Mechanics and Mechanical Engineering
Vol. 15, No. 4 (2011) 11–20
© Technical University of Lodz

Modeling of Estimation Tasks of the Year Long Noise Indicators

Wojciech Batko
AGH - University of Science and Technology
Department of Mechanics and Vibroacoustics
Al. Mickiewicza 30, 30–059 Krakow, Poland
phone +12 617 36 24

Renata Bal
State Higher Vocational School in Krosno
Politechnical Institute
Rynek 1, 38–400 Krosno
phone +13 437 55 00

Received (16 December 2011)
Revised (17 January 2012)
Accepted (23 February 2012)

Mathematical modeling of the estimation of the long–term noise indicators on the basis of random "momentary" environment control investigations, is presented in the paper. An attention was directed towards deficiencies of the obligatory estimation procedures of their representation, which is given by assessments of the average of the random test and the standard deviation value of the environment test results. Possible ways of looking for the problem solution were discussed. The proposed method of modeling the results of a random, "momentary", test of control investigations, leading to the determination of the expected values of the long–term noise indicators and to the assessment of their uncertainty, was described. The selection and realization of the proposed solution was related to the theory and methods of analysis of time series. Investigations, analyses and verifying procedures accompanying the proposed mathematical formalization, were given. Exemplifying contents of the presented paper were related to the assessment of the traffic noise at one of the main arteries of Krakow. They are considered in a context of the practice of the "momentary" measurements and the estimation of the long-term noise indicators performed on their basis.

Keywords: Autonomous mobile robots, rapid prototyping, robot control

1. Introduction

The grounds of environment monitoring in the scope of acoustic protection constitute the rules contained in the methodology of acoustic maps preparation. They are mentioned in the legal acts based on the Directive 2002/49/WE [10], imposing on the member countries the unified method of the acoustic climate assessment and the activity scenario based on them: preventing, limiting and decreasing an acoustic strenuousness in the environment.

Assessments of acoustic hazards in requiring protection zones and related to them selection of acoustic protection solutions, is conditioned by the knowledge of the long–term year–average sound levels $L_{LT}^{(j)}$ at times: day – $j = 1$, evening – $j = 2$ and night – $j = 3$, during the whole calendar year.

Their values $L_{LT}^{(j)}$ for individual times of the day: $j = 1, 2, 3$ are determined as logarithmic mean:

$$L_{LT}^{(j)} = 10 \log \left[\frac{1}{365} \sum_{k=1}^{365} L_{AeqLTk}^{(j)} \right] \tag{1}$$

from the time-equivalent ($L_{Aeq,T}$) sound A level [dB] in the kth day of the calendar year, in the considered reference time $T^{(j)}$, proper for the day–time: $j = 1,2,3$.

The long–term day–evening–night L_{DEN} level is calculated on the basis of the values{ $L_{AeqLTk}^{(1)} = L_D$, $L_{AeqLTk}^{(2)} = L_E$, $L_{AeqLTk}^{(3)} = L_N$}:

$$L_{DWN} = 10 \log \left[\frac{1}{24} (12 \cdot 10^{0.1 L_D} + 4 \cdot 10^{0.1 L_W + 5} + 8 \cdot 10^{0.1 L_N + 10}) \right] \tag{2}$$

This L_{DWN} value is the base indicator for activities related to the environment acoustic protection.

The year–long equivalent sound levels are determined by computational methods at taking into account:

- Numerous input parameters characterizing the noise emission into the analyzed zones by the considered sources - including their acoustic power levels as well as the equivalent sound levels for periods of their activity in the calendar year;
- Variables, describing conditions of the sound propagation into the analyzed zones;
- Measurement conditions in the analyzed locations.

Essential elements of the analysis of the correctness of the computational model of environment acoustic hazards are verifying investigations based on random control tests. The results of random control tests constitute the bases of statistic inferences, in which the assessment of the expected value of noise indicators and their uncertainty estimation is being done.

The standard way of drawing up the random results $x_i \, i = 1, 2, ..., n$ of the environment acoustic control consists of calculating the mean:

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

from the standard deviation of the control results: $s(x_i)$ during observation:

$$s(x_i) = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2} \tag{3}$$

and the mean standard deviation:

$$s(\overline{x}) = \frac{s(x_i)}{\sqrt{n}} \tag{4}$$

which – at the possibility of the systematic error neglecting – can be equated with the uncertainty of the control process. In relation to the mean value, calculated in such a way, the possible error interval is defined taking into account the required for it confidence level:

$1 - \alpha$, connected with the assigned distribution quantile $k(\alpha)$ of the estimation error:

$$U = \mp k(\alpha) u_c = \mp k(\alpha) s(\overline{x}) \tag{5}$$

of the expected value of the analyzed control variable.

This way of estimation of the long–term noise indicators is in conformity with the procedures contained in the international document [12], issued by seven most important metrological organizations. Its calculation rules are related to assumptions of: the equivalence of the random control test results and normality of their probability distribution, and also to a condition that subsequent observations x_i, $i = 1, 2, ..., n$ are statistically independent of each other, which means that they are uncorrelated.

In issues of the environment acoustic control the applicable references of such approach to the noise indicators estimation can be found. However, this estimation is done on the bases of random control tests without any broadened analysis and discussion of the possibility of their likelihood application. Especially they are deprived of the analysis of the reliable acceptation possibility of the assumptions related to them. This mainly concerns the correctness of the assumption of the probability distribution normality, the sound level measurements results as well as the condition of the lack of their mutual correlation.

The hereby paper is an attempt of a critical look at the correctness of such assessments. Especially the assumptions and resulting from them needs of looking for other model formalisms for the estimation of the expected values of the long–term noise indicators and assessments of their uncertainties – on the bases of random, 'momentary' environment control tests – will be critically assessed. The possible searching ways for solving this problem will be provided.

2. Probabilistic properties of the results of random acoustic tests of the environment

Statistics, being the decision–taking tool under the uncertainty condition related to the limited random test results, is utilized in all domains of a human activity. This concerns also assessments of noise indicators and their errors in the activities of environment protection authorities, which tasks are defined as statistic. Sources of the related to them errors constitute possible inaccuracies of the results, a lack of the technical possibility of obtaining full measurement data or environment disturbances occurring during testing. Those factors determine significantly the randomness of assessments and related to them need of the estimation of the environment acoustic hazards accuracy.

The available observation results sequence $\{x_1, x_2, ..., x_n\}$ – determined by the analyzed noise indicator $L_{LT}^{(j)}$ related to the considered time of the day: $j = 1, 2, 3$ – is usually recorded in the order of the performed control tests.

In case of the estimation of the long–term day–evening–night level L_{DEN} on the bases of random control tests, one is dealing with a random test x_i, $I = 1, 2, ..., n$ in which successive assessments are determined by adding a new control data to calculations $L_{LT}^{(j)}$ from the previous control. Thus, in the computational procedure the random test x_i, $i = 1, 2, ..., n$ of the long–term noise indicators is widened by the repeated calculation of $L_{LT}^{(j)}$ – necessary for the assessment of the long–term day–evening–night noise indicator L_{DEN} – together with the last control result:

$$\begin{aligned}
&x_1 = L_{LT1}^{(j)}\\
&x_2 = 10\log\frac{1}{2}\left(10^{0.1L_{LT_1}^{(j)}} + 10^{0.1L_{LT_2}^{(j)}}\right)\\
&x_3 = 10\log\frac{1}{3}\left(10^{0.1L_{LT_1}^{(j)}} + 10^{0.1L_{LT_2}^{(j)}} + 10^{0.1L_{LT_3}^{(j)}}\right)\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&x_n = 10\log\frac{1}{n}\left(10^{0.1L_{LT_1}^{(j)}} + 10^{0.1L_{LT_2}^{(j)}} + \ldots + 10^{0.1L_{LT_n}^{(j)}}\right)
\end{aligned} \tag{6}$$

This is the procedure of the determination of the new (last) random long–term year–average noise indicator based on the 24–hour control tests with taking into account the new observation result.

This means that the random test results $\{x_1, x_2, ..., x_n\}$, being the bases for the calculation of the long–term average day–evening–night noise indicator L_{DEN}, are correlated. Values of each element x_i of the random test input sequence are used for calculation of the consecutive value of the random test, on which bases the expected value of the long–term average day–evening–night noise indicator L_{DEN} is estimated. This fact contradicts the possibility of the likelihood usage of the standard estimation methods, which corresponds to the assumption of independence of the random test results used for its estimation and uncertainty assessments.

The second basic assumption, that control assessments are burdened with a random error of a normal distribution, is also without theoretical grounds and wider empirical reasons. This is confirmed by the results presented in several papers [5, 6, 11, 14]. Doubts related to two out of three basic assumptions of the methodology of the noise indicators estimation on the bases of random control tests – referred to classic methods [12] – generate the need of working out new model formalisms for the realization of such tasks.

3. New estimation idea of the long–term noise indicators on the basis of random environment control tests

As it turns out, from the analysis presented in the previous chapter, there is a need of departing from the current solutions with their limiting assumptions for formalisms with not full information on the probability distribution of the investigated indicators and their interconnections in the sequence of control observations.

Various directions of the methodology of statistic investigations of effects, of this specificity – colloquially called: "not classic statistic methods" – are currently being developed [15, 20].

These are methods of:

- Nuclear estimators [16],
- Bootstrap analysis [17, 18],
- Bayes' analysis [19, 20]
- Time series [21]

The analysis of the possible application and adaptation of these solutions for the estimation of the long-term noise indicators was the subject of the research in the Department of Mechanics and Vibroacoustics and the prepared there Ph.D. Thesis [1, 13]. Their results are presented in several papers [2–4, 7–9].

The contents of the hereby paper is limited to sketching the model formalism for the estimation of the expected value and variance of the long–term noise indicator, related to modeling the sequence of results of the 'momentary' control observations $\{x_1, x_2, ..., x_n\}$ by the time series, which means the sequence of random values of variable X.

It is assumed in the modeling process of control data that the probabilistic structure of the control results changes can be shaped by the mechanism:

$$X_t = \mu_t + \varphi_t + \xi_t, \quad t = 1, 2, ..., n$$

where: trend μ_t – describes a constant tendency shaping the level of the analysed noise indicators, cyclic component φ_t – representing periodical changes related to recurrent characteristic forcing, which influence changes of the controlled noise indicators, residue component ξ_t – representing random disturbances (inaccuracies) of a normal distribution $N(0, \sigma_\xi^2)$.

In contrast to the classic model of the random control test, in which it is assumed that consecutive observations are random variables of normal distribution, in this approach the presence of a certain mechanism forcing changes of control results is assumed. This mechanism is subjected to Gaussian disturbances of the expected value being zero and variance: σ_ξ^2. The estimation problem of the expected value and variance of the analyzed noise indicator is reduced – in this approach – to the identification of the time series structure. This requires the determination of the proper approximation $\hat{\mu}_t$, $\hat{\varphi}_t$ for components and μ_t, which should secure the right description of variability (with a random Gaussian error ε_t of the expected value being zero and variance σ_ε^2), of successively observed results of control tests.

The proper selection of the approximation for the changes of the controlled noise indicators values requires an identification of its properties. Especially important are statements concerning:

1. Stationary of the analyzed time series;
2. Cyclic component presence;
3. Homogeneity of the observation set, and properties (including variance) of the random component.

This knowledge is helpful in selecting the correct modeling of time series formed from the control tests results. The basis for the decisive tests constitutes the knowledge of their basic characteristics: process average, variance (standard deviation), autocorrelation function, periodic diagram and process spectrum. The autocorrelation function – giving information concerning relations between various observations and used also in the stationary analysis – as well as the spectral analysis – helpful in detecting the cyclic component presence – are especially important in the analysis of the variability of the time series formed from the control tests results. Assuming that in the variability of the day-evening-night indicator during the calendar year long–, middle– and short–term changes can occur, the spectral function knowledge allows assessing their participation in the total variability of the analyzed process. The solution, originated from the chaos theory, based on the Hurst exponent analysis [1] can be a helpful tool in the analysis of the randomness of the noise indicators changes. Filters of the variable mean are also useful in the assessment of the modeled process components.

Realization examples of such analyses, in relation to the time series formed from the results of multiyear continuous noise monitoring recorded at one of the main arteries in Krakow, are contained in the Ph.D. Thesis of R. Bal [1]. They supplied recommendations for the selection of the proper model for the estimation of the long–term noise indicators.

In relation to the selection of the model formalism intended for calculations of the expected value and variance of the time series, formed from the results of the environment acoustic control tests, several postulates – resulting from application conditions – can be formulated.

The following requirements are significant:

1. Acceptability of model assumptions, supplemented with the condition of its easy verification;

2. Quantitative proper functioning of the accepted model already at the relatively not numerous time series;

3. Adaptability of the model (ability to improve the accuracy of the model process when new data are available);

4. Simplicity of the computational algorithm (recurrent calculation forms) .

The performed analysis of the basic time series models, related to the described above time series of environment acoustic control assessments, indicated that the adaptation exponential smoothing model of Brown R.G. and Mayer F.F. [22] can constitute the proper tool [1–4] of modeling random mechanisms of changes of the long-term noise indicators in successive controls. It belongs to the autoregression AR (1) class of processes.

Its realization assumes, that the calculated - in successive stages of investigations - value of the estimated noise indicator $x_t, t = 1, 2, ..., n$, can be approximated {with accuracy ε_t of the Gaussian residue process $N(0, \sigma_\varepsilon^2)$ }; expansion of the order p, of the selected function by function $\hat{\mu}_t$ into the Taylor series, on the time interval in between successive controls:

$$\mu_{t+\tau} = \hat{\mu}_t^{(0)} + \left(\frac{d\hat{\mu}}{dt} \mid \bar{t}\right)\tau + ... + \frac{1}{p!}\left(\frac{d^{(p)}\hat{\mu}}{dt^p} \mid \bar{t}\right)\tau^p + \varepsilon_t \tag{7}$$

Coefficients $a_t^{(p)}$ present in this expansion

$$\mu_{t+\tau} = a_t^{(0)} + a_t^{(1)}\tau + ... + \frac{1}{p!}a_t^{(p)}\tau^p + \varepsilon_t \tag{8}$$

are linear functions:

$$\begin{aligned} a_t^{(0)} &= L_0(S_t^{(1)}),\ S_t^{(2)},\ ...,\ S_t^{(p+1)} \\ a_t^{(1)} &= L_1(S_t^{(1)}),\ S_t^{(2)},\ ...,\ S_t^{(p+1)} \\ &.................................. \\ a_t^{(p)} &= L_p(S_t^{(1)}),\ S_t^{(2)},\ ...,\ S_t^{(p+1)} \end{aligned} \tag{9}$$

of operators of the exponential smoothing $\mathbf{S} = [S_t^{(1)},\ ...,\ S_t^{(p+1)}]$ of successive orders $k = 1,\ 2,\ ...,\ p+1$

$$S_t^{(k)}(x) = \sum_{j=0}^{\infty} \alpha_k (1-\alpha)^j \binom{k+j-1}{k-1} x_{t-j} \tag{10}$$

calculated recurrently :

$$S_t^{(1)}(x) = \alpha x_t + (1-\alpha)S_{t-1}^{(1)}(x) \quad \text{dla} \quad k = 1 \tag{11}$$

$$S_t^{(k)}(x) = \alpha S_{t-1}^{(k)}(x) + (1-\alpha)S_t^{(k)}(x) \quad \text{dla} \quad k = 2,\ 3, ...,\ p+1, \quad t \geq 2 \tag{12}$$

Occurring in the above expression parameter , called the smoothing constant, assumes values from the interval (0, 1) and in a similar fashion as the degree p of the assumed approximation multinomial is rated as the so–called "*difficult parameter*" of the method. Their selection is often realized by experimental methods. Functions L_i, $i = 0,\ 1,\ ...,\ p$ connecting coefficients of the Taylor series $a_t^{(p)}$ with the exponential smoothing operators $S_t^{(k)}$ of the appropriate order present in the relation (12) result from the Brown–Mayer rule and are presented in paper [22].

At the preliminary analysis stage of the time series, formed from the estimation results of the controlled noise indicators, it was found that there is a possibility of limiting the approximation to the zero degree multinomial. Under these conditions the approximated model of the description of changes of the estimated noise indicators values $\{\ \boldsymbol{X}_i\} = \{x_1,\ x_2,\ ...,\ x_n\ \}$; $i = 1,\ 2,\ ...,\ n$ is given by equations:

$$\mu_t = \overline{\mu}_t^{(0)} + \varepsilon_t \tag{13}$$

where:

$$E[x_t] = \alpha x_t + 91 - \alpha)S_t^{(1)}; \quad S_1^{(0)} = x_1 \tag{14}$$

$$E[\text{var}(x_t)] = \sigma_x^2 = \frac{\alpha}{2-\alpha}\sigma_\varepsilon^2 \tag{15}$$

Such model is stipulated by the assumption that the error process $\varepsilon{+}x_t{-}E[x_t]$: is the Gaussian process $\varepsilon_t \sim N(0,\ \sigma_\varepsilon^2)$. Thus, at each estimation stage of the analyzed noise indicators, it requires the verification of its likelihood. This process can be realized on the bases of numerous statistic tests. The authors of paper [3] proposed the recurrent control procedure convenient in practice. Their description together with examples from the acoustic monitoring are illustrating their applications.

By means of the proposed estimation method of the year–averaged long–term noise indicators and – based on utilizing the approximation of the time series of the control results by the R.G. Brown's adaptation model of exponential smoothing – its predictive properties for the correct assessments of their expected values and variances were tested. The verification reference base constituted the results of, the mentioned above, continuous noise monitoring recorded for several years at one of the main arteries in Krakow. Values of the long–term average $\{L_{DEN}, L_N\}$ noise indicators were calculated on the bases of the randomly selected days {8, 12, 18, 27} of the chosen calendar year and then compared with the real values calculated on the bases of all days of the same calendar year.

Examples of the obtained results are presented in Tables below

Test	Number of days	The long–term average indicator L$_{DEN}$ [dB] from the selected days	The long–term average indicator L$_{DEN}$ [dB] from estimations of the assumed model	$\sigma_\varepsilon\,(\alpha)$	$\sigma\,(L_{DWN})$
1	8	76.6	76.9	0.12	0.109
2	12	76.7	77.2	0.11	0.095
3	18	76.9	77.2	0.10	0.090
4	27	77.0	77.3	0.10	0.090

Test	Number of days	The long-term average noise indicator L$_{night}$ [dB] from the selected days	The long-term average noise indicator L$_{night}$ [dB] from estimations of the assumed model	$\sigma_\varepsilon\,(\alpha)$	$\sigma\,(L_{noc})$
1	8	68.8	69.1	0.027	0.024
2	12	68.9	69.5	0.024	0.021
3	18	69.1	69.5	0.017	0.015
4	27	69.3	69.6	0.017	0.015

The data given in Tables indicate that in all tests the estimated results of the long–term (average) noise indicators calculated according to the proposed methodology provided satisfactory results.

4. Final remarks

The problem of dilemma related to the currently widely used methods of the uncertainty estimation of the long-term (year-average) noise indicators – on the bases of random tests of 'momentary' environment controls – was discussed in the presented paper.

Especially the estimation conditioning – being the bases for questioning the reliability of realization assumptions of the current calculation procedures – was shown. The possible directions of searching for the problem solution were formulated by indicating useful model formalisms. The useful solution originated from the group of adaptation modeling methods of time series, including the exponential smoothing R.G. Brown's method, was sketched.

The empirical illustration of its application was shown. It allowed to state that the application of the model formalism of the time series analysis for the estimation of the expected value and variance of the long–term noise indicators, on the bases of the random test of environment controls, constitute the perspective tool for such tasks. This was confirmed by the verifying experimental tests, which provided fully satisfying results. Using them allows giving up rather unreliable (often untrue) model assumptions of the current estimation procedures.

It is worth noticing that the presented problem can have essential consequences in the administrative decision process corresponding to the management of the environment acoustic protection.

This study is dedicated to my Teacher, Professor Józef Giergiel, PhD, Eng. Honorary Doctor - on the occasion of the Jubilee of his 80–th birthday.

References

[1] **Bal, R.**: Analysis of estimation conditions of the long–term noise indicators, Ph.D. Thesis, AGH, (in Polish), **2010**.

[2] **Batko, W. and Bal, R.**: Analysis of usefulness of R.G. Brown model for estimation process of long term noise ratio, *Archives of Acoustics*, Vol. 33, No. 3, **2008**.

[3] **Batko, W. and Bal, R.**: Controlling of Violations of Basic Assumptions of the Recurrent model of the long–term noise indicators assessments, *Archives of Acoustics*, **2009**.

[4] **Batko, W. and Bal, R.**: Selection of Model Approximation for the Uncertainty Estimation of the Long–term Noise Indicators LDEN and LN, *Archives of Acoustics*, **2009**.

[5] **Batko, W. and Przysucha, B.**: Determination of the Probability Distribution of the Mean Sound Level, *Archives of Acoustics*, Vol. 35, No 4, p. 543–550, **2010**.

[6] **Batko, W., Przysucha, B.**: Determination of the distribution form for sound level measurements results, (sent for publication in OSA 11 Proceedings), (in Polish).

[7] **Batko, W, Stępień, B.**: Application of the Bayesian inference for estimation of the long-term noise indicators and their uncertainty, *Acta Physica Polonica A*, Vol. 119, p. 916–920, **2011**.

[8] **Batko, W, Stępień, B.**: Application of the bootstrap estimator for uncertainty analysis of the long–term noise indicators and their uncertainty, *Acta Physica Polonica A*, Vol.118, p.11–16, **2010**.

[9] **Batko, W. and Stępień, B.**: Non–parametric methods of estimation of type A uncertainty of the environmental noise hazard indices, *Archives of Acoustics*, Vol. 34, No 3, p. 295–303, **2009**.

[10] Directive 2002/49/WE of the European Parliament and of the Council of 25 June 2002, relating to the assessment and management of environmental noise, *Official Journal of the European Communities*, **2002**.

[11] **Gałuszka, M.**: Statistic distributions of levels and energy in a road traffic noise, *Environment Monitoring Conference*, 24–25.05, Kraków, (in Polish), **2010**.

[12] Guide to the Expression of Uncertainty Measurement. International Organization for Standardization, ISBN 92–67–10188–9, 1995, Polish Edition: Wyrażanie niepewności pomiaru. Przewodnik. *The Central Office of Measurements*, **1999**.

[13] **Stępień, B.**: Estimation of uncertainties of the environment noise hazard indicators, Ph. D. Thesis, *AGH*, (in Polish), **2011**.

[14] **Wszołek, T.**: Uncertainty analysis in environment noise investigations, *Proceedings of the XXXIV-th Winter School of Vibroacoustic Hazards Control*, p. 205–216, **2006**.

[15] **Robert, L., Launer, R.L. and Wilkinson, G.N.**: Robustness in Statistics, *Academic Pres*, New York, **1979**.

[16] **Kulczycki, P.**: Nuclear estimators in the system analysis, *WNT*, Warszawa, (in Polish),**2005**.

[17] **Efron, B.**: Bootstrap Methods: Another Look at Jacknife, *The Annals of Statistics*, No 7, **1979**.

[18] **Efron, B. and Tibshirani**: An Introduction to the Bootstrap, *Chapmann& Hall*, New York, **1993**.

[19] **Krzysko, M.**: Mathematical Statistics, Edited by *UAM*, Pozna, 2–nd edition, (in Polish), **2004**.

[20] **Domański, C. and Pruska, K.**: Not classic statistic methods, *PWE*, Warszawa, (in Polish), **2000**.

[21] **Box G.E.P,Jenkins G.M.**: Analysis of time series : forecasting and control, *PWN*, (in Polish), **1983**.

[22] **Brown, R.G. and Mayer, R.F.**: The Fundamental Theorem of Exponential Smoothing, *Operations Research*, Vol.9, **1961**.