z)W^A(x,\xi,t)\ , \\ &\partial_1\tilde{w}_g(x,z,\xi,t) = \partial_1 w^0(x,\xi,t) + \partial_1 g^A(z)W^A(x,\xi,t) \\ &\partial_2\tilde{w}_g(x,z,\xi,t) = \partial_2 w^0(x,\xi,t) + g^A(z)\partial_2 W^A(x,\xi,t) \\ &\partial_{11}\tilde{w}_g(x,z,\xi,t) = \partial_{11} w^0(x,\xi,t) + \partial_{11} g^A(z)W^A(x,\xi,t) \\ &\partial_{12}\tilde{w}_g(x,z,\xi,t) = \partial_{21}\tilde{w}_g = \partial_{12} w^0(x,\xi,t) + \partial_1 g^A(z)\partial_2 W^A(x,\xi,t) \\ &\partial_{22}\tilde{w}_g(x,z,\xi,t) = \partial_{22} w^0(x,\xi,t) + g^A(z)\partial_{22} W^A(x,\xi,t) \\ &\dot{\tilde{w}}_g(x,z,\xi,t) = \dot{w}^0(x,\xi,t) + g^A(z)\dot{W}^A(x,\xi,t) \end{aligned} \tag{12}$$

for every $x \in \bar{\Omega}$, almost every $z \in \Delta(x)$ and every $(\xi,t) \in \Xi$.

Setting $u_{h\alpha} \equiv u_\alpha$, $w_g \equiv w$, we obtain from (2) lagrangian
$L_{hg}(x, \partial_\beta u_{h\alpha}, \dot{u}_{h\alpha}, \partial_{\alpha\beta} w_g, \partial_\alpha w_g, w_g, \dot{w}_g)$, $x \in \bar{\Omega}$

Since L_{hg} is highly oscillating with respect to x then there exists a periodic approximation

$\tilde{L}_{hg}(z, \partial_\beta \tilde{u}_{h\alpha}, \dot{\tilde{u}}_{h\alpha}, \partial_{\alpha\beta}\tilde{w}_g, \partial_\alpha \tilde{w}_g, \tilde{w}_g, \dot{\tilde{w}}_g)$, $z \in \Delta(x)$
of L_{hg} in every $\Delta(x)$. Substituting the right hand sides of approximations (11), (12) into $\tilde{L}_{hg}$ and using *tolerance averaging formula* (5) with (6), (7), we arrive at *the tolerance averaging of* L_{hg} *in* $\Delta(x)$ *under micro–macro decomposition* (9). Introducing *the extra approximation* $1 + \lambda/r \approx 1$, the obtained result has the form

$$\begin{aligned}
&< L_{hg} > (\partial_\beta u^0_\alpha, \partial_2 U^a_\alpha, U^a_\alpha, \dot{u}^0_\alpha, \dot{U}^a_\alpha, \partial_{\alpha\beta} w^0, \partial_\beta w^0, w^0, \partial_{22} W^A, \partial_2 W^A, W^A, \dot{w}^0, \dot{W}^A) \\
&= \frac{1}{2}[< D^{\alpha\beta\gamma\delta} > \partial_\beta u^0_\alpha \partial_\delta u^0_\gamma + 2 < D^{\alpha\beta\gamma 1}\partial_1 h^a > \partial_\beta u^0_\alpha U^a_\gamma \\
&+ < D^{\alpha 11\gamma}\partial_1 h^a \partial_1 h^b > U^a_\gamma U^b_\alpha + \underline{< D^{\alpha 22\gamma} h^a h^b >} \partial_2 U^b_\gamma \partial_2 U^a_\alpha \\
&+ 2r^{-1}[< D^{\alpha\beta 11} > \partial_\beta u^0_\alpha w^0 + < D^{\alpha 111}\partial_1 h^a > w^0 U^a_\alpha] \\
&+ r^{-2} < D^{1111} > w^0 w^0 + < B^{\alpha\beta\gamma\delta} > \partial_{\alpha\beta} w^0 \partial_{\gamma\delta} w^0 \\
&+ 2(< B^{\alpha\beta 11}\partial_{11} g^A > \partial_{\alpha\beta} w^0 W^A + \underline{< B^{\alpha\beta 22} g^A >} \partial_{\alpha\beta} w^0 \partial_{22} W^A \\
&+ \underline{< B^{1122} g^A \partial_{11} g^B >} \partial_{22} W^B W^A) + 4\underline{< B^{1212}\partial_1 g^A \partial_1 g^B >} \partial_2 W^A \partial_2 W^B \\
&+ < B^{1111}\partial_{11} g^A \partial_{11} g^B > W^A W^B + \underline{< B^{2222} g^A g^B >} \partial_{22} W^A \partial_{22} W^B \\
&+ \bar{N}^{\alpha\beta}\partial_\alpha w^0 \partial_\beta w^0 + 2\bar{N}^{\alpha 2} < g^A > \partial_\alpha w^0 \partial_2 W^A + \bar{N}^{22}\underline{< g^A g^B >} \partial_2 W^A \partial_2 W^B) \\
&- \bar{N}^{11}\underline{< \partial_1 g^A \partial_1 g^B >} W^A W^B - < \mu a^{\alpha\alpha}(\dot{u}^0_\alpha)^2 > - < \mu(\dot{w}^0)^2 > \\
&- \underline{< \mu h^a h^b >} a^{\alpha\alpha}\dot{U}^a_\alpha \dot{U}^b_\alpha - \underline{< \mu g^A g^B >} \dot{W}^A \dot{W}^B]
\end{aligned} \tag{13}$$

Functional

$$A_{hg}(u^0_\alpha, U^a_\alpha, w^0, W^A) = \int_0^{L_1}\int_0^{L_2}\int_{t_0}^{t_1} < L_{hg} > dt d\xi dx \tag{14}$$

where $< L_{hg} >$ is given by (13), is called *the tolerance averaging of functional* $A(u_\alpha, w)$ defined by (1) *under decomposition* (9). The underlined terms in (13) depend on microstructure length parameter λ.

The second step in the tolerance modeling of Euler–Lagrange equations (3) is to apply the principle of stationary action to A_{hg} given above.

The principle of stationary action applied to A_{hg} leads to the following system of equations for $u^0_\alpha, w^0, U^a_\alpha, W^A$ as the basic unknowns

$$\begin{aligned}
&\partial_\beta \frac{\partial < L_{hg} >}{\partial(\partial_\beta u^0_\alpha)} + \frac{\partial}{\partial t}\frac{\partial < L_{hg} >}{\partial \dot{u}^0_\alpha} = 0 \\
&-\partial_{\alpha\beta}\frac{\partial < L_{hg} >}{\partial(\partial_{\alpha\beta} w^0)} + \partial_\alpha \frac{\partial < L_{hg} >}{\partial(\partial_\alpha w^0)} - \frac{\partial < L_{hg} >}{\partial w^0} + \frac{\partial}{\partial t}\frac{\partial < L_{hg} >}{\partial \dot{w}^0} = 0 \\
&\frac{\partial}{\partial t}\frac{\partial < L_{hg} >}{\partial \dot{U}^a_\alpha} - \frac{\partial < L_{hg} >}{\partial U^a_\alpha} + \partial_2 \frac{\partial < L_{hg} >}{\partial(\partial_2 U^a_\alpha)} = 0 \\
&\frac{\partial}{\partial t}\frac{\partial < L_{hg} >}{\partial \dot{W}^A} - \frac{\partial < L_{hg} >}{\partial W^A} + \partial_2 \frac{\partial < L_{hg} >}{\partial(\partial_2 W^A)} - \partial_{22}\frac{\partial < L_{hg} >}{\partial(\partial_{22} W^A)} = 0
\end{aligned} \tag{15}$$

Combining (15) with (16) we arrive finally at the explicit form of *the tolerance model equations under micro–macro decomposition* (9). We shall write these equations in the form of constitutive equations

$$\begin{aligned}
&N^{\alpha\beta} =< D^{\alpha\beta\gamma\delta} > \partial_\delta u^0_\gamma + \frac{1}{r} < D^{\alpha\beta 11} > w^0 + < D^{\alpha\beta\gamma 1}\partial_1 h^b > U^b_\gamma \\
&M^{\alpha\beta} =< B^{\alpha\beta\gamma\delta} > \partial_{\gamma\delta} w^0 + < B^{\alpha\beta 11}\partial_{11} g^B > W^B + \underline{< B^{\alpha\beta 22} g^B >}\partial_{22} W^B \\
&H^{a\beta} =< \partial_1 h^a D^{\beta 1\gamma\delta} > \partial_\delta u^0_\gamma + \frac{1}{r} < \partial_1 h^a D^{\beta 111} > w^0 + < \partial_1 h^a D^{\beta 11\gamma}\partial_1 h^b > U^b_\gamma \\
&-\underline{< h^a D^{\beta 22\gamma}\, h^b >}\partial_{22} U^b_\gamma \qquad (16)\\
&G^A =< \partial_{11} g^A B^{11\alpha\beta} > \ \partial_{\alpha\beta} w^0 + \underline{< g^A B^{\alpha\beta 22} >}\ \partial_{\alpha\beta 22} w^0 \\
&+ < \partial_{11} g^A B^{1111}\partial_{11} g^B > W^B + [\underline{< \partial_{11} g^A B^{1122} g^B >} + \underline{< g^A B^{1122}\partial_{11} g^B} > \\
&-4\underline{< \partial_1 g^A B^{1212}\partial_1 g^B >}]\ \partial_{22} W^B + \underline{< g^A B^{2222}\, g^B >}\ \partial_{2222}\partial W^B
\end{aligned}$$

and the dynamic equilibrium equations

$$\begin{aligned}
&\partial_\alpha N^{\alpha\beta} - < \mu > a^{\beta\beta}\ddot{u}^0_\beta = 0 \\
&\partial_{\alpha\beta} M^{\alpha\beta} + \frac{1}{r} N^{11} + \bar{N}^{\alpha\beta}(t)\partial_{\alpha\beta} w^0 + \bar{N}^{\alpha 2}(t)\underline{< g^A >}\partial_{\alpha 2} W^A + \ < \mu > \ddot{w}^0 = 0 \\
&\underline{< \mu\, h^a h^b >} a^{\beta\beta}\,\ddot{u}^b_\beta + H^{a\beta} = 0\ , \qquad a, b = 1, 2, ..., n \qquad (17)\\
&\underline{< \mu\, g^A\, g^B >}\ddot{W}^B + G^A - \bar{N}^{11}(t)\,\underline{< \partial_1 g^A \partial_1 g^B >} W^B + \\
&\bar{N}^{\alpha 2}(t)\,(\underline{< g^A g^B >}\partial_{\alpha 2} W^B + \underline{< g^A >}\partial_{\alpha 2} w^0) = 0\ , \qquad A, B = 1, 2, ..., N.
\end{aligned}$$

Equations (16) and (17) together with *micro–macro decomposition* (9) and *physical reliability conditions* (10) constitute *the tolerance model for analysis of selected dynamic stability problems for uniperiodically stiffened shells under consideration.* In contrast to starting equations (4) with discontinuous, highly oscillating and periodic coefficients, the tolerance model equations derived here *have constant coefficients.* Moreover, *some of them depend on microstructure length parameter* λ (underlined terms). Hence, the tolerance model makes it possible to *describe the effect of length scale on the shell behavior.*

It has to be emphasized that solutions to selected initial/boundary value problems formulated in the framework of the tolerance model have a physical sense only if conditions (10) hold for the pertinent tolerance parameter δ. These conditions can be also used for the *a posteriori* evaluation of tolerance parameter δ and hence, for the verification of the physical reliability of the obtained solutions.

For a homogeneous shell $D^{\alpha\beta\gamma\delta}(x), B^{\alpha\beta\gamma\delta}(x),\ \mu(x)$ are constant and because $< \mu h >=< \mu g >= 0$, we obtain $< h >=< g >= 0$, and hence

$$< \partial_1 h >=< \partial_1 g >=< \partial_{11} g >= 0$$

In this case equations $(17)_{1,2}$ reduced to the well known shell equations of motion for averaged displacements

$$u^0_\alpha(x, \xi, t), w^0(x, \xi, t)$$

and independently for fluctuation amplitudes

$$U^a_\alpha(x, \xi, t), W^A(x, \xi, t)$$

we arrive at a system of equations, which under initial conditions

$$U^a_\alpha(x,\xi,t_0) = W^A(x,\xi,t_0) = 0$$

has only trivial solution

$$U^a_\alpha = W^A = 0$$

Hence, from decomposition (9) it follows that

$$u_\alpha = u^0_\alpha, \qquad w = w^0$$

It means that equations (9), (16), (17) generated by tolerance averaged Lagrange function (13) reduce to the starting equations (4) generated by Lagrange function (2).

5. Asymptotic Model

Asymptotic modeling procedure for Euler–Lagrange equations (3) is realized in two steps.

The first step is *the consistent asymptotic averaging of lagrangian* L occurring in (1). To this end we have to introduce *the consistent asymptotic decomposition* of displacements $u_\alpha = u_\alpha(z,\xi,t)$, $w = w(z,\xi,t)$, $z \in \Delta(x)$, $(\xi,t) \in \Xi$, in an arbitrary cell $\Delta(x)$, $x \in \Omega$

$$\begin{aligned} & u_{\varepsilon\alpha}(z,\xi,t) \equiv u_\alpha(z/\varepsilon,\xi,t) = u^0_\alpha(z,\xi,t) \\ & +\varepsilon h^a_\varepsilon(z) U^a_\alpha(z,\xi,t) \quad a = 1,..,n, \\ & w_\varepsilon(z,\xi,t) \equiv w(z/\varepsilon,\xi,t) = w^0(z,\xi,t) \\ & +\varepsilon^2 g^A_\varepsilon(z) W^A(z,\xi,t) \quad A = 1,..,N, \\ & z \in \Delta_\varepsilon(x), \quad (\xi,t) \in \Xi \end{aligned} \tag{18}$$

where summation convention over a and A holds, and

$$\begin{aligned} & \varepsilon = 1/m, \quad m = 1,2,..., \quad \Delta_\varepsilon \equiv (-\varepsilon\lambda/2,\ \varepsilon\lambda/2), \quad \Delta_\varepsilon(x) \equiv x + \Delta_\varepsilon, \ \ x \in \bar{\Omega} \\ & h^a_\varepsilon(z) \equiv h^a(z/\varepsilon) \in HO^1_\delta(\Omega,\Delta) \quad g^A_\varepsilon(z) \equiv g^A(z/\varepsilon) \in HO^2_\delta(\Omega,\Delta) \end{aligned}$$

Functions h^a, g^A have been defined in the previous Section; we recall that they are postulated *a priori* in every problem under consideration. Unknown functions u^0_α, U^a_α in (18) are assumed to be continuous and bounded together with their first derivatives. Unknown functions w^0, W^A in (18) are assumed to be continuous and bounded together with their derivatives up to the second order.

Moreover u^0_α, U^a_α, w^0, W^A are assumed to be independent of ε. This is the main difference between the asymptotic approach under consideration and approach which is used in the known homogenization theory, cf. [4, 9].

Due to the fact that lagrangian L defined by (2) is highly oscillating with respect to x there exists for every $x \in \bar{\Omega}$ lagrangian $\tilde{L}(z, \partial_\beta u_\alpha, \dot{u}_\alpha, \partial_{\alpha\beta} w, \partial_\alpha w, w, \dot{w})$ which constitutes a Δ-periodic approximation of lagrangian L in $\Delta(x)$, $z \in \Delta(x)$. Let $\tilde{L}_\varepsilon$

be a family of functions given by

$$\begin{aligned}\tilde{L}_\varepsilon &= \tilde{L}(z/\varepsilon, \partial_\beta u_{\varepsilon\alpha}, \dot{u}_{\varepsilon\alpha}, \partial_{\alpha\beta} w_\varepsilon, \partial_\alpha w_\varepsilon, w_\varepsilon, \dot{w}_\varepsilon)\\ &= \frac{1}{2}[D^{\alpha\beta\gamma\delta}(z/\varepsilon)\partial_\beta u_{\varepsilon\alpha}\partial_\delta u_{\varepsilon\gamma} + \frac{2}{r}D^{\alpha\beta 11}(z/\varepsilon) w_\varepsilon \partial_\beta u_{\varepsilon\alpha}\\ &+\frac{1}{r^2}D^{1111}(z/\varepsilon) w_\varepsilon w_\varepsilon + B^{\alpha\beta\gamma\delta}(z/\varepsilon)\partial_{\alpha\beta} w_\varepsilon \partial_{\gamma\delta} w_\varepsilon + \bar{N}^{\alpha\beta}(t)\partial_\alpha w_\varepsilon \partial_\beta w_\varepsilon\\ &-\mu a^{\alpha\alpha}(\dot{u}_{\varepsilon\alpha})^2 - \mu(\dot{w}_\varepsilon)^2]\end{aligned} \tag{19}$$

Substituting the right-hand sides of (18) into (19) and taking into account that if $\varepsilon \to 0$ then every continuous and bounded function $f(z,\xi,t)$, $z \in \Delta_\varepsilon(x)$, $(\xi,t) \in \Xi$, tends to function $f(x,\xi,t)$, $x \in \bar{\Omega}$, cf. [26], as well as after neglecting terms $O(\varepsilon)$, $O(\varepsilon^2)$ we arrive at

$$\begin{aligned}&\tilde{L}_\varepsilon = \tilde{L}(z/\varepsilon, \partial_1 u^0_\alpha(x,\xi,t) + \partial_1 h^a(z/\varepsilon) U^a_\alpha(x,\xi,t), \partial_2 u^0_\alpha(x,\xi,t), \dot{u}^0_\alpha(x,\xi,t),\\ &\partial_{11} w^0(x,\xi,t) + \partial_{11} g^A(z/\varepsilon) W^A(x,\xi,t), \partial_{12} w^0(x,\xi,t), \partial_{21} w^0(x,\xi,t),\\ &\partial_{22} w^0(x,\xi,t), \partial_\alpha w^0(x,\xi,t), w^0(x,\xi,t), \dot{w}^0(x,\xi,t))\end{aligned} \tag{20}$$

Moreover, if $\varepsilon \to 0$ then, by means of a property of the mean value, cf. [9], the obtained result tends weakly to $L_0(\partial_\beta u^0_\alpha, U^a_\alpha, \dot{u}^0_\alpha, \partial_{\alpha\beta} w^0, \partial_\alpha w^0, w^0, W^A, \dot{w}^0)$, where

$$L_0 = \frac{1}{\lambda}\int\limits_{-\lambda/2}^{\lambda/2} \tilde{L}(z, \partial_\beta u^0_\alpha, U^a_\alpha, \dot{u}^0_\alpha, \partial_{\alpha\beta} w^0, \partial_\alpha w^0, w^0, W^A, \dot{w}^0) dz, \;\; z \in \Delta(x), \;\; x \in \bar{\Omega}$$

It follows that

$$\begin{aligned}&L_0(\partial_\beta u^0_\alpha, U^a_\alpha, \dot{u}^0_\alpha, \partial_{\alpha\beta} w^0, \partial_\alpha w^0, w^0, W^A, \dot{w}^0)\\ &= \frac{1}{2}[< D^{\alpha\beta\gamma\delta}(z) > \partial_\beta u^0_\alpha \partial_\delta u^0_\gamma + 2 < D^{\alpha\beta\gamma 1}(z)\partial_1 h^a(z) > \partial_\beta u^0_\alpha U^a_\gamma\\ &+ < D^{\alpha 11\gamma}(z)\partial_1 h^a(z)\partial_1 h^b(z) > U^a_\gamma U^b_\alpha + 2r^{-1} < D^{\alpha\beta 11}(z) > \partial_\beta u^0_\alpha w^0\\ &+r^{-2} < D^{1111}(z) > (w^0)^2 + < B^{\alpha\beta\gamma\delta}(z) > \partial_{\alpha\beta} w^0 \partial_{\gamma\delta} w^0\\ &+2 < B^{\alpha\beta 11}(z)\partial_{11} g^A(z) > \partial_{\alpha\beta} w^0 W^A\\ &+ < B^{1111}(z)\partial_{11} g^A(z)\partial_{11} g^B(z) > W^A W^B\\ &+\bar{N}^{\alpha\beta}(t)\partial_\alpha w^0 \partial_\beta w^0 - < \mu > a^{\alpha\alpha}(\dot{u}^0_\alpha)^2 - < \mu > (\dot{w}^0)^2], \qquad z \in \Delta(x), \quad x \in \bar{\Omega}\end{aligned} \tag{21}$$

where denotation (8) has been used.

Function L_0, given above, is *the averaged form of lagrangian L defined by* (2) *under consistent asymptotic averaging.*

In the framework of consistent asymptotic modelling we introduce *the consistent asymptotic action functional* defined by

$$A^0_{hg}(u^0_\alpha, U^a_\alpha, w^0, W^A) = \int\limits_0^{L_1}\int\limits_0^{L_2}\int\limits_{t_0}^{t_1} L_0 dt d\xi dx$$

where L_0 is given by (21).

Under assumption that $\partial L_0/\partial(\partial_\beta \bar{u}_\alpha)$, $\partial L_0/\partial(\partial_{\alpha\beta}\bar{w})$ are continuous, from the principle of stationary action for A^0_{hg}, we obtain the Euler–Lagrange equations

$$
\begin{aligned}
&\partial_\beta \frac{\partial L_0}{\partial(\partial_\beta u^0_\alpha)} + \frac{\partial}{\partial t}\frac{\partial L_0}{\partial \dot{u}^0_\alpha} = 0 \\
&-\partial_{\alpha\beta}\frac{\partial L_0}{\partial(\partial_{\alpha\beta} w^0)} + \partial_\alpha \frac{\partial L_0}{\partial(\partial_\alpha w^0)} - \frac{\partial L_0}{\partial w^0} + \frac{\partial}{\partial t}\frac{\partial L_0}{\partial \dot{w}^0} = 0 \\
&\frac{\partial L_0}{\partial U^a_\alpha} = 0, \qquad a = 1,2,...,n, \qquad \frac{\partial L_0}{\partial W^A} = 0, \qquad A = 1,2,...,N.
\end{aligned}
\tag{22}
$$

Combining (22) with (21) we arrive at the explicit form of *the consistent asymptotic model equations* for $u^0_\alpha(x,\xi,t), w^0(x,\xi,t), U^a_\alpha(x,\xi,t), W^A(x,\xi,t),\ x \in \Omega,\ (\xi,t) \in \Xi$

$$
\begin{aligned}
&< D^{\alpha\beta\gamma\delta} > \partial_{\alpha\delta} u^0_\gamma + r^{-1} < D^{\alpha\beta 11} > \partial_\alpha w^0 + < D^{\alpha\beta\gamma 1}\partial_1 h^b > \partial_\alpha U^b_\gamma \\
&- < \mu > a^{\beta\beta}\ddot{u}^0_\beta = 0 \\
&< B^{\alpha\beta\gamma\delta} > \partial_{\alpha\beta\gamma\delta} w^0 + < D^{\alpha\beta 11}\partial_{11} g^B > \partial_{\alpha\beta} W^B \\
&+ r^{-1} < D^{11\gamma\delta} > \partial_\delta u^0_\gamma + r^{-2} < D^{1111} > w^0 \\
&+ r^{-1} < D^{111\gamma}\partial_1 h^b > U^b_\gamma + \bar{N}^{\alpha\beta}\partial_{\alpha\beta} w^0 - < \mu > \ddot{w}^0 = 0 \\
&< \partial_1 h^a D^{\beta 11\gamma}\partial_1 h^b > U^b_\gamma = - < \partial_1 h^a D^{\beta 1\gamma\delta} > \ \partial_\delta u^0_\gamma - r^{-1} < \partial_1 h^a D^{\beta 111} > w^0 \\
&< \partial_{11} g^A B^{1111}\partial_{11} g^B > W^A = - < \partial_{11} g^B B^{11\gamma\delta} > \partial_{\gamma\delta} w^0
\end{aligned}
\tag{23}
$$

It can be observed that we have obtained the linear algebraic equations for extra unknowns U^a_α, W^A, cf. Eqs $(23)_{3,4}$. It can be shown that linear transformations $\mathbf{G}$, $\mathbf{E}$ given by $G^{ab}_{\beta\gamma} =< \partial_1 h^a D^{\beta 11\gamma}\partial_1 h^b >$, $E^{AB} =< \partial_{11} g^A B^{1111}\partial_{11} g^B >$, respectively, are invertible. Hence, solutions U^b_γ, W^A to $(23)_{3,4}$ can be written in the form

$$
\begin{aligned}
&U^b_\gamma = -(G^{-1})^{bc}_{\gamma\eta}\left[< \partial_1 h^c D^{1\eta\mu\vartheta} > \ \partial_\vartheta u^0_\mu + \frac{1}{r} < \partial_1 h^c D^{1\eta 11} > w^0\right] \\
&W^A = -(E^{-1})^{AB} < \partial_{11} g^B B^{11\gamma\delta} > \partial_{\gamma\delta} w^0
\end{aligned}
\tag{24}
$$

where $\mathbf{G}^{-1}$ and $\mathbf{E}^{-1}$ are the inverses of the linear transformations $\mathbf{G}$, $\mathbf{E}$, respectively. Substituting (24) into $(23)_{1,2}$ and setting

$$
\begin{aligned}
&D^{\alpha\beta\gamma\delta}_h \equiv < D^{\alpha\beta\gamma\delta} > - < D^{\alpha\beta\eta 1}\ \partial_1 h^a > (G^{-1})^{ab}_{\eta\zeta} < \partial_1 h^b D^{1\zeta\gamma\delta} > \\
&B^{\alpha\beta\gamma\delta}_h \equiv < B^{\alpha\beta\gamma\delta} > - < B^{\alpha\beta 11}\partial_{11} g^A > (E^{-1})^{AB}\ < \partial_{11} g^B B^{11\gamma\delta} >
\end{aligned}
\tag{25}
$$

we arrive finally at the following form of Euler–Lagrange equations for u^0_α, w^0

$$
\begin{aligned}
&D^{\alpha\beta\gamma\delta}_h \partial_{\alpha\delta} u^0_\gamma + r^{-1} D^{\alpha\beta 11}_h \partial_\alpha w^0 - < \mu > a^{\beta\beta}\ddot{u}^0_\beta = 0 \\
&B^{\alpha\beta\gamma\delta}_h \partial_{\alpha\beta\gamma\delta} w^0 + r^{-1} D^{11\gamma\delta}_h \partial_\delta u^0_\gamma + r^{-2} D^{1111}_h w^0 + \bar{N}^{\alpha\beta}\partial_{\alpha\beta} w^0 + < \mu > \ddot{w}^0 = 0
\end{aligned}
\tag{26}
$$

Equations (26) have to be considered together with decomposition of

$$u_\alpha(x,\xi,t),\, w(x,\xi,t) \quad \text{in} \quad \Omega\times\Xi$$

$$\begin{aligned} u_\alpha(x,\xi,t) &= u_\alpha^0(x,\xi,t) + h^a(x)U_\alpha^a(x,\xi,t) \\ w(x,\xi,t) &= w^0(x,\xi,t) + g^A(x)W^A(x,\xi,t)\ , \quad x\in\Omega,\ \ (\xi,t)\in\Xi \end{aligned} \tag{27}$$

with U_α^a, W^A given by (24). Contrary to (9), the above formula is not a micro–macro decomposition since in the consistent asymptotic approach it is not assumed that functions $u_\alpha^0, w^0, U_\alpha^a, W^A$ are slowly–varying.

Equations (26) together with formula (27) represent *the consistent asymptotic model* of Euler–Lagrange equations (4) derived from lagrangian (2). Coefficients in equations (26) are constant in contrast to coefficients in equations (4) which are discontinuous, highly oscillating and periodic. The above model is not able to describe the length–scale effect on the overall shell dynamics and stability being independent of the microstructure cell size.

The subsequent analysis dealing with a certain dynamical stability problem will be based on tolerance model equations (16), (17) and consistent model equations (26).

6. Dynamical Stability of Uniperiodically Stiffened Shells

Now, the tolerance model equations (16), (17) will be applied to derivation of frequency equation being a starting point in the analysis of parametric vibrations and dynamic stability of periodically stiffened shells under consideration. In order to evaluate the effect of a cell size on this equation the results obtained from the tolerance model will be compared with those derived from an asymptotic model (26).

6.1. *Formulation of the problem*

The object of considerations is a closed circular cylindrical shell with r, d as its midsurface curvature radius and its constant thickness, respectively. It means that now $L_1 = 2\pi r$. The shell is reinforced by two families of longitudinal stiffeners, which are periodically and densely distributed in circumferential direction, cf. Fig. 2. The stiffeners of both kinds have constant rectangular cross–sections with A_1, A_2 as their areas and with I_1, I_2 as their moments of inertia. The gravity centers of the stiffener cross-sections are situated on the shell midsurface. Let a_1, a_2 be the widths of the ribs. It is assumed that both the shell and stiffeners are made of homogeneous isotropic materials. Denote by E, ν Young's modulus and Poisson's ratio of the shell material, respectively, and by E_1, E_2 Young's moduli of the stiffener materials. At the same time μ_0 stands for the constant shell mass density per midsurface unit area and μ_1, μ_2 stand for the constant mass densities of the stiffeners per the stiffener unit length, cf. Fig. 3.

In agreement with considerations in Section 2, we define λ as a period of the stiffened shell structure in $x \equiv x^1$–direction, which represents the distance between axes of two neighbouring stiffeners belonging to the same family, cf. Figs 2 and 3.

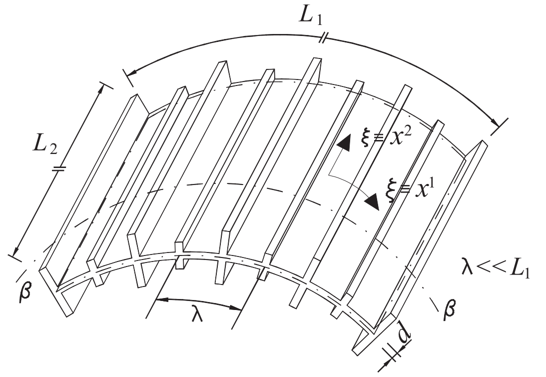

Figure 2 A fragment of a shell with two families of uniperiodically spaced stiffeners

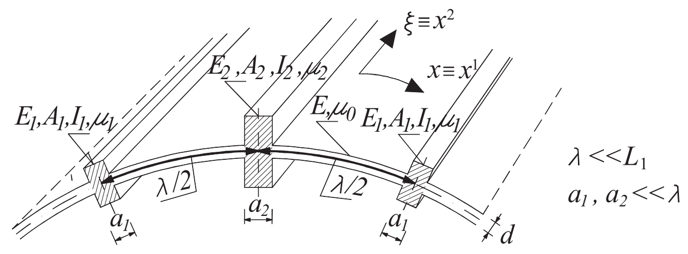

Figure 3 fragment of the stiffened shell cross–section $\beta - \beta$

We recall that the period λ has to satisfy conditions: $\lambda/d_{\max} >> 1, \quad \lambda/r << 1$ and $\lambda/L_1 << 1$. Moreover, under additional assumption that $L_2 > L_1$ the condition $\lambda/L_2 << 1$ holds. We also recall that *the basic cell* Δ and *the cell distribution* (Ω, Δ) assigned to $\Omega = (0, L_1)$ are defined by

$$\Delta \equiv [-\lambda/2,\ \lambda/2] \qquad (\Omega, \Delta) \equiv \{\Delta(x) \equiv x + \Delta,\ x \in \bar{\Omega}\}$$

The basic cell is shown in Fig. 4. Setting $z \equiv z^1 \in [-\lambda/2,\ \lambda/2]$, we assume that the cell Δ has a symmetry axis for $z = 0$. It means that inside the cell, the geometrical, elastic and inertial properties of the stiffened shell are described by symmetric (i.e. even) functions of argument z. At the same time, these functions are independent of argument $\xi \equiv x^2$

Tensile $E_1A_1,\ E_2A_2$ and bending $E_1I_1,\ E_2I_2$ rigidities of the stiffeners are constant. We assume that widths of the ribs $a_1\,, a_2 << \lambda$ and hence the torsional rigidity of stiffeners can be neglected. The rigidities $D_0^{\alpha\beta\gamma\delta},\ B_0^{\alpha\beta\gamma\delta}$ of the shell (without ribs) are also constant and described by: $D_0^{\alpha\beta\gamma\delta} = DH^{\alpha\beta\gamma\delta}$, $B_0^{\alpha\beta\gamma\delta} = BH^{\alpha\beta\gamma\delta}$, where $D = Ed/(1-\nu^2)$, $B = Ed^3/(12(1-\nu^2))$, and the nonzero components of $H^{\alpha\beta\gamma\delta}$are: $H^{1111} = H^{2222} = 1$, $H^{1122} = H^{2211} = \nu$, $H^{1212} = H^{1221} = H^{2121} = H^{2112} = (1-\nu)/2$.

The shell under consideration is simply supported on edges $\xi = 0$, $\xi = L_2$, cf. [10]. We shall neglect the rotational inertia effect on the dynamics shell behavior.

In agreement with notations introduced in Section 2, we denote by $D^{\alpha\beta\gamma\delta}(x)$, $B^{\alpha\beta\gamma\delta}(x)$ and $\mu(x)$ the stiffness tensors and the mass density of the reinforced shell under consideration, respectively. This periodically densely ribbed shell will be treated as a non–stiffened shell with a constant thickness d, made of a certain Δ–periodically non-homogeneous material. The shell's tensile $D^{2222}(\cdot)$ and bending $B^{2222}(\cdot)$ rigidities in the axial direction are λ-periodic functions in x, being independent of ξ. The remaining components of the shell stiffness tensors are constant and given by: $D^{\alpha\beta\gamma\delta} = D_0^{\alpha\beta\gamma\delta}$, $B^{\alpha\beta\gamma\delta} = B_0^{\alpha\beta\gamma\delta}$. The shell mass density $\mu(\cdot)$ is also a λ-periodic function in x, being independent of ξ.

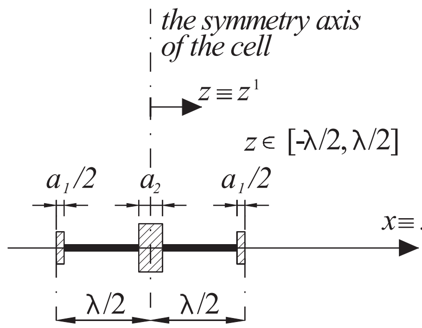

Figure 4 The basic cell of the shell under consideration a_1, $a_2 << \lambda$

Inside the cell Δ, functions $D^{2222}(z)$, $B^{2222}(z)$ and $\mu(z)$, $z \equiv z^1 \in [-\lambda/2,\ \lambda/2]$, take the form

$$D^{2222}(z) = \begin{cases} D_0^{2222} = D & \text{for} \quad z \in (-\lambda/2,\ \lambda/2) - \{0\} \\ E_1 A_1/2 & \text{for} \quad z = -\lambda/2 \quad \text{and} \quad z = \lambda/2 \\ E_2 A_2 & \text{for} \quad z = 0 \end{cases}$$

$$B^{2222}(z) = \begin{cases} B_0^{2222} = B & \text{for} \quad z \in (-\lambda/2,\ \lambda/2) - \{0\} \\ E_1 I_1/2 & \text{for} \quad z = -\lambda/2 \quad \text{and} \quad z = \lambda/2 \\ E_2 I_2 & \text{for} \quad z = 0 \end{cases}$$

$$\mu(z) = \begin{cases} \mu_0 & \text{for} \quad z \in (-\lambda/2,\ \lambda/2) - \{0\} \\ \mu_1/2 & \text{for} \quad z = -\lambda/2 \quad \text{and} \quad z = \lambda/2 \\ \mu_2 & \text{for} \quad z = 0 \end{cases}$$

Taking into account definition (5) we obtain for functions $D^{2222}(z)$, $B^{2222}(z)$ and $\mu(z)$, given above, the following averages values

$$\begin{aligned} &< D^{2222} >= D + (E_1A_1 + E_2A_2)/\lambda \\ &< B^{2222} >= B + (E_1I_1 + E_2I_2)/\lambda \\ &< \mu >= \mu_0 + (\mu_1 + \mu_2)/\lambda \end{aligned} \tag{28}$$

In order to analyze the problem of dynamical stability, we assume that the shell is uniformly compressed in axial direction by the time-dependent forces $\bar{N}(t) \equiv \bar{N}^{22}(t)$; hence $\bar{N}^{12} = \bar{N}^{21} = \bar{N}^{11} = 0$. Moreover, the forces of inertia in directions tangential to the shell midsurface will be neglected.

The investigated problem is rotationally symmetric with a period λ/r; hence $u_1^0, U_1^a = 0$ and the remaining unknowns u_2^0, U_2^a, w^0, W^A of the tolerance and asymptotic models (but not displacements u_2, w in decompositions (9) and (27)!) are only the functions of ξ–midsurface parameter. Moreover, we will neglect forces of inertia in an axial direction.

For the sake of simplicity, we shall confine ourselves to the simplest forms of the tolerance and asymptotic models in which $a = n = A = N = 1$, taking into account only one fluctuation shape function $h(z) \equiv h^1(z) \in HO_\delta^1(\Omega, \Delta)$, being antisymmetric on the cell, and only one fluctuation shape function $g(z) \equiv g^1(x) \in HO_\delta^2(\Omega, \Delta)$, being symmetric on the cell. We will take into account the following approximate forms of these functions: $h(z) = \lambda \sin(2\pi z/\lambda)$, $g(z) = \lambda^2[\cos(2\pi z/\lambda) + c]$, where constant c is calculated from condition $< \mu g >= 0$, cf. [21]. These functions relate to the smallest eigenvalues of certain periodic eigenvalue problems on the cell. Hence, they are referred to the lowest natural vibration modes in directions tangent and normal to the shell midsurface, respectively.

In the sequel denotations $U_2(\xi, t) \equiv U_2^1(\xi, t)$, $W(\xi, t) \equiv W^1(\xi, t)$ will be used.

Bearing in mind assumptions given above, the effect of the microstructure size on dynamic stability of stiffened shell under consideration will be analyzed by using both the tolerance model given by equations (16), (17) and the asymptotic model represented by equations (26).

6.2. *Analysis in the framework of the tolerance model*

Now, the system of tolerance equations (17) is separated into independent equation for $U_2(\xi, t)$:

$$< D^{2222}h^2 > \partial_{22}U_2 - D(1-\nu)2^{-1} < (\partial_1 h)^2 > U_2 = 0$$

which yields $U_2(\xi, t) = 0$, and the following system of three equations for $u_2^0(\xi, t)$, $w^0(\xi, t)$, $W(\xi, t)$

$$\begin{aligned} &< D^{2222} > \partial_{22}u_2^0 + D\nu\, r^{-1}\, \partial_2 w^0 = 0 \\ &D\nu\, r^{-1}\partial_2 u_2^0 + < B^{2222} > \partial_{2222}w^0 + \bar{N}^{22}\partial_{22}w^0 + Dr^{-2}w^0 + < \mu > \ddot{w}^0 \\ &+\lambda^2 < B^{2222}\bar{g} > \partial_{2222}W + \bar{N}^{22}\lambda^2 < \bar{g} > \partial_{22}W = 0 \\ &\lambda^2 < B^{2222}\bar{g} > \partial_{2222}w^0 + \bar{N}^{22}\lambda^2 < \bar{g} > \partial_{22}w^0 \\ &+\lambda^4 < B^{2222}\, (\bar{g}\,)^2 > \partial_{2222}W - 2B\,\lambda^2 < (\partial_1\tilde{g})^2 > \partial_{22}W \\ &+B < (\partial_{11}g)^2 > W + \lambda^4 < \mu\, (\bar{g}\,)^2 > \ddot{W} + \bar{N}^{22}\lambda^4 < (\bar{g})^2 > \partial_{22}W = 0 \end{aligned} \tag{29}$$

where

$$\tilde{g} = \lambda^{-1} g, \qquad \bar{g} = \lambda^{-2} g$$

Some terms in (29) depend explicitly on microstructure length parameter λ. All coefficients of equations (29) are constant.

Separating variables ξ and t,the solutions to Eqs.(29) satisfying boundary conditions for the simply supported shell on the edges $\xi = 0$, $\xi = L_2$ can be assumed in the form

$$\begin{aligned} u_2^0(\xi,t) &= \sum_{m=1}^{\infty} T_m^U(t)\cos(\alpha_m\xi) \qquad w^0(\xi,t) = \sum_{m=1}^{\infty} T_m(t)\sin(\alpha_m\xi) \\ W(\xi,t) &= \sum_{m=1}^{\infty} T_m^W(t)\sin(\alpha_m\xi) \qquad \alpha_m = m\pi/L_2 \end{aligned} \tag{30}$$

Substituting (30) into (29), assuming that $m = 1$ and taking into account that $\alpha\lambda << 1$, $d/\lambda << 1$ and hence neglecting some terms as small compared to 1, then assuming that the compressive axial forces $\bar{N}(t) \equiv N^{22}(t)$ are given as $\bar{N}(t) \equiv N^{22}(t) = \bar{N}_b \cos(pt)$, where p is the oscillating frequency of these forces and $\bar{N}_b$ is constant, as well as introducing the following denotations

$$\begin{aligned} &\tilde{\eta} \equiv < B^{2222} > + D(r^2\alpha^4)^{-1}[1 - D\nu^2 < D^{2222} >^{-1}] \\ &\tilde{\chi} \equiv D(r\alpha)^{-2}(1 - D\nu < D^{2222} >^{-1}) < \bar{g} > + \alpha^2 < B^{2222}\bar{g} > \\ &\tilde{\kappa} \equiv Dr^{-2}(-D\nu < D^{2222} >^{-1} < \bar{g} >^2 + < (\bar{g})^2 >) + \alpha^4 < B^{2222}(\bar{g})^2 > \\ &\tilde{\zeta} \equiv Dr^{-2}(-D\nu < D^{2222} >^{-1} < \bar{g} > + < \bar{g} >) + \alpha^4 < B^{2222}\bar{g} > \end{aligned} \tag{31}$$

$$\begin{aligned} &\omega^2 \equiv \alpha^4 < \mu >^{-1} \tilde{\eta}, \quad \vartheta_*^2 \equiv \alpha^4 < \mu >^{-1} \lambda^2 \tilde{\chi}, \quad \tilde{\omega}_*^2 \equiv \tilde{\zeta}(\lambda\alpha)^{-2} < \mu(\bar{g})^2 >^{-1} \\ &\tilde{\vartheta}_*^2 \equiv (B\lambda^{-4} < (\partial_{11}g)^2 > + 2B\lambda^{-2}\alpha^2 < (\partial_1\tilde{g})^2 > + \tilde{\kappa}) < \mu(\bar{g})^2 >^{-1} \\ &\tilde{N}_{cr} \equiv \alpha^2\tilde{\eta}, \quad \bar{S}_{cr} \equiv \tilde{\chi} < \bar{g} >^{-1}, \quad \widehat{S_{cr}} \equiv \tilde{\zeta}\alpha^{-2} < \bar{g} >^{-1} \\ &\bar{N}_{*cr} \equiv (B\lambda^{-4} < (\partial_{11}g)^2 > + 2B\lambda^{-2}\alpha^2 < (\partial_1\tilde{g})^2 > + \tilde{\kappa})\alpha^{-2} < (\bar{g})^2 >^{-1} \end{aligned} \tag{32}$$

we arrive at *the system of frequency equations*

$$\begin{aligned} &\frac{d^2T}{dt^2} + \omega^2\left[1 - \frac{N_b}{\tilde{N}_{cr}}\cos(pt)\right]T + \vartheta_*^2\left[1 - \frac{N_b}{\bar{S}_{cr}}\cos(pt)\right]T^W = 0 \\ &\frac{d^2T^W}{dt^2} + \tilde{\vartheta}_*^2\left[1 - \frac{N_b}{\bar{N}_{*cr}}\cos(pt)\right]T^W + \tilde{\omega}_*^2\left[1 - \frac{N_b}{\widehat{S}_{cr}}\cos(pt)\right]T = 0 \end{aligned} \tag{33}$$

In equations (33) the parameter λ is comprised in *the new additional higher free vibration frequencies* ϑ_*^2, $\tilde{\vartheta}_*^2$, $\tilde{\omega}_*^2$ and in *the new additional higher critical force* $\bar{N}_{*cr}$. In equations (33) we also deal with *the lower free vibration frequency* ω^2 and *the lower static critical forces* $\tilde{N}_{cr}$, $\bar{S}_{cr}$, $\widehat{S_{cr}}$, which are independent of a cell size λ.

The above system of two the second–order ordinary differential equations for two unknown functions of time coordinate is a starting point of the analysis of dynamic stability of the shell under consideration in the framework of the non–asymptotic tolerance model. Some parameters of equations (33) depend on the period length λ and hence *they make it possible to investigate the length–scale effect on parametric vibrations and dynamical stability of periodic shells under considerations.* It must be emphasized that *result* (33) *is a certain generalization of the known Mathieu equation* being the second–order ordinary differential equation, cf. [10]. Neglecting in (33) the terms with λ the Mathieu equation is obtained. It has to be emphasized that result (33) can be easily generalized on the case in which we deal with a shell reinforced by more then two families of stiffeners which are periodically and densely distributed in circumferential direction.

In order to evaluate the obtained results let us analyze this same problem in the framework of a model without the length–scale effect, represented by equations (26)

6.3. *Analysis in the framework of the consistent asymptotic model*

Under assumptions introduced in Subsection 6.1, equations (26) yield

$$\begin{aligned} &< D^{2222} > \partial_{22} u_2^0 + D\nu\, r^{-1}\,\partial_2 w^0 = 0 \\ &D\nu\, r^{-1}\partial_2 u_2^0 + < B^{2222} > \partial_{2222} w^0 + \bar{N}^{22}\partial_{22} w^0 + Dr^{-2} w^0 + < \mu > \ddot{w}^0 = 0 \end{aligned} \tag{34}$$

The above model is not able to describe the length–scale effect on the dynamic shell stability being independent of the period length λ. It is easy to see that there are not fluctuation amplitudes in model equations (34) derived here. Thus, from decomposition (27) it follows that $u_2^0 = U_2$, $w^0 = W$. Hence the governing equations (34), with averages $< D^{2222} >$, $< B^{2222} >$ and $< \mu >$ given by means of (28), coincide with the well–known equations of the orthotropic theory for stringer–stiffened cylindrical shells; see [5].

The solutions to Eqs. (34) can be assumed in the form $(30)_{1,2}$. Substituting $(30)_{1,2}$ into (34) and using denotations introduced in Subsection 6.2, after some manipulations, *the frequency equation of the consistent asymptotic model* takes the form

$$\frac{d^2T}{dt^2} + \omega^2[1 - \frac{N_b}{\tilde{N}_{cr}}\cos(pt)]T = 0 \tag{35}$$

It has to be observed that all parameters of the above equation are independent of the cell size. In the framework of the asymptotic model it is not possible to determine *the additional higher free vibration frequencies* and *the additional higher critical forces*, caused by the periodic structure of the shell. The result (35) neglecting the length-scale effect has a form of the known Mathieu equation, which describes dynamic stability and parametric vibrations of different structures, cf. [10].

7. Final Remarks and Conclusions

The new results obtained here lead to the following conclusions and remarks:

- Thin linear–elastic Kirchhoff–Love–type circular cylindrical shells with a periodically inhomogeneous structure along the circumferential direction are objects under consideration. Shells of this kind are termed *uniperiodic*. As an example we can mention cylindrical shells with periodically spaced families of longitudinal stiffeners as shown in Fig.1. Dynamic and stability behavior of such shells are described by Euler–Lagrange equations (3) generated by the well known Lagrange function (2). The explicit form of (3), given by (4), coincides with the governing equations of the simplified Kirchhoff–Love second–order theory for elastic shells. For periodic shells coefficients of these equations are highly oscillating non–continuous periodic functions. That is why the direct application of equations (4) to investigations of specific problems is non–effective even using computational methods.

- *The new mathematical non–asymptotic model* for analysis of selected dynamic and dynamical stability problems for periodic shells under consideration has been formulated by applying *the tolerance modeling procedure* given in [26]. The *tolerance approach* is based on the notions of *tolerance parameter, cell distribution, tolerance periodic function, slowly–varying function and highly–oscillating function* as well as on the concept of *the tolerance averaging of a tolerance periodic function.* Following the book [26], the definitions of these basic notions were outlined in Section 3 of this paper. The fundamental assumption imposed on the lagrangian under consideration in the framework of *the tolerance averaging approach* is called *the micro–macro decomposition.* It states that the displacement fields occurring in this lagrangian have to be *the tolerance periodic functions* in periodicity direction. Hence, they can be decomposed into *unknown averaged displacements* being *slowly–varying functions* and *fluctuations* represented by *known highly–oscillating functions* called *fluctuation shape functions* and by *unknown slowly–varying fluctuation amplitudes*. The tolerance modeling technique is realized in two steps. The first step is based on *the tolerance averaging of lagrangian* (2) *under micro–macro decomposition* (9). The resulting tolerance averaged form of lagrangian (2) is given by (13). In the second step, applying the principle of stationary action to *the tolerance averaged action functional* (14) defined by means of averaged lagrangian (13) we arrive at Euler–Lagrange equations (15). After combining (15) with (13) we obtain finally the explicit form of *the tolerance model equations under micro–macro decomposition* (9). These equations are written in the form of constitutive relations (16) and dynamic balance equations (17). Contrary to the "exact" shell equations (4) with highly oscillating non–continuous periodic coefficients, *the obtained tolerance model equations have constant coefficients which depend on microstructure length parameter* λ. It means that the proposed tolerance model equations describe *the effect of the cell size on the overall shell dynamics and dynamical stability.*

- *The new mathematical asymptotic model* for analysis of selected dynamic and dynamical stability problems for periodic shells under consideration has been formulated by applying *the new consistent asymptotic modeling procedure* given in [26]. This approach is based on the notion of *highly–oscillating function.* The fundamental assumption imposed on the lagrangian under consideration in the framework of this approach is called *the consistent asymptotic decomposition.* It states that the displacement fields occurring in the lagrangian have to be replaced by families of fields depending on small parameter $\varepsilon = 1/m,\ m = 1, 2, ...$ and defined in an arbitrary cell. These families of displacements are decomposed into averaged part described by unknown functions being continuously bounded in periodicity direction and highly–oscillating part depending on ε. This highly–oscillating part is represented by known fluctuation shape functions and by unknown functions being continuously bounded in direction of periodicity. Asymptotic modeling procedure for Euler–Lagrange equations (3) is realized in two steps. The first step is *the consistent asymptotic averaging of lagrangian* (2) under *consistent asymptotic decomposition* (18). The resulting averaged form of lagrangian (2) is given by (21). In the second step, applying the principle of stationary action to *the consistent asymptotic action functional* defined by means of averaged lagrangian (21) we arrive at Euler–Lagrange equations (22). After combining (22) with (21) we obtain finally the explicit form of *the consistent asymptotic model equations* given either by (23) or (26). The resulting equations have to be considered together with decomposition (27). *Coefficients in these equations are constant.* Contrary to the tolerance model, the presented consistent asymptotic model is not able to describe the length–scale effect on the overall shell dynamics and stability being independent of the microstructure cell size.

- Taking into account the effect of the microstructure length on a dynamic stability of the shells under consideration we arrive at *the system of two the second–order ordinary differential frequency equations* (33) *for the unknown functions of time coordinate, which can be treated as a certain generalization of the known Mathieu equation,* cf. [10]. This system reduces to the Mathieu equation provided that the period length λ is neglected. On the contrary, within the consistent asymptotic model the known Mathieu equation (35) is obtained.

- In the framework of the tolerance model, proposed here, the fundamental lower and *new additional higher free vibration frequencies* as well as the fundamental lower and *new additional higher critical forces can be derived and analyzed. The higher free vibration frequencies and the higher critical forces depend on a cell size and hence cannot be determined applying asymptotic models commonly used for investigations of the shell stability.*

The application of the obtained frequency equations (33) and (35) to evaluation of the effect of microstructure length parameter λ on parametric vibrations and on boundaries of dynamical instability regions for periodically stiffened shells under consideration is reserved for a separate paper.

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