Mechanics and Mechanical Engineering
Vol. 15, No. 4 (2011) 31–41

Formulating of Diverse Task of Chosen Class of Vibrating Mechatronic Systems

Andrzej Buchacz
Silesia University of Technology
Institute of Engineering Processes Automation
and Integrated Manufacturing Systems
Konarskiego 18A, 44-100 Gliwice, Poland
andrzej.buchacz@polsl.pl

Received (16 December 2011)
Revised (17 January 2012)
Accepted (23 February 2012)

In this paper the modeling by different category graphs and analysis of vibrating clamped – free mechatronic system by the approximate method called Galerkin's method has been presented. The frequency – modal analysis and assignment of amplitude - frequency characteristics of the mechatronic system is considered. The aim was to nominate the relevance or irrelevance between the characteristics obtained by exact – only for shaft – and approximate method. Such formulation especially concerns the relevance the relevance of the natural frequencies-poles of characteristics both of mechanical subsystem and the discrete – continuous clamped – free vibrating mechatronic system. This approach is a fact, that approximate solutions fulfill all conditions for vibrating mechanical and/or mechatronic systems and can be an introduction to synthesis of these systems modeled by different category graphs. Using of the hypergraph methods of modeling and synthesis methods of torsionally vibrating bars to the synthesis of discrete–continuous mechatronic systems is originality of such formulation problems.

Keywords: Discrete–continuous vibrating mechatronic system, approximate method, graphs and structural numbers, modeling, synthesis

1. Introduction

In the research Centre in Gliwice the problems of analysis of vibrating beam systems, discrete and discrete–continuous mechanical systems by means of the structural numbers methods modeled by the graphs, hypergraphs, have been investigated (e.g. [4, 5, 10, 21]). The synthesis[1] of a selected class of continuous, discrete - continuous discrete mechanical systems and active mechanical systems have been dealt in [3-9].

The approximate method of analysis, that means the orthogolization method [15] and Galerkin's method [16], has been used to obtain the frequency–modal

[1]The analysis and synthesis of electrical systems were presented in monograph [1].

characteristics. The continuous–discrete torsional and transverse vibrating mechatronic systems were considered in [10, 11]. Transformations of hypergraps of flexibly vibrating beams were presented in [14]. To compare the obtained dynamical characteristics – dynamical flexibilities only for mechanical torsional vibrating bar and transverse vibrating beam being a parts of complex mechatronic systems, an exact method and the Galerkin's method were used [12, 13, 16]. Such formulation can be an introduction to synthesis of vibrating mechatronic systems which will lead to generating the vibrations with require parameters.

2. Characteristics of torsional discrete–continuous vibrating mechatronic system

Therefore it becomes necessary to search the new solutions, having on aim the reduction of movable elements as well as compiled and long kinematic chains. From here in last years it is clear that there is a huge development on the market, especially in field of new technologies basing on phenomenon of piezoelectricity, electro - and the magnetostriction {e.g. [17, 19]}. The piezoelectric elements are used to eliminate the oscillation [18].

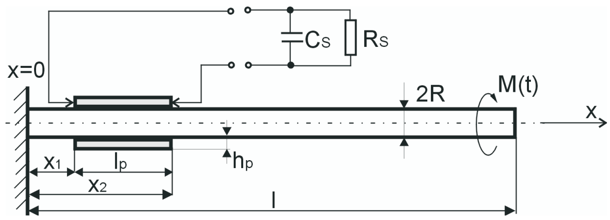

Figure 1 The torsional vibrating mechatronic systems with mechanical excitation

Considered vibrating systems has been shown in Fig. 1. The equation of torsional vibrating shaft with ideally attached piezotransducer showed in Fig. 1 is following:

$$\rho I_o \frac{\partial^2 \varphi}{\partial t^2} - G I_o \frac{\partial^2 \varphi}{\partial x^2} = \frac{-\lambda^*}{l} U \left[\delta\left(x - x_1\right) - \delta\left(x - x_2\right)\right] + \frac{M_o}{l} \sin \omega t \delta\left(x - l\right) \tag{1}$$

where:

$\delta(.)$ – Dirac's function,
G – the Kirchoff's modulus,
ρ – the mass density ρ,
I_0 – the polar moment of inertia for a shaft,
l – length of shaft,
$\lambda^* = \frac{2}{3}\pi G_p \left[\left(R + h_p\right)^3 - R^3\right] \frac{d_{15}}{l_p}$,
G_p – the Kirchoff's modulus of piezoelectric.

Sentence (1) is coupling with equation transducer, which can be written in form:

$$\frac{dU}{dt} + \frac{1}{R_s C_p} U + \frac{2\pi R^2 h_p d_{15} G_p}{l_p C_p} \dot{\varphi}(l_p, t) = 0 \tag{2}$$

where: $C_p = 2\pi R h_p \frac{e_l}{l_p}\left(1 - \frac{2d_{15}G_p}{e_1}\right) + C_x$
C_x – additional capacity in short circuit system.

The expressions (1) and (2) can write in form of:

$$\begin{cases} \ddot{\varphi} - a^2\varphi_{xx} - bU[\delta(x-x_1) - \delta(x-x_2)] = cM\delta(x-l) \\ \dot{U} + \alpha_1 U + \alpha_2\dot{\varphi}(l_p,\, t) = 0 \end{cases} \tag{3}$$

where:
$a = \sqrt{\frac{G}{\rho}}$
$b + \frac{-\lambda}{I_0 l\rho}$
$c = \frac{1}{I_0 l\rho}$
$\alpha_1 = \frac{1}{R_s C_p}$
$\alpha_2 = \frac{2\pi R_2 h)pd_{15}G_p}{l_p C_p}$

The set of equations (3) it be solved with Galerkin's method, in which the solution has the form

$$\varphi(x,\, t) = A\sum_{n=1}^{\infty}\sin\left[(2n-1)\frac{\pi}{2l}x\right]\cos\omega t \tag{4}$$

The considered shaft is exited by harmonic moment as

$$M = M_0\cos\omega t \tag{5}$$

The tension, generated on clamps, piezotransducer will have harmonic character, because extortion has the same character:

$$U = B\sin\omega t \tag{6}$$

The sentence (4) has to fulfill boundary conditions as:

$$\begin{matrix} \varphi(0,\, t) = 0, & X(0)T(t) = 0 \Rightarrow X(0) = 0 \\ \frac{\partial\varphi}{\partial t}|_{x=1}, & X'(l)T(t) = 0 \Rightarrow X'(l) = 0 \end{matrix} \tag{7}$$

After calculation of suitable derivatives (4) and their substitution to the equations describing vibration and the state of mechatronic system the set of equations (3) takes the following form

$$\begin{cases} A\sin kx\cos\omega t\left[a^2\left(\frac{\pi}{2l}\right)^2 - \omega^2\right] - Bb\delta(x)\sin\omega t = cM\cos\omega t\delta(x-l) \\ B\omega\cos\omega t + \alpha_1 B\sin\omega t - \alpha_2 A\omega\sin\left((2n-1)\frac{\pi}{2l}l_p\right)\sin\omega t = 0 \end{cases} \tag{8}$$

where:
$k = (2n-1)\frac{\pi}{2l}x$
$\delta(x) = \delta(x-x_1) - \delta(x-x_2)$
or using the Euler' theorem and after transformations in matrix form

$$\begin{bmatrix} \sin kx\left[a^2\left(\frac{\pi}{2l}\right)^2 - \omega^2\right] & -\frac{1}{e^{i\pi/2}}b\delta(x) \\ -\alpha_2\frac{1}{e^{i\pi/2}}\omega\sin l_p & \omega + \frac{\alpha_1}{e^{i\pi/2}} \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} cM_0\delta(x-l) \\ 0 \end{bmatrix} \tag{9}$$

that is

$$\mathbf{W}\mathbf{A} = \mathbf{F} \tag{10}$$

In (10) the value of main determinant of matrix $\mathbf{W}$ is equal

$$|\mathbf{W}| = \sin kx \left[a^2 \left(\frac{\pi}{2l}\right)^2 - \omega^2\right] \left(\omega + \frac{\alpha_1}{e^{i\pi/2}}\right) - \frac{b}{(e^{i\pi/2})^2} \delta(x) \alpha_2 \omega \sin kl_p \tag{11}$$

Substituting in (9) first column, by column of free words, the determinant $\mathbf{W}_A$ was received

$$|\mathbf{W}_A| = cM_0 \delta(x-l) \left(\omega + \frac{\alpha_1}{e^{i\pi/2}}\right) \tag{12}$$

The amplitude A_n is determined as

$$A_n = \frac{|\mathbf{W}_{A_n}|}{|\mathbf{W}|} \tag{13}$$

and after substitution of it to (4), the dynamic flexibility after transformations takes the form

$$Y_{xl} = \sum_{n=1}^{\infty} Y_{xl}^{(n)} \tag{14}$$

In (14) $Y_{xl}^{(n)}$ is equal

$$Y_{xl}^{(n)} = \frac{c\delta(x-l)\left(\omega + \frac{\alpha_1}{e^{i\pi/2}}\right)}{\sin kx \left[a^2 \left(\frac{\pi}{2l}\right)^2 - \omega^2\right] \left(\omega + \frac{\alpha_1}{e^{i\pi/2}}\right) - \frac{b}{\left(e^{i\pi/2}\right)^2} \delta(x) \alpha_2 \omega \sin kl_p} \tag{15}$$

The transient of absolute value of flexibility in considered range of frequency and flexibility for three first vibration modes (14) – after further formal transformations and after putting of the numerical values of parameters and when $x = l$, that is $\alpha_Y = |Y_{ll}|$ – it was showed in Fig. 2.

3. Transformations of characteristics of torsional vibrating subsystems of mechatronic systems

The problem consists in modeling of torsional vibrating multiple–segment with mechanical bar systems as subsystems of mechatronic systems in the form of models with uniformly distributed parameters and constant section in the segment.

In the modeling of the considered class of systems, the dependence between the amplitudes of *generalized forces* ${}_2s_k \in {}_2S$ and *generalized displacements* ${}_1s_i \in {}_1S$ can by described by *dynamical flexibility* Y_{ik} [4, 5]. In other words, dynamical flexibility is the assigned amplitude of generalized displacement in the direction of i–th generalized coordinate caused by generalized force in the form of harmonic function with unitary amplitude, in relation with k–th generalized coordinate, so

$${}_1s_i = Y_{ij} \, {}_2s_j \tag{16}$$

where: ${}_2s_j = Q_j$, $\sin \omega t = 1e^{j\omega t}$, ω – frequency.

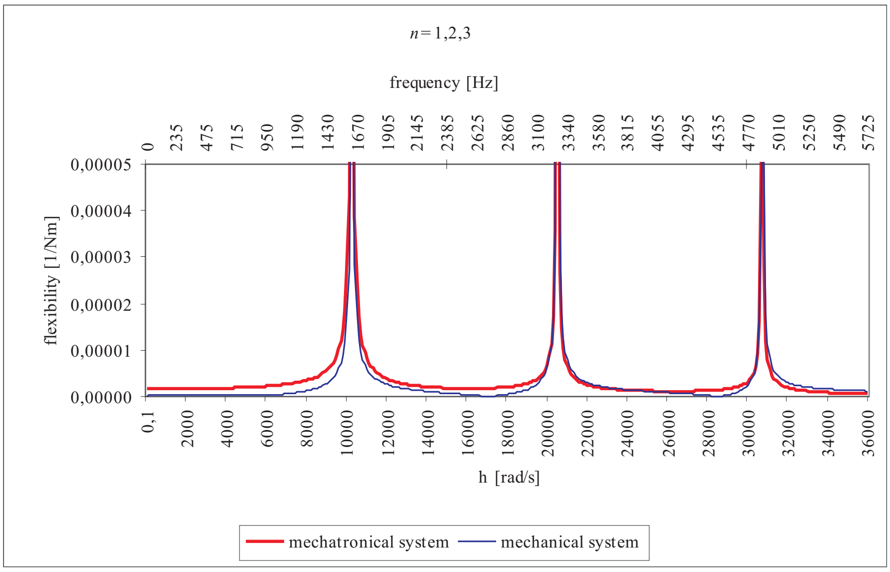

Figure 2 Transient of the sum for n=1, 2, 3 vibration mode

A characteristic - dynamical flexibility of mechanical subsystem of mechatronic system is given in form

$$Y(s) = \frac{\sum\limits_{i=0}^{k} c_i t h^j \Gamma s}{\sum\limits_{j=0}^{l} d_j t h^j \Gamma s} \tag{17}$$

After transformations of flexibility into mobility [4, 5, 15, 16] the mobility function has been obtained as

$$V(r) = \frac{\sum\limits_{i=0}^{k} c_i r^i}{\sum\limits_{j=0}^{l} d_j r^j} \tag{18}$$

where:

$c_i,\ c_{i-1},\ \ldots,\ c_0,\ d_j,\ d_{j-1},\ \ldots,\ d_0$ are any real numbers,

$\Gamma = \sqrt{\frac{\rho}{G}}L = \sqrt{\frac{\rho^{(i)}}{G^{(i)}}}L^{(i)}$, – mass density,

$L = L^{(i)}$ – length of basic element [4, 5],

$s = j\omega,\ j = \sqrt{-1},\ i,\ j,\ k,\ l$ – natural numbers, $k - l = 1$.

4. Modeling the subsystems of mechatronic systems by means the graphs

The review of essential concepts of graph theory, to fix the meaning of necessary terms and symbols have been presented before modeling the torsional vibrating continuous bar systems as subsystems of mechatronic systems and problems connected with it. Weighted hypergraphs (called in this paper also weighted block graphs or weighted graphs of category k) have been applied to modeling of the considered systems. Definitions of graphs, as mathematical objects, have been presented on the basis of the literature. The bibliography of this subject is very extensive and regards the theory as well as its applications (see [1, 2, 4]).

A following couple (using the symbols introduced in papers [4, 5, 20, 21])

$$X = ({}_1X,\ {}_2X) \tag{19}$$

is called a *graph,* where:

${}_1X = \{{}_1x_0, {}_1x_1, {}_1x_2, ..., {}_1x_n\}$ – finite set of vertices,
${}_2X = \{{}_2x_1, {}_2x_2, ..., {}_2x_m\}$ – family of edges,
being two–element subsets of vertices, in the form of
${}_2x_k = ({}_1x_i, {}_1x_j)$ $(i, j = 0,$ 1,..., $n)$.

The *hypergraph* is called a couple

$$^kX = \left({}_1X, {}^k_2X\right) \tag{20}$$

where: ${}_1X$ is the set as in (20), and ${}^k_2X = \left({}^k_2X^{(i)}/i \in N\right)$, ($k$=2,3, ... ∈N) is a family of subsets of set ${}_1X$; the family ${}^k_2X$ is called a *hypergraph* over ${}_1X$ as well, and ${}^k_2X = \left\{{}^k_2X^{(1)}, {}^k_2X^{(2)}, ..., {}^k_2X^{(m)}\right\}$ is a set of edges [2], called *hyperedges* or *blocks,* if

$$\begin{cases} {}^k_2X \neq \emptyset (i \in I) \\ \bigcup\limits_{i \in I} {}^k_2X^{(i)} = {}_2X \end{cases} \tag{21}$$

Graphs X and hypergraphs ^kX have been shown in their geometrical representation on plane. Sets of edges ${}_2X$ have been marked by lines, subsets of family ${}^k_2X$ (hyperedges or blocks) – two–dimensional continuum with enhanced vertices, in the shape of circles.

In this paper hypergraphs – graphs of category k – ^kX $(k = 2, 3)$ are used, which will be clearly mentioned each time, as well as graphs X, called also graphs of the first category – ^1X (see [4, 5]). The basic notions are shown in literature (e.g. [4, 5, 15, 16]).

In the example of torsional vibration of the subsystem (i) with constant cross–section and constant torsional rigidity $(GJ_0)^{(i)}$ (where $G^{(i)}$ – Kirchhoff's modulus of a bar structure, $J_0^{(i)}$ – polar moment of inertia of bar cross–section as well as length $l^{(i)}$, the model in the form of a determined and continuous system is introduced. In this model, generalized displacements ${}_1s_1^{(i)}$ and ${}_1s_2^{(i)}$ – angles of rotation correspond to its extreme points. These displacements are measured in the inertial system of reference. Moreover, the origin of the inertial system of reference has generalized coordinate ${}_1s_0^{(i)} = 0$ assigned to it (see e.g. [4,5]).

In this way a set of generalized displacements of a torsional vibrating subsystem of mechatronic system can be formulated as follows: ${}_1S^{(i)} = \left\{{}_1s_0^{(i)}, {}_1s_1^{(1)}, {}_1s_2^{(i)}\right\}$, while its dynamical flexibilities set may be denoted as $Y^{(i)} = \left\{Y_{11}^{(i)}, Y_{22}^{(i)}, Y_{12}^{(i)}\right\}$, $\left(Y_{12}^{(i)} = Y_{21}^{(i)}\right)$.

Determining one–to–one transformation, that:

$$f :_1 S^{(i)} \rightarrow {}_1X^{(i)}, \quad {}_1s_j^{(i)} \in {}_1S^{(i)}, \quad {}_1x_j^{(i)} \in {}_1X^{(i)}, \quad j = 0,\, 1,\, 2 \tag{22}$$

the *hypergraph of bar – subsystem of mechatronic system* is obtained

$$\underset{f}{{}^2X}^{(i)} = \left[{}^2X^{(i)},\, f\right] \tag{23}$$

where: ${}^2X^{(i)} = \left({}_1X^{(i)},\, {}^k_2X^{(i)}\right), {}_1X^{(i)} = \left\{{}_1x_0^{(i)}, {}_1x_1^{(i)}, {}_1x_2^{(i)}\right\},\ {}^k_2X^{(i)}$ – one–element family – three–element subset of vertices ${}_1X^{(i)}$.

Investigating $i - -$th *segment* in a *n–segment* mechanical bar as a subsystem of mechatronic system with constant section, the hypergraph model $\underset{f}{{}^2X}^{(i)}$ is introduced.

All the elements of hypergraph and all the generalized displacements, material coefficients and geometric coefficients should be denoted by subscript (i) placed to the right of the stem symbol, whereas subscript $k = 2$ should be placed to the left of the stem symbol.

On the basis of this assumption, geometrical representation of mapping (23) has been shown in [4, 5].

The assignment f_3 to the edges of weighted Lagrange skeleton ${}^2\underset{12}{\vec{X}}{}_0^{(i)}$ of hypergraph for the model of i-th bar - $\underset{f}{{}^2X}^{(i)}$, making couples of numbers – respectively – generalized coordinates and generalized forces ${}_2S = \left[\left|{}_2s_1^{(i)}\right|,\, \left|{}_2s_2^{(i)}\right|\right]$ so that:

$$f_3\left(\left\{{}_1x_0^{(i)}, {}_1x_1^{(i)}\right\}, \left\{{}_1x_0^{(i)}, {}_1x_2^{(i)}\right\}\right) = \left[\left\{\left|{}_1s_1^{(i)}\right|,\, \left|{}_2s_1^{(i)}\right|\right\}, \left\{\left|{}_1s_2^{(i)}\right|,\, \left|{}_2s_2^{(i)}\right|\right\}\right] \tag{24}$$

a *polar graph* is obtained

$$\underset{00}{\vec{X}}{}^{(i)} = \underset{3}{\vec{X}}{}_0^{(i)} = \left[{}^2\underset{f}{\vec{X}}{}_0^{(i)},\ f_3\right] \tag{25}$$

Polar equation [4,5], in the case of oriented polar graph $\underset{00}{\vec{X}}{}^{(i)}$, can be formulated as:

$$\begin{bmatrix} {}_1s_1^{(i)} \\ {}_1s_2^{(i)} \end{bmatrix} = \begin{bmatrix} Y_{11}^{(i)} & 0 \\ 0 & Y_{22}^{(i)} \end{bmatrix} \begin{bmatrix} {}_2s_1^{(i)} \\ {}_2s_2^{(i)} \end{bmatrix} \tag{26}$$

The set of equations (26) is a particular case of equation (16).

In the case of analysis of n–segment model of the system, composed of subsystems with constant section, vibrating torsional subsystem of mechatronic system, it is modeled by the loaded graph of the second category with n three–vertices–blocks, connected to those vertices to which the corresponding generalized coordinates are assigned (see i.e. [4, 5]).

In this way the weighted hypergraph (as a model of torsional vibrating mechanical and/or mechatronic system) may provide to the basis for the formalization which is the necessary condition of numerical discretization of the considered class of continuous mechanical systems as a part of discrete – continuous mechatronic system.

5. Review of synthesis methods of the mechanical and/or mechatronic systems represented by different category graphs

In this paper is showed how one of the methods, which were applied in order to synthesize the dynamical characteristic of the torsional vibrating mechanical system, may be applied to synthesis the mechatronic system with cascade structure as well. This is continued fraction expansion method distribution of characteristic represented by different category graphs [4–6].

5.1. *The synthesis of the mechanical subsystem of mechatronic system by the continued fraction expansion method*

Using notion of graph and of hypergraph [2] and system of notation [4, 5, 20], methods of mechanical subsystem as task of the synthesis of dynamical characteristic – mobility has been presented.

If characteristic - dynamical flexibility is given in form (17) then after transformations $V(s) = sY(s)$ and Richards' transformation $r = \mathrm{th}\Gamma s$ [4–9] the mobility (17) has been obtained as (18).

The method of the synthesis of transformed mobility function $V(r)$ is presented here, assuming the even number of elements, and when k is an even natural number, then $V(r)$ takes form

$$V(r) = \frac{c_k r^k + c_{k-1} r^{k-1} + ... + c_0}{d_{k-1} r^{k-1} + d_{k-3} r^{k-k} + ... + d_1 r}. \tag{27}$$

After dividing in (27) the numerator by denominator – it is a first step of the synthesis – the equation below is obtained

$$\begin{aligned} V(r) &= V_r^{(1)}(r) + \frac{L_{k-2}(r)}{M_{k-1}(r)} = V_r^{(1)}(r) + \frac{1}{\frac{M_{k-1}(r)}{L_{k-2}(r)}} \\ &= V_r^{(1)}(r) + \frac{1}{U_2(r)} = \frac{r}{c_r^{(1)}} + \frac{1}{U_2(r)} \end{aligned} \tag{28}$$

where: $c_r^{(1)}$ is value of "i" synthesized discrete elastic element.

The second step is the realization of the function $U_2(r)$ into (28). When dividing $M_{k-1}(r)$ by $L_{k-2}(r)$, $U_2(r)$ takes form

$$\begin{aligned} U_2(r) &= U_z^{(2)}(r) + \frac{M_{k-3}(r)}{L_{k-2}(r)} = U_z^{(2)}(r) + \frac{1}{\frac{L_{k-2}(r)}{M_{k-3}(r)}} \\ &= U_z^{(2)}(r) + \frac{1}{V_3(r)} = J_z^{(2)} r + \frac{1}{V_3(r)} \end{aligned} \tag{29}$$

where:$J_z^{(i)}$ –value of "i" synthesized discrete inertial element.

The graphs of synthesized mechanical bar or/and mechatronic system after operation (28) and (29) are shown in Fig. 3 and 4.

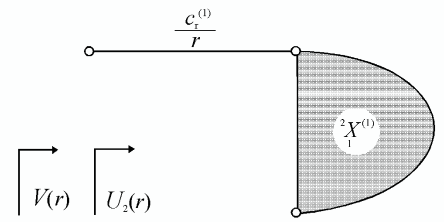

Figure 3 Graphical illustration of equation (28)

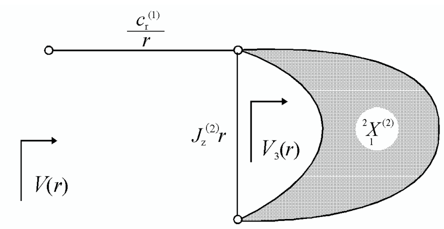

Figure 4 Graphical illustration of equations (28) and (29)

The process of the synthesis after steps (28) and (29) is to be continued until the function $U_k(r)$ will take form

$$U_k(r) = U_z^{(k)}(r) = J_z^{(k)} r \tag{30}$$

Finally the mobility (27) as a continued fraction is obtained in form

$$V(r) = V_r^{(1)} \cfrac{1}{U_z^{(2)}(r) + \cfrac{1}{V_r^{(3)}(r) + \cfrac{1}{U_z^{(4)}(r) + \cdots + \cfrac{1}{V_r^{(k-1)}(r) + \cfrac{1}{U_z^{(k)}(r)}}}}} = \frac{r}{c_r^{(1)}} + \cfrac{1}{J_z^{(2)}(r) + \cfrac{1}{\frac{r}{c_r^{(3)}} + \cfrac{1}{J_z^{(4)} r + \cdots + \cfrac{1}{\frac{r}{c_r^{(k-1)}} + \frac{1}{J_z^{(k)}}}}}} \tag{31}$$

The form (31) corresponds with mobility function (27) of a polar graph $\underset{00}{X}$ (see Fig. 5). The mobility determined at the point indicated by the arrow is identical with (27). This graph is a model of discrete system but after transformation it is a continuous system (comp. [4]).

Next causes realized after transformations as continuous torsional vibrating mechanical systems as subsystems of mechatronic systems were considered in [6].

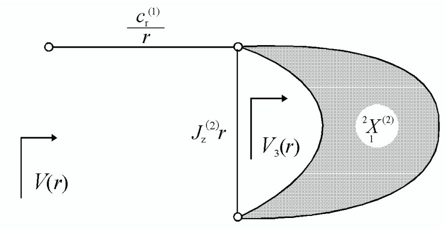

Figure 5 Graphical illustration of equation (31)

6. Last remarks

Applied method and received results can make up the introduction to the synthesis of considered class systems – torsional vibrating mechatronic ones with constant changeable cross–section. The problems will be presented in future works.

Acknowledgements

This work has been conducted as a part of research project N N502 452139 supported by the Ministry of Science and Higher Education in 2010-2013.

References

[1] **Bellert, S. and Woźniacki, H.**: The analysis and synthesis of electrical systems by means of the method of structural numbers, (in Polish), *WNT*, Warszawa, **1968**.

[2] **Berge, C.**: Graphs and hypergraphs, *American Elsevier Publishing Co.*, Inc., New York/ North Holland Publishing Co., Amsterdam–London, **1973**.

[3] **Białas, K., Buchacz, A. and Dzitkowski, T.**: Synthesis of active mechanical systems with dumping inview of polar graphs and structural numbers, Monograph No. 230. *Silesian University of Technology Press*, Gliwice, (in Polish), **2009**.

[4] **Buchacz, A.**: The synthesis of vibrating bar-systems represented by graph and structural numbers, *Scientific Letters of Silesian University of Technology*, MECHANICS, z. 104, Silesian University of Technology Press, (in Polish), **1991**.

[5] **Buchacz, A.**: Modeling, synthesis and analysis of bar systems characterized by a cascade structure represented by graphs, *Mech. Mach. Theory*, 30, 7, 969–986, Pergamon, **1995**.

[6] **Buchacz, A.**: Modification of Synthesised Vibration Bar-Systems Represented by Graphs by Means the Continued Fraction Expansion Method, Donetsk State Technical University, *International Journal of Proceedings–Machine–Buildings and Systems*, Donetsk, Vol. 19, p.265–272, **2002**.

[7] **Buchacz, A., Dymarek, A. and Dzitkowski T.**: Design and examining of sensitivity of continuous and discrete–continuous mechanical systems with required frequency spectrum represented by graphs and structural numbers, Monograph No. 88. *Silesian University of Technology Press*, Gliwice, (in Polish), **2005**.

[8] **Buchacz, A.**: The expansion of the synthesized structures of mechanical discrete systems represented by polar graphs, *Journal of Materials Processing Technology*, Vol. 164-165, Complete Elsevier, p.1277–1280, **2005**.

[9] **Buchacz, A. and Żurek K.**: Reverse task of dynamice of active mechanical systems by means the graphs and structural numbers methods, *Monograph No 81. Silesian University of Technology Press*, Gliwice, (in Polish), **2005**.

[10] **Buchacz, A.**: Modeling, synthesis, modification, sensitivity and analysis of mechanic and mechatronic systems, *Journal of achievements in materials and manufacturing engineering*, International OCOSCO World Press, Vol. 24, Issue 1, p. 198–207, **2007**.

[11] **Buchacz, A.**: Dynamical flexibility of discrete-continuous vibrating mechatronic system, *Journal of Achievements in Materials and Manufacturing Engineering*, International OCOSCO World Press,Vol. 28, Issue 2, p. 159–166, **2008**.

[12] **Buchacz, A.**: Characteristics of discrete–continuous flexibly vibrating mechatronic system, *Journal of Achievements in Materials and Manufacturing Engineering*, International OCOSCO World Press,Vol. 28, Issue 1, p. 43–46, **2008**.

[13] **Buchacz, A.**: Calculation of flexibility of vibrating beam as the subsystem of mechatronic system by means the exact and approximate methods, *PAMM - Proc. Appl. Math. Mech. 9*, Issue 1, 373 – 374, WILEY–VCH Verlag GmbH & Co. KgaA, Weinheheim, Published Online: DOI 10.1002/pamm.200910160, **2010**.

[14] **Buchacz, A.**: Transformations of hypergraphs of beams as models in synthesis of flexibly vibrating continuous mechanical systems, *Vestnik Krymskoy Akademii Nauk – Sevastopolskie otdelenie*, Release 1, Sevastopol, p. 35–38, **2010**.

[15] **Buchacz, A.**: Orthogonalization method of analysis of mechanical and mechatronic systems, *LES PROBLÈMES DU TECHNOSHÈRE ET DE LA FORMATION DES CADRES D'INGÉNIEURS CONTEMPORAINS, Recueil des exposés des participants du III Séminaire international scientifique et méthodique*, Sousse (Tunisie), Donetsk , p. 85–88, —textbf2009.

[16] **Buchacz, A.**: Comparison of characteristics of subsystems and systems as introduction to synthesis of torsionally vibrating mechatronic systems, *LES PROBLÈMES DU TECHNOSHÈRE ET DE LA FORMATION DES CADRES D'INGÉNIEURS CONTEMPORAINS, Recueil des exposés des participants du IV Séminaire international scientifique et méthodique*, Hammamet (Tunisie), Donetsk,p. 10–13, **2010**.

[17] **Callahan, J. and Baruh, H.**: Vibration monitoring of cylindrical shells using piezoelectric sensors, *Finite Elements in Analysis and Design*, 23, 303–318, **1996**.

[18] **Kurnik, W.**: Damping of mechanical vibrations utilizing shunted piezoelements, *Machine Dynamics Problems*, Vol. 28, No 4, 15–26, **2004**.

[19] **Lu, P., Lee, K. H. and Lim, S. P.**: Dynamical analysis of a cylindrical piezoelectric transducer, *Journal of Sound and Vibration*, 259(2), 427–443, **2003**.

[20] **Wojnarowski, J.**: Application of graphs in analysis of vibration mechanical systems, *PWN*, Warszawa–Wrocław, (in Polish) **1981**.

[21] **Wojnarowski, J. and Buchacz, A.**: Use of hypergraphs and complete structural numbers in the analysis of vibrating beam systems with non-linearly changing cross–sections, *VIBRATION ENGINEERING, Hemisphere Publishing Corp.*, 3, 4, 593–598, **1989**.