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Practical Possibilities of Utilization of Tangential Longitudinal Vibrations for Controlling the Friction Force and Reduction of Drive Force in Sliding Motion

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This work presents the results of simulation analyses and experimental investigation of the influence of tangential-longitudinal vibrations on the drive force and friction force in sliding motion. The numerical analyses were carried out with the use of classical Coulomb model of friction and with the use of Dahl and Dupont dynamic models. It has been demonstrated that the use in numerical analyses of Dahl friction model, facilitates precise projection of changes in these forces taking place in real systems under the influence of imposed tangential-longitudinal vibrations.

Keywords: Friction force, friction models, tangential vibrations, reduction of drive force

1. Introduction

The influence of vibrations on the static friction force and on friction force in sliding motion is known and practically utilised by humans for years. The effect of a change in this force taking place under the influence of vibrations in the area of contact of two bodies is the reduction of drive force essential for initiating the sliding motion of such body and sustaining it. In literature this effect is frequently denoted as a reduction of the average friction force.

The level of reduction of drive force (average friction force) in sliding motion can vary. There may be no reduction at all, but it may also be so large that the magnitude of the drive force in sliding motion in the presence of forced vibrations would be many–fold lesser than the magnitude of such force in sliding motion over a motionless support. It follows from the results of investigations published by Littmann et al. [1, 2] and from experimental investigations and simulation analyses conducted by the authors of this work [3–5] that the mandatory condition for the reduction of drive force in the presence of tangential longitudinal contact vibrations is the fulfilment of a requirement that the amplitude v_a of the velocity of these vibrations is greater than the drive velocity v_d ($v_a > v_d$).

The amplitude of vibrations velocity in harmonic motion is a product of amplitude of vibrations and their frequency. Hence, attaining reduction of friction resistance in sliding motion at a given nominal sliding velocity requires an appropriate selection of those two parameters. However, the magnitude of changes of friction forces in sliding motion, and hence the magnitude of changes in drive force under influence of vibrations depends not only on the above mentioned vibrations parameters but also on the contact stiffness which, in turn, is dependent on a variety of factors including the roughness of surfaces in contact and normal stresses at the contact surface.

Such multitude of factors influencing the ultimate effect of forced vibrations on the friction force and hence, on the drive force in sliding motion resulted in the fact that regardless of massive interest of many research centres around the world interested in this topic, and regardless of many works dedicated to theoretical analyses and experimental investigation of this subject, until now the mechanism of reduction of the drive force (average friction force) under influence of vibrations is not yet fully investigated nor described.

This work presents the results of simulation analyses and experimental investigations on the reduction of drive force in sliding motion under the influence of tangential longitudinal vibrations. The investigations were conducted using a specifically designated and constructed experimental stand whose details are provided in work [5]. The simulation analyses were carried out in the Matlab/Simulink environment utilising an earlier developed computational model. Comparing the results of simulation and experimental investigations it has been demonstrated that the use in numerical analyses of Dahl dynamic friction model facilitates precise presentation of changes in friction force and drive force in sliding motion caused by the imposed vibrations.

2. Experimental investigations

The scheme of the test stand used in experimental investigations is given in Fig. 1. Its main component is a sliding pair whose upper sample moves over the ball–bearing supported slide constituting a lower sample which, in turn, can be at any time excited into vibrations aligned with the motion direction of the upper part. Both samples are contacting on two flat rectangular surfaces of dimensions 12x50 [mm] each, and spaced symmetrically in relation the movement axis.

Movement of the upper sample is realised through a system comprising a linear guide *EPX40*, a step-motor with a gear, a driver and the unit powering and controlling the work of the motor. The lower sample is excited into vibrations by a piezoelectric element. The signal controlling it is generated by the *DS1104* control board. A ring dynamometer with glued resistance tensometers is used for the measurement of drive force. During investigations, apart from the drive force, the accelerations of both samples are measured due to which it is possible to control the transfer of vibrations from the lower sample to the upper one and to determine the inertia force of the upper sample at any moment of its motion. Accelerations are measured by accelerometers *PCB 352B10*. Experiments can be conducted at a variable normal load of the contact, which can be controlled by a designated pre–tensioning system.

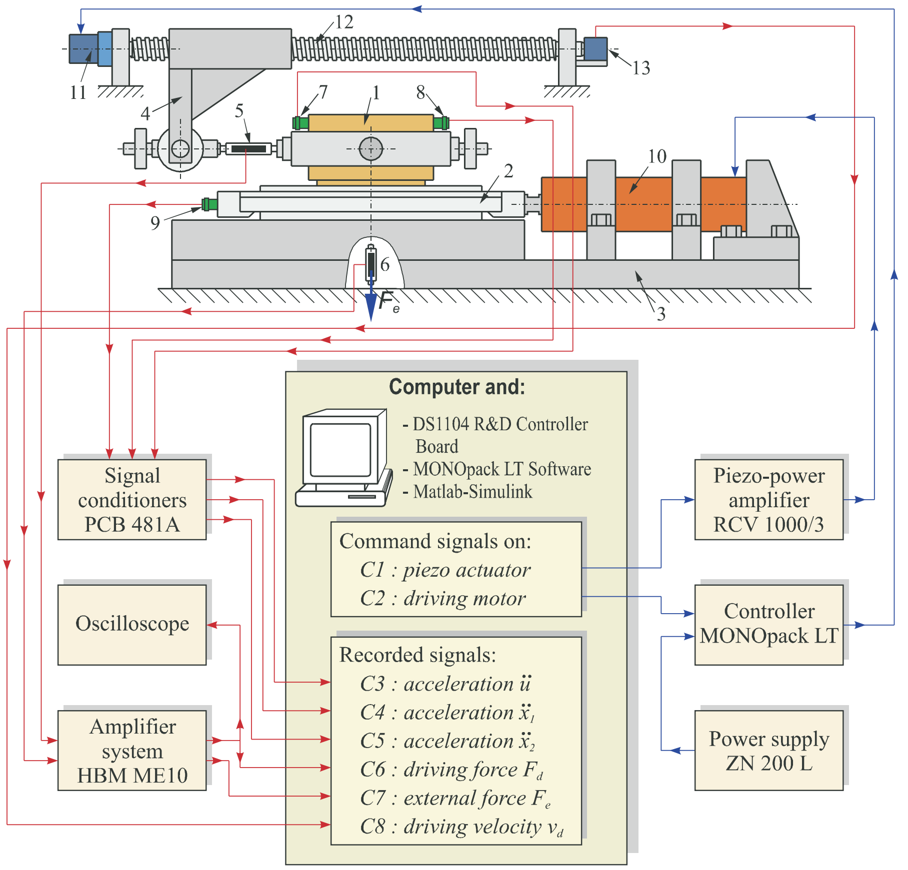

Figure 1 Scheme of the mechanical part of the test stand: 1 – upper sample, 2 – bottom sample, 3 – fixed base, 4 – driver, 5, 6 – ring dynamometers, 7,8,9 – accelerometers, 10 – vibration exciter, 11– step motor with a gear, 12 – linear guide, 13 – encoder

Surface roughness was measured for each pair used in experiments, whilst the coefficient of tangential stiffness was determined at each change of the normal load of the contact.

Fig. 2 presents the experimentally determined drive force characteristic during sliding of an upper sample at the nominal drive force speed $v_d = 0.62$ [mm/s]. At an initial and final stage of experiments, the sliding was conducted over a fixed lower sample, whilst at intermediate stages, on the sample excited into vibration motion at the frequency $f = 3900$ [Hz] and variable amplitude v_a of the vibrations velocity. At the first stage of those intermediate stages the amplitude was less than the drive velocity, and was set at $v_a = 0.20$ [mm/s]. At the successive stages the vibrations velocity amplitude of the lower sample was increased. During each of these stages it was greater than the drive speed and was pre–set as follows: 0.84, 1.06, 1.94, 3.20 i 6.46 [mm/s]. Investigations were carried out at dry contact, for a steel–steel friction pair, at normal stresses $p = 0.031$ [N/mm^2]. Contact surfaces of the samples have been polished.

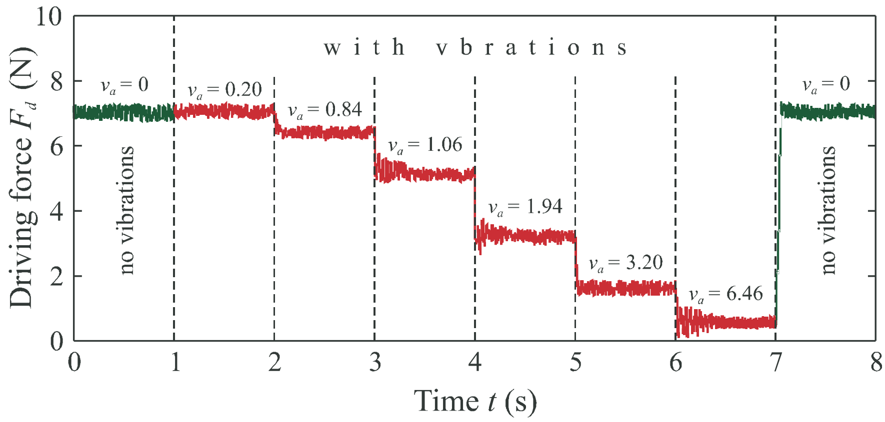

Figure 2 The driving force characteristic at sliding motion realised on a fixed and on a vibrating support; $v_d = 0.62$ [mm/s], $f = 3900$ [Hz], $p = 0.031$ [N/mm^2]

It is apparent from the characteristics presented that at each stage of the motion when the condition $v_a > v_d$ is satisfied, the imposed tangential vibrations resulted in the reduction of the force required for sustaining the sliding motion of the moving sample. The greater was the difference between the amplitude of vibration velocity and the nominal drive velocity, the greater was the reduction of this force.

Having the experimentally determined characteristics of acceleration of the moving and simultaneously vibrating upper sample it was possible to determine the profiles of its force of inertia. Knowing then the characteristic of the force of inertia and that of the drive force it is possible, utilising Newton's second Law of Dynamics, to determine the characteristic of friction force at the contact surface of the moving sample and vibrating support.

The gathered experimental data have been used for verification of computational procedures developed in the Matlab/Simulink environment which facilitate the conduct of simulation analyses of changes in the drive force and friction force taking place in sliding motion under the influence of imposed tangential longitudinal vibrations.

3. Assumed models and developed computational procedures

In the analyses conducted, a model has been assumed in which a body of mass m is moved over the vibrating support (Fig. 3a) utilizing a drive system with a known stiffness. It has been then assumed that the point of attachment of drive force moves at a known velocity v_d (Fig. 3b). It has been also assumed that known are the elastic-plastic properties of the contact zone. The distribution of forces acting on the moved body is shown in Fig. 3c.

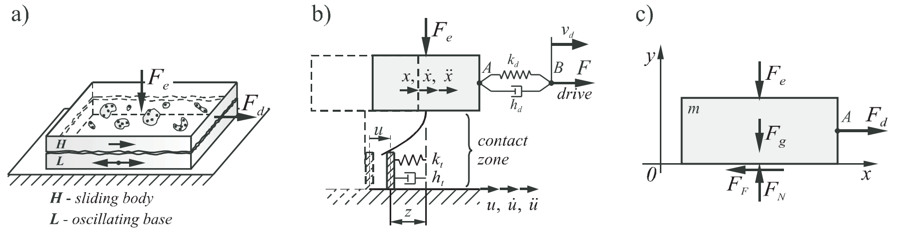

Figure 3 A model assumed in conducted analyses: a) a true contact, b) modelling of the contact zone and the drive system, c) distribution of forces acting on the moved body

A vector equation of sliding motion of this body in a stationary reference system $0xy$ (Fig. 3c) has the following form:

$$m \cdot \vec{a} = \vec{F}_d + \vec{F}_e + \vec{F}_g + \vec{F}_F + \vec{F}_N \tag{1}$$

where: m – mass, $\vec{a}$ – acceleration, $\vec{F}_d$ – drive force, $\vec{F}_e$ – external load perpendicular to the surface of sliding, $\vec{F}_g$ – force of gravity, $\vec{F}_F$ – friction force, $\vec{F}_N$ – normal reaction.

Hence, for motion along the $0x$ axis we have:

$$m\ddot{x} = F_d - F_F \tag{2}$$

Assuming a linear characteristic of the driving system elasticity, the magnitude of the drive force is proportional to the magnitude of elastic deformation of its components. In the computational model, the magnitude of this deformation relates to the relative displacement of A and B–ends of an elastic component modelling this drive (Fig. 3b). The magnitude of the drive force can hence be estimated from the following relationship:

$$F_d = k_d \left(v_d t - x \right) \tag{3}$$

where: k_d – the coefficient of stiffness of drive system elements.

The value of friction force in numerical analyses depends on the friction model assumed. In the procedures developed, three friction models have been utilised. As a reference, the Coulomb model has been adopted. It is assumed in this model, that friction force is described by the following relationship:

$$F_F = \mu F_N sgn \left(v_r \right) \tag{4}$$

where: μ – coefficient of static friction, v_r – relative velocity of sliding body in relation to the vibrating support.

In dynamic analyses carried out, this model was shown to be relatively inaccurate. Hence, for further investigations two dynamic models have been adopted, i.e.: Dahl's [6, 7] and Dupont's [8, 9], which in contrast with the static models consider the true elasto-plastic properties of the contact zone, and in particular its contact compliance and a phenomenon of so called pre–sliding effect. In both models it is assumed that the magnitude of friction force is associated, in a real manner, with the elastic strain z of the contact measured in the direction of sliding (Fig. 3b) and is described through the following relationship:

$$F_F = k_t z \tag{5}$$

where: k_t – coefficient of stiffness in the tangential direction, and z – elastic strain of the contact in the tangential direction.

In Dahl's and Dupont's models it is assumed then that a relationship exists between the velocity of elastic deformation of the contact, $\dot{z} = dz/dt$, and the relative velocity v_r of sliding body in relation to the support. This relationship, in the case of Dahl's model, is expressed by the following relationship [6, 7]:

$$\dot{z} = \frac{dz}{dt} = v_r \left[1 - \frac{k_t}{\mu F_N} sgn\left(v_r\right) z\right]^{\alpha} \tag{6}$$

whilst in the case of Dupont's model, by the relationship [8, 9]:

$$\dot{z} = \frac{dz}{dt} = v_r \left[1 - \beta\left(z, v_r\right) \frac{k_t}{\mu F_N} sgn\left(v_r\right) z\right]^{\alpha} \tag{7}$$

where: α - parameter determining the shape of relationship between tangential displacements and tangential force [10].

The method for determining the $\beta\left(z, v_r\right)$ function in consecutive zones of tangential displacements is described in work [8]. In simulation computations it has been assumed that vibrating motion is a harmonic one in the following form:

$$u = u_0 \sin\left(\omega t\right) \tag{8}$$

where: u_0 – amplitude of forced vibrations, whilst $\omega = 2\pi f$, where f – is the vibrations frequency.

Hence, the amplitude of vibrations velocity is:

$$v_a = \omega u_0 = 2\pi f u_0 \tag{9}$$

Based on equations (1–9), a computational program in the Matlab/Simulink environment has been developed which facilitates simulation analyses of changes in friction force and drive force taking place during sliding motion under the influence of imposed tangential longitudinal vibrations. Fig. 4 presents a computational module for the case when friction force is determined in accordance with the Dahl model.

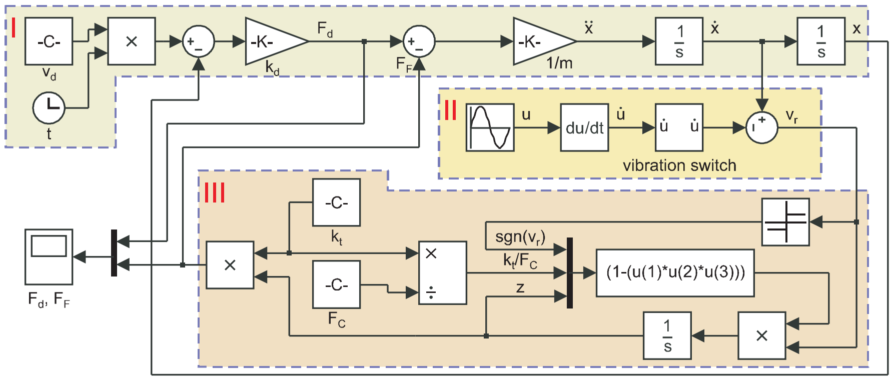

Figure 4 Matlab/Simulink program for analysis of changes in friction and drive forces under the influence of tangential longitudinal vibrations; I – differential equation of motion solving module, II – relative velocity determination module, III – friction force calculation module

4. Simulation analyses and their experimental verification

The models and computational procedures developed were designated to facilitate the conduct of effective simulation analyses of the influence of tangential longitudinal vibrations on the friction force and drive force in sliding motion. Hence, they required experimental verification prior to ensuing practical use.

Simulation analyses were conducted based on the three models of friction, as described above. In these analyses, using computational paths developed in the Matlab/Simulink program for each of these models, time characteristics were determined with respect to the variability of friction force F_F, and drive force F_d caused in sliding motion through imposing the tangential longitudinal vibrations of a variable amplitude of the vibrations velocity v_a. Investigations, for which the results are presented in this work were conducted at the frequency $f = 3900$ [Hz], drive velocity $v_d = 0.62$ [mm/s] and normal stresses $p = 0.056$ [N/mm^2]. A value of the coefficient of friction was assumed as $\mu = 0.22$, and that of the coefficient of contact's tangent stiffness, $k_t = 0.78$ [N/μm]. Both of these parameters were experimentally determined for the contact investigated. For the Dahl and Dupont models of friction, the value of coefficient α was assumed as $\alpha = 1$.

The results of simulation analyses were verified by comparing them with the results of earlier accomplished experimental investigations. Fig. 5 presents an example of friction force characteristics F_F determined from numerical analyses for all three friction models analysed, compiled together with those determined experimentally.

It is clear that a noticeable divergence between compared characteristics is observed in the case of Coulomb and Dupont models, whilst for the Dahl model the profile of the characteristic generated through numerical analyses constitutes an almost ideal reflection of that determined experimentally.

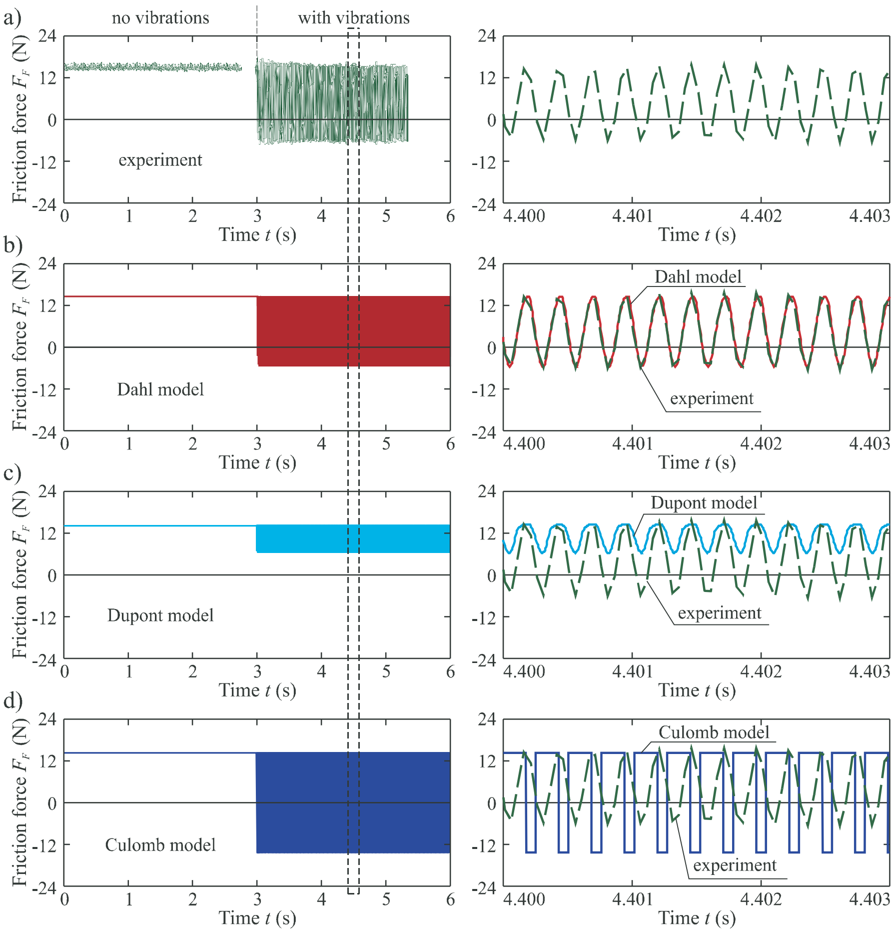

Figure 5 Variability of friction force under the influence of tangential longitudinal vibrations: a) experiment, b) Dahl model, c) Dupont model, d) Coulomb model; $f = 3900$ [Hz], $v_d = 0.62$ [mm/s], $p = 0.056$ [N/mm^2], $k_t = 78$ [N/μm]

In Fig. 6, in turn, the sets of drive force F_d characteristics are compared. Also in this case an excellent consistence between the experimental and analytical results is observed when Dahl model has been used, and the lack of this consistence for the two remaining models.

It needs to be pointed out that in investigations carried out with the use of the same samples but under different parameters of vibrations whilst using different sliding velocities, and at the normal stresses within the range of $p = 0.031 - 0.081$ [N/mm^2] the extent of consistence between simulation analyses and experimental results for analysed friction models was similar to that illustrated in Fig. 5 and 6.

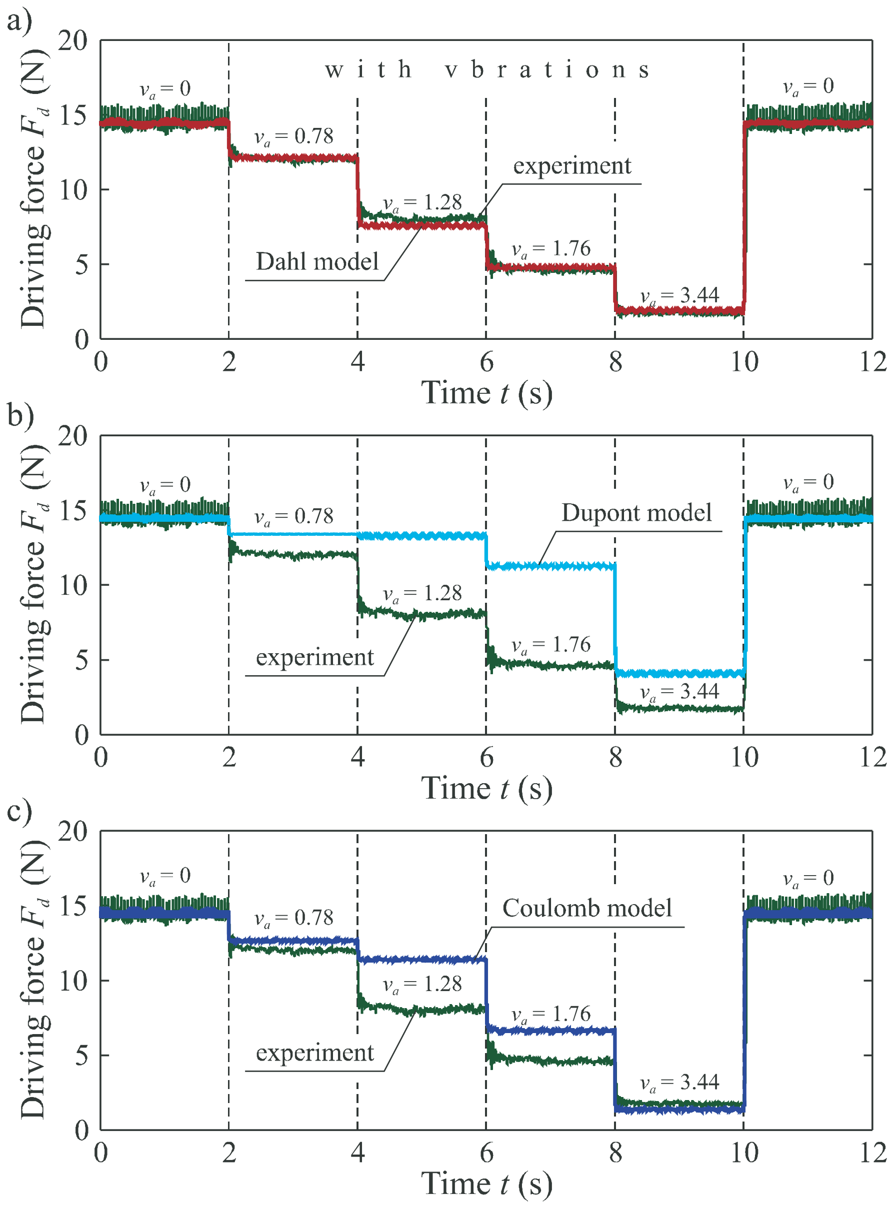

Figure 6 Comparison of results of simulation analyses and experimental tests concerning variability of drive force under the influence of tangential longitudinal vibrations: a) Dahl model, b) Dupont model, c) Coulomb model; f = 3900 [Hz], v_d = 0.62 [mm/s], p = 0.056 [N/mm^2], k_t = 78 [N/μm]

5. Summary

The results of simulation analyses as well as those achieved from experimental investigations on the influence of tangential longitudinal vibrations on the drive force clearly demonstrate that the effect of the reduction of this force in sliding motion is observed upon fulfilling the condition that the amplitude of the velocity of vibrations is greater than the drive velocity ($v_a > v_d$).

It is seen from the comparison of numerical analyses results with those achieved experimentally that the accuracy of simulation analyses depends strongly on the friction model adopted. In the case of analyses carried out at low normal stresses ($p = 0.031 - 0.081$ [N/mm^2]) the best consistence is achieved when the Dahl model is used. It may be a consequence of the fact that at such low stresses a non-linear dependence of contact deformation upon tangential loads may be already occurring in the initial phase of the contact loading in the tangential direction, as it is assumed in the Dahl model.

It is necessary, however, to emphasise that at large normal stresses the tangential deformation of the contact may, in the initial region of loading, be linearly dependent on those loads, as the Dupont model assumes, and in such case the improvement in the consistence of results of numerical estimations carried out with the use of Dupont model with experimental results, whilst the decrease of such consistence in the case of the Dahl model needs to be anticipated.

The feasibility of precise modelling of the friction and drive force variability caused in sliding motion by the imposed tangential longitudinal vibrations indicates, that these vibrations can be effectively used for controlling those forces in the real systems.

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