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A Note on the Generalized Hohmann Transfer Time

Osman M. Kamel
Astronomy and Space Science Dept.
Faculty of Sciences
Cairo University, Giza, Egypt

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We calculate the transfer time from the inner to the outer elliptic planetary orbits of a space vehicle for the four feasible configurations and for the circular case. We find that the least time of transfer t_T corresponds to the second configuration.

Keywords: Astrodynamics, Hohmann transfer orbits.

1. Methods and Results

Generally the period of time spent in making the transfer from the inner planetary orbit to the outer one is given by [1]:

$$t_T = \pi \left[\frac{a_T^3}{\mu} \right]^{1/2} \tag{1}$$

For circular inner and outer orbit case, we have:

$$a_T = \frac{a_1 + a_2}{2} \tag{2}$$

Where is the semi – major axis of the elliptic transfer orbit: a_1, a_2 are the two terminal orbits radii.

From previous literature of Kamel & Soliman [2], we discover that there are four feasible configurations for the generalized Hohmann transfer.

For the first configuration (when the apo – apse of the transfer orbit coincides with the apo – apse of the final orbit) a_T is given by:

$$a_T = \frac{a_1(1 - e_1) + a_2(1 + e_2)}{2} \tag{3}$$

For the second configuration (when the apo – apse of the transfer orbit coincides with the peri – apse of the final orbit) ; third configuration (when the peri – apse of the transfer orbit coincides with the peri – apse of the final orbit) and fourth configuration (when the apo – apse of the transfer orbit coincides with the apo – apse of the final orbit), we have respectively:

$$a_T = \frac{a_1(1-e_1) + a_2(1-e_2)}{2} \tag{4}$$

$$a_T = \frac{a_1(1+e_1) + a_2(1-e_2)}{2} \tag{5}$$

$$a_T = \frac{a_1(1+e_1) + a_2(1+e_2)}{2} \tag{6}$$

After little reduction, we get for the four configurations, successively:

$$t_T = \frac{\pi}{\sqrt{8\mu}} \left[a_1(1-e_1) + a_2(1+e_1)\right]^{3/2} \tag{7}$$

$$t_T = \frac{\pi}{\sqrt{8\mu}} \left[a_1(1-e_1) + a_2(1-e_1)\right]^{3/2} \tag{8}$$

$$t_T = \frac{\pi}{\sqrt{8\mu}} \left[a_1(1+e_1) + a_2(1-e_1)\right]^{3/2} \tag{9}$$

$$t_T = \frac{\pi}{\sqrt{8\mu}} \left[a_1(1+e_1) + a_2(1+e_1)\right]^{3/2} \tag{10}$$

Eqs (2), (7 – 10) show that we acquire different results for the different configurations, and also we get a different result when we compare the time of the elliptic terminal orbits with the circular terminal ones, since from Eqs (1), (2) where $e_1 = 0$, $e_2 = 0$, we may write for the fifth configuration

$$t_T = \frac{\pi}{\sqrt{8\mu}} \left[a_1 + a_2\right]^{3/2} \tag{11}$$

For the case of Earth – Mars transfer, we calculated the value of t_T for the five configurations, where μ is the product of the Sun's mass and the gravitational constant. We may $\mu = 1$ assume for analytical developments, and a_1, a_2 are the semi – major axes of the Earth and Mars respectively, whilst e_1, e_2 are the eccentricities of the Earth and Mars.

The calculations indicate that the least value of t_T corresponds to the second configuration.

For the case of Earth – Mars transfer, we have:

$a_1 = 1$; a_2= 1.5237 ; $e_1 = 0.0167$; $e_2 = 0.0934$ [3]

Fig	**Eq.**	t_T
1	**(7)**	**4.7896**
2	**(8)**	**4.0389**
3	**(9)**	**4.1248**
4	**(10)**	**4.8805**
Circle	**(11)**	**4.4531**

References

[1] **Roy, A. E.**: Orbital Motion, Fourth Edition, *IOP publication Ltd.*, Bristol and Philadelphia, **2005**.

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[3] **Murray, C. D. and Dermott, S. F.**: Solar System Dynamics, *Cambridge University Press*, **1999**.