Mechanics and Mechanical Engineering
Vol. 20, No. 3 (2016) 263–277

Internal Heat Source in Temperature Rate Dependent Thermoelastic Medium with Hydrostatic Initial Stress

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Received (15 February 2016)
Revised (16 February 2016)
Accepted (17 April 2016)

The present work is devoted to study the effect of hydrostatic initial stress in an infinite isotropic generalized thermoelastic medium with the dependence of modulus of elasticity and thermal conductivity on the reference temperature. In view of calculating general problems, a numerical solution technique is to be used. For this purpose , the normal mode analysis method is chosen. The results for the displacement components, force stress and temperature distribution are illustrated graphically with some comparisons. The numerical results are given and presented graphically for Lord–Shulman theory of thermoelasticity when mechanical force is applied.

Keywords: thermoelasticity, hydrostatic initial stress, internal heat source, normal–mode, modulus of elasticity, thermal conductivity.

1. Introduction

In recent years increasing attention has been directed towards the generalized theory of thermoelasticity, which was found to give more realistic results than the coupled or uncoupled theories of thermoelasticity, especially when short–time effects or step temperature gradients are considered. The absence of any elasticity term in the heat conduction equation for uncoupled thermoelasticity appears to be unrealistic, since due to the mechanical loading of an elastic, the strain so produced causes variation in the temperature field. Moreover, the parabolic type of heat conduction equation results in an infinite velocity of the strain–rate term in the uncoupled heat conduction equation. Biot [1] extended the analysis to incorporate coupled thermoelasticity. In this way, although the first shortcoming was over, there remained the parabolic–type partial differential equation of heat conduction, which leads to the paradox of the infinite velocity of the thermal wave.

To take care of the paradox in Biot's theory, Kaliski [2] proposed a possible physical model, involving a finite velocity of heat propagation as actually required in nature. Lord and Shulman [3] developed a theory in which they modified the Fourier's law of heat conduction with the introduction of a thermal relaxation time parameter. The theory of generalized thermoelasticity with two relaxation times was first introduced by Muller [4]. A more explicit version was then introduced by Green and Laws [5], Green and Lindsay [6] and independently by Suhubi [7]. In this theory, the temperature rates are considered among the consecutive variables. This theory also predicts finite speeds of propagation as in Lord and Shulman's theory of generalized thermoelasticity with one relaxation time [3]. It differs from the latter in that Fourier's law of heat conduction is not violated if the body under consideration has a center of symmetry.

A generalized thermoelastic problem in an infinite cylinder under initial stress has been discussed by El–Naggar and Abd–Alla [8]. because of the inclusion of thermal relaxation parameters, the basic governing equations involved in the generalized theories of thermoelasticity are all of hyperbolic type differential equation and these theories are also referred to as hyperbolic thermoelasticity theories(Chandrasekharaiah [9]). Chandrasekharaiah [10] studied free plane harmonic waves without energy dissipation in an unbounded body. Chandrasekharaiah and Srinath[11, 12] have studied cylinderical/spherical waves due to:

1. a load applied to the boundary of the cylindrical/spherical cavity in an unbounded body,

2. a line/point heat source in an unbounded body.

Sharma and Chauhan[13] tackled a problem on thermoelastic interaction without energy dissipation due to body forces and heat sources. Recently Green and Nagdhi [14–16] and chandrasekharaiah [17] have formulated three different models of thermoelasticity in an alternative way. Mukhopadhyay [18] dealt with a problem concerning the thermoelastic interactions without energy dissipation in an unbounded medium with a spherical cavity subjected to a thermal shock.

Initial stresses in solids have significant influence on the mechanical response of the material from an initially–stressed configuration and have applications in geophysics, engineering structures and in the behaviour of soft biological tissues. Initial stress arises from processes, such as manufacturing or growth, and is present in the absence of applied loads. Montanaro [19] formulated the isotropic thermoelasticity with hydrostatic initial stress. Singh et al.[20], Othman et al. [21]. Singh [22], and many others have applied Mantanro [19] theory to study the plane harmonic waves in context of generalized thermoelasticity.

Modern structure elements are often subjected to temperature changes of such magnitude that their materials properties may no longer be regarded as having constant values even in approximate sense. The thermal and mechanical properties of materials vary with temperature, so that the temperature dependence of material properties must be taken into consideration in the thermal stress analysis of these elements, Youssef [23] constructed a model of the dependence of the modulus of elasticity and thermal conductivity on the reference temperature and solved a problem of an infinite material with a spherical cavity. Ezzat et al. [24] studied generalized thermoelasticity with temperature dependent modulus of elasticity under three theories. Tianhu and Shuanhu [25] studied the effect of temperature–dependent properties on thermoelastic problems with thermal relaxation. Chakravorty and Chakravorty [26] discussed the transient disturbances in a relaxing thermoelastic half space due to moving stable internal heat source. Kumar and Devi [27] studied thermomechanical interactions in porous generalized thermoelastic material permeated with heat source. Lotfy [28] have studied the transient disturbance in a half–space under generalized magneto–thermoelasticity with a stable internal heat source. Lotfy [29] discussed the transient thermo-elastic disturbances in a visco–elastic semi–space due to moving internal heat source. Othman [30] studied the state space approach to generalized thermoelastic problem with temperature elastic moduli and internal heat source.

The present paper is concerned with the investigations related to effect of hydrostatic initial stress and temperature rate dependent material in an infinite isotropic generalized thermoelastic medium with internal heat source. The normal mode analysis is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are represented graphically.

2. Formation of the problem

We consider an infinite isotropic generalized thermoelastic medium with the dependence of modulus of elasticity and thermal conductivity on the reference temperature under hydrostatic initial stress. All quantities considered are functions of the time variable t and of the coordinates x and y.

A rectangular cartesian coordinate system $(x,\ y,\ t)$ having origin on the surface $y=\ 0$ and y–axis pointing normally into the medium is introduced. We assume the displacement vector as

$$\overrightarrow{u}(x, y, t) = (u_1, u_2, 0) \tag{1}$$

To analyze the displacement components, stresses and temperature distribution at the interior of the medium, the continuum is divided into two half spaces defined by:

1. half space I $|x| <\infty$, $-\infty<y \leq 0$, $|z| <\infty$,
2. half space II $|x| <\infty$, $0 \leq y<\infty$, $|z| <\infty$,

if we restrict our analysis to the plane strain parallel to xy-plane with displacement vector $\overrightarrow{u}= (u_1, u_2, 0)$, then the field equations and constitutive relations for such a medium in the absence of body forces are written as:

$$\frac{\partial\ t_{11}}{\partial\ x}+\frac{\partial\ t_{12}}{\partial\ y}=\rho\frac{\partial^2 u_1}{\partial\ t^2} \tag{2}$$

$$\frac{\partial\ t_{21}}{\partial\ x}+\frac{\partial\ t_{22}}{\partial\ y}=\rho\frac{\partial^2 u_2}{\partial\ t^2} \tag{3}$$

where:

$$t_{11}= -p+(\lambda+2)\frac{\partial\, u_1}{\partial\, x}+\lambda\frac{\partial\, u_2}{\partial\, y}-\vartheta(1+\vartheta_0\frac{\partial}{\partial\, t})T \tag{4}$$

$$t_{12}= (\mu-\frac{p}{2})\frac{\partial\, u_2}{\partial\, x}+(\mu+\frac{p}{2})\frac{\partial\, u_1}{\partial\, y} \tag{5}$$

$$t_{21}= (\mu+\frac{p}{2})\frac{\partial\, u_2}{\partial\, x}+(\mu-\frac{p}{2})\frac{\partial\, u_1}{\partial\, y} \tag{6}$$

$$t_{22}= -p+\lambda\frac{\partial\, u_1}{\partial\, x}+(\lambda+2\mu)\frac{\partial\, u_2}{\partial\, y}-\vartheta(1+\vartheta_0\frac{\partial}{\partial\, t})T \tag{7}$$

where $\vartheta= (3\lambda+2\mu)\alpha_t$.

Using equations (4)–(7) in equations (2)–(3) we obtain:

$$(\lambda+2\mu)\frac{\partial^2 u_1}{\partial\, x^2}+(\mu-\frac{p}{2})\frac{\partial^2 u_1}{\partial\, y^2}+(\lambda+\mu+\frac{p}{2})\frac{\partial^2 u_2}{\partial\, x\partial\, y}-\vartheta(1+\vartheta_0\frac{\partial}{\partial\, t})\frac{\partial\, T}{\partial\, x}=\rho\frac{\partial^2 u_1}{\partial\, t^2} \tag{8}$$

$$(\lambda+\mu+\frac{p}{2})\frac{\partial^2 u_1}{\partial\, x\partial\, y}+(\mu-\frac{p}{2})\frac{\partial^2 u_2}{\partial\, x^2}+(\lambda+2\mu)\frac{\partial^2 u_2}{\partial\, y^2}-\vartheta(1+\vartheta_0\frac{\partial}{\partial\, t})\frac{\partial\, T}{\partial\, y}=\rho\frac{\partial^2 u_2}{\partial\, t^2} \tag{9}$$

The heat conduction equation is given by:

$$K^*(n^*+t_1\frac{\partial}{\partial\, t})(\frac{\partial^2 T}{\partial\, x^2}+\frac{\partial^2 T}{\partial\, y^2}) =\rho C^*(n_1\frac{\partial}{\partial\, t}+\tau_0\frac{\partial^2}{\partial\, t^2})T$$

$$+\vartheta T_0(n_1\frac{\partial}{\partial\, t}+\tau_0 n_0\frac{\partial^2}{\partial\, t^2})(\frac{\partial\, u_1}{\partial\, x}+\frac{\partial\, u_2}{\partial\, y}) - (n_1+n_0\tau_0\frac{\partial}{\partial\, t})Q \tag{10}$$

The use of thermal relaxation times τ_0, ϑ_0 and the parameters n^*, n_1 and n_0 helps to make the above mentioned fundamental equations possible for three different theories as:

L–S theory due to internal heat source, when:

$$n^*=n_1=n_0= 1 \qquad t_1=\vartheta_0= 0, \quad \tau_0> 0 \tag{11}$$

G–L theory due to internal heat source, when:

$$n^*=n_1= 1 \quad n_0= 0 \qquad t_1= 0, \quad \vartheta_0 \geq \tau_0> 0 \tag{12}$$

where ϑ_0, τ_0 are the two relaxation times.

Our aim is to investigate the effect of temperature dependence of modulus of elasticity keeping the other elastic and thermal parameters as constant. Therefore we may assume:

$$\begin{gathered}\lambda=\lambda_0(1-\alpha^* T_0) \quad \mu=\mu_0(1-\alpha^* T_0) \quad \alpha=\alpha_0(1-\alpha^* T_0) \quad p=p_0(1-\alpha^* T_0)\\ K=K_0(1-\alpha^* T_0) \quad k^*=k^*_0(1-\alpha^* T_0) \quad \vartheta=\vartheta_0(1-\alpha^* T_0)\end{gathered} \tag{13}$$

where λ_0, μ_0, α_0, p_0, K_0, k_0^*, ϑ_0 are considered constants, α^* is called empirical material constant, in case of the reference temperature independent of elasticity moduli and thermal conductivity $\alpha^*= 0$.

To facilitate the solution, following dimensionless quantities are introduced:

$$\{x', y'\}=\frac{\omega^*}{c_1}\{x,y\} \quad \{u_1', u_2'\}=\frac{\rho c_1 \omega^*}{\tau T_o}\{u_1,u_2\} \quad T'=\frac{T}{T_0} \quad t_{ij}'=\frac{t_{ij}}{\vartheta T_o}$$
$$t'=\omega^* t \quad t_1'=\omega^* t_1 \quad \tau_0'=\omega^*\tau_0 \quad \vartheta_0'=\omega^*\vartheta_0 \quad p'=\frac{p}{\vartheta T_0} \quad Q_0'=\frac{1}{\lambda\omega^*}Q_0 \tag{14}$$

where:

$$c_1^2=\frac{(\lambda+2\mu)}{\rho} \qquad \omega^*=\frac{\rho C^* c_1^2}{K_1^*}$$

Using the expression relating displacement components $u_1(x,y,t)$ and $u_2(x,y,t)$ to the scalar potential functions $\phi(x,y,t)$ and $\psi(x,y,t)$ in dimensionless form:

$$u_1=\frac{\partial \phi}{\partial x}-\frac{\partial \psi}{\partial y} \qquad u_2=\frac{\partial \phi}{\partial y}+\frac{\partial \psi}{\partial x} \tag{15}$$

Equations (8)–(10), with the help of equations (13)–(15) may be recast into dimensionless form after suppressing the primes as:

$$(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-A^*\frac{\partial^2}{\partial t^2})\phi-(1+\vartheta_0\frac{\partial}{\partial t})T= 0 \tag{16}$$

$$(\zeta_2\frac{\partial^2}{\partial x^2}+\zeta_2\frac{\partial^2}{\partial y^2}-A^*\frac{\partial^2}{\partial t^2})\psi= 0 \tag{17}$$

$$(n^*+t_1\frac{\partial}{\partial t})(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2})T= (n_1\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2})T$$
$$+\zeta_3(n_1\frac{\partial}{\partial t}+\tau_0 n_0\frac{\partial^2}{\partial t^2})(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2})\phi-\zeta_4(n_1+n_0\tau_0\frac{\partial}{\partial t})Q \tag{18}$$

3. Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form:

$$[\phi,\psi,T,t_{ij}](x,y,t) = [\overline{\phi},\overline{\psi},\overline{T},\bar{t}_{ij}](y)e^{(\omega t+iax)} \tag{19}$$

$$Q=Q_0 e^{(\omega t+iax)} \qquad \overline{Q}=Q_0 \tag{20}$$

where $[\overline{\phi},\overline{\psi},\overline{T},\bar{t}_{ij}]$ are the magnitude of the functions, ω is the complex time constant and a is the wave number in x–direction and Q_0 is the magnitude of stable internal heat source.

Using (19)–(20), in equations (16)–(18) we obtain:

$$[\nabla^2 - \zeta_8]\overline{\phi} - \zeta_9 \overline{T} = 0 \tag{21}$$

$$[\zeta_7 \nabla^2 - \zeta_7 a^2]\overline{\phi} - [\zeta_5 \nabla^2 - \zeta_5 a^2 - \zeta_6]\overline{T} = \varepsilon Q_0 \tag{22}$$

$$[\zeta_2(\nabla^2 - \zeta^2) - A^* \omega^2]\overline{\psi} = 0 \tag{23}$$

where:

$$\zeta_1 = (\lambda + \mu + \frac{\vartheta T_0 p}{2})\frac{1}{\rho c_1^2} \quad \zeta_2 = (\mu - \frac{\vartheta T_0 p}{2})\frac{1}{\rho c_1^2} \quad \zeta_3 = \frac{\vartheta^2 T_0}{\rho K^* \omega^*} \quad \zeta_4 = \frac{\lambda c_1^2}{\omega^* K^*}$$

$$\zeta_5 = (n^* + t_1 \omega) \quad \zeta_6 = (n_1 \omega + \tau_0 \omega^2) \quad \zeta_7 = a_3(n_1 \omega + n_0 \tau_0 \omega^2) \quad \zeta_8 = (a^2 + A^* \omega^2)$$

$$\zeta_9 = (1 + \vartheta_0 \omega) \quad \zeta_{10} = \frac{\lambda}{\rho c_1^2} \quad \zeta_{11} = (\mu + \frac{\vartheta T_0 p}{2})\frac{1}{\rho c_1^2} \tag{24}$$

with:

$$A^* = \frac{1}{(1 - \alpha^* T_0)}$$

Eliminating $\overline{T}$ from equations (21)–(22) we obtain:

$$[\nabla^4 - \lambda_1 \nabla^2 + \lambda_2](\overline{\phi}(y)) = \varepsilon \zeta_9 Q_0 \tag{25}$$

where:

$$\nabla = \frac{d}{dy}$$

$$\lambda_1 = (\zeta_5 a^2 + \zeta_6 + \zeta_5 \zeta_9 + \zeta_7 \zeta_9)$$

$$\lambda_2 = (a^2(\zeta_5 \zeta_8 + \zeta_7 \zeta_9) + \zeta_6 \zeta_8)$$

The solution of equation (25) is given by:

$$\overline{\phi}(y) = \sum_{j=1}^{2} S_j(a,\omega) e^{-k_j y} + \sum_{j=1}^{2} R_j(a,\omega) e^{k_j y} + f_1 \tag{26}$$

In a similar way, we get

$$\overline{T}(y) = \sum_{j=1}^{2} \xi_j^* S_j(a,\omega) e^{-k_j y} + \sum_{j=1}^{2} \xi_j^* R_j(a,\omega) e^{k_j y} + f_2 \tag{27}$$

The solution of equation (23) is given by:

$$\overline{\psi}(y) = S_3(a,\omega) e^{-k_3 y} + R_3(a,\omega) e^{k_3 y} \tag{28}$$

where:

$$f_1 = \frac{\varepsilon \zeta_9 Q_0}{\zeta_9} \quad f_2 = \frac{\varepsilon Q_0 (k_j^2 - \zeta_8)}{\zeta_5} \quad k_3^2 = \frac{\zeta_2 a^2 + A^* \omega^2}{\zeta_2} \quad \xi_j^* = \frac{k_j^2 - \zeta_8}{\zeta_9} \quad j = 1, 2 \tag{29}$$

where $S_j(a,\omega)$, $R_j(a,\omega)$ are some parameters depending on a and ω; k_j^2 $(j = 1, 2)$ are the roots of the characteristic equation (21).

4. Applications

The boundary conditions at the interface $y = 0$ subjected to an arbitrary normal force P_1 are:

$$\begin{aligned}
&1.\ t_{22}(x,0^+,t)-t_{22}(x,0^-,t) = -P_1 e^{(\omega t+iax)}\\
&2.\ t_{21}(x,0^+)-t_{21}(x,0^-) = 0\\
&3.\ u_1(x,0^+) = u_1(x,0^-)\\
&4.\ u_2(x,0^+) = u_2(x,0^-)\\
&5.\ v)T(x,0^+) = T(x,0^-)\\
&6.\ \frac{\partial T}{\partial y}(x,0^+) = \frac{\partial T}{\partial y}(x,0^-)
\end{aligned} \tag{30}$$

where P_1 is the magnitude of mechanical force. Using equations(14) and (6)–(7) on the non–dimensional boundary conditions and then using (26)–(28), we get the expressions of displacement, force stress and temperature distributions for isotropic generalized thermoelastic medium as:

$$\begin{aligned}
u_1 =& \{\sum_{j=1}^{2} iaS_j(a,\omega)e^{-k_j y} + \sum_{j=1}^{2} iaR_j(a,\omega)e^{k_j y}\\
&+k_3S_3e^{-k_3 y} - k_3R_3e^{k_3 y}\}e^{(\omega t+iax)} + iaf_1
\end{aligned} \tag{31}$$

$$\begin{aligned}
u_2 =& \{-\sum_{j=1}^{2} k_jS_j(a,\omega)e^{-k_j y} + \sum_{j=1}^{2} k_jR_j(a,\omega)e^{k_j y}\\
&+iaS_3e^{-k_3 y} + iaR_3e^{k_3 y}\}e^{(\omega t+iax)}
\end{aligned} \tag{32}$$

$$\begin{aligned}
t_{22} =& \{\sum_{j=1}^{2} \beta_j^* S_j(a,\omega)e^{-k_j y} + \sum_{j=1}^{2} \beta_j^* R_j(a,\omega)e^{k_j y} + L_1S_3e^{-k_3 y}\\
&-N_1R_3e^{k_3 y}\}e^{(\omega t+iax)} + L_2
\end{aligned} \tag{33}$$

$$\begin{aligned}
t_{21} =& \{\sum_{j=1}^{2} \gamma_j^* S_j(a,\omega)e^{-k_j y} - \sum_{j=1}^{2} \gamma_j^* R_j(a,\omega)e^{k_j y}\\
&+L_3S_3e^{-k_3 y} + L_3R_3e^{k_3 y}\}e^{(\omega t+iax)}
\end{aligned} \tag{34}$$

$$T = \{\sum_{j=1}^{2} \xi_j^* S_j(a,\omega)e^{-k_j y} + \sum_{j=1}^{2} \xi_j^* R_j(a,\omega)e^{k_j y}\}e^{(\omega t+iax)} + f_2 \tag{35}$$

where:

$$\beta_j^*= (k_j^2-a^2a_{11}-a_9a_j^*) \quad \gamma_j^*=iak_j(a_{12}-a_2) \quad L_1=iak_3(\xi_{13}-1)$$

$$L_2=a^2a_{11}f_1-a_9f_2 \quad L_3=iak_3(a_2+a_{12})$$

Invoking the boundary conditions (30) at the surface $y= 0$, we obtain a system of six equations, and applying the inverse of matrix method, we obtain the values of six constants R_j and S_j, $j= 1, 2, 3$, as:

$$R_1=\frac{\Delta_1}{\Delta}, \quad R_2=\frac{\Delta_2}{\Delta}, \quad R_3=\frac{\Delta_3}{\Delta}, \quad S_1=\frac{\Delta_4}{\Delta}, \quad S_2=\frac{\Delta_5}{\Delta}, \quad S_3=\frac{\Delta_6}{\Delta}.$$

where Δ, Δ_i, $i= 1, 2, 3, .., 6.$ are defined in appendix A.

5. Particular cases

5.1. *Isotropic generalized thermoelastic medium with internal heat source*

Letting $p \to 0$, in the system of equations (26)–(28), we obtain the components of displacements, force stress and temperature distribution in isotropic generalized thermoelastic medium with internal heat source and temperature rate dependent material .

For all the cases discussed above the components of displacement, stresses and temperature distribution for the region $-\infty< y \leq 0$, are obtained by inserting $R_1 = R_2 = R_3 = 0$ in Eqs. (31)–(35).

Similarly for the region $0 \leq y <\infty$, the components are obtained by inserting $S_1 = S_2 = S_3 = 0$ in Eqs. (31)–(35).

Taking $\alpha^*= 0$, we obtain the corresponding expressions in isotropic generalized thermoelastic half-space with internal heat sourse under hydrostatic initial stress.

6. Numerical results

To study the effect of initial stress and temperature-dependent material, we now present some numerical results. For this purpose, the values of physical constants are taken as Sharma [31]:

$$\lambda= 8.2 \times 10^{10}Nm^{-2} \quad \mu= 4.2 \times 10^{10}Nm^{-2} \quad \rho= 8.950 \times 10^3kgm^{-3}$$

$$K^*= 1.13 \times 10^2calm^{-1}s^{-1}K^{-1} \quad \alpha_T= 1.0 \times 10^{-8}K \quad T_0= 300K$$

$$\omega^*= 4.347 \times 10^{13}sec^{-1}$$

The comparison are carried out for: $\alpha^*= 0.051/K$. The computations are carried out for the value of non–dimensional time $t= 0.1$ in the range $0 \leq x \leq 10$ and on the surface $y= 1.0$. Using this data the value of the physical quantities are evaluated and absolute values of displacement, force stress, temperature distribution are plotted in Figs. 1-3 in the context of the L–S theory for mechanical force with $P_1 = 1.0$, $p= 2.0$, $\omega = \omega_0 + i\zeta$, $\omega_0 = 2.3$, $\zeta = 0.1$, $a= 2.1$ and $Q_0 = 10$ for an:

(a) Isotropic generalized thermoelastic medium with hydrostatic initial stress and temperature rate dependent property (IGTHTD)by solid line.

(b) Isotropic generalized thermoelastic medium with hydrostatic initial stress and temperature rate independent propety(IGTHTI) by solid line with centered symbol (*).

(c) Isotropic generalized thermoelastic medium without hydrostatic initial stress and with temperature rate dependent property(IGTWHTD) by dashed line.

(d) Isotropic generalized thermoelastic medium(IGTWHTI) by dashed line with centered symbol (*).

These graphical results represent the solutions obtained for the generalized theory with one relaxation time (L-S-theory) by taking $\tau_0 =$ 0.02.

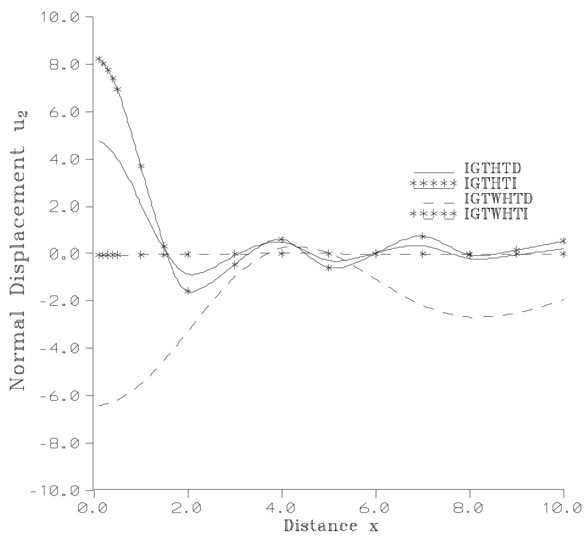

Figure 1 Variations of normal displacement u_2 with distance x

7. Discussions

Fig. 1 depicts the variations of normal displacement u_2 with distance x. The variations of normal displacement u_2 for IGTHTD and IGTHTI show similar patterns with different degree of sharpness. i.e. the values for IGTHTD and IGTHTI increases and decreases alternately with distance x. The value of normal displacement u_2 for IGTWHTI lie in a very short range. Further normal displacement u_2 shows small variations near to zero value in the whole range for IGTWHTD.

Fig. 2 depicts the variations of normal force stress t_{22} with distance x. The pattern observed for IGTHTD and IGTWHTI are opposite in nature near the point of application of source. The value of t_{22} for IGTWHTD decreases, then follow an oscillatory pattern with decreasing magnitude. It is also noticed that IGTHTI show small variations about origin.

The variations of temperature distribution T with distance x is depicted in Fig. 3. It is interesting to observe from Fig. 3, that the behaviour of variations of temperature distribution T with reference to x is same i.e. oscillatory for (IGTHTD, IGTHTI) with difference in their magnitude.

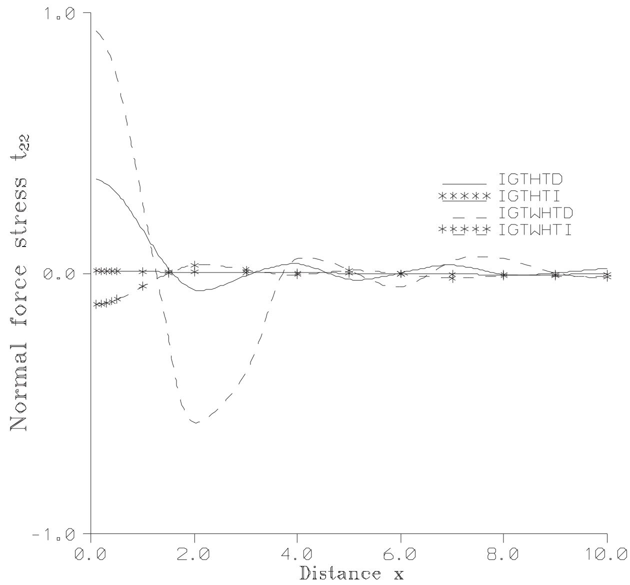

Figure 2 Variations of normal force stress t_{22} with distance x

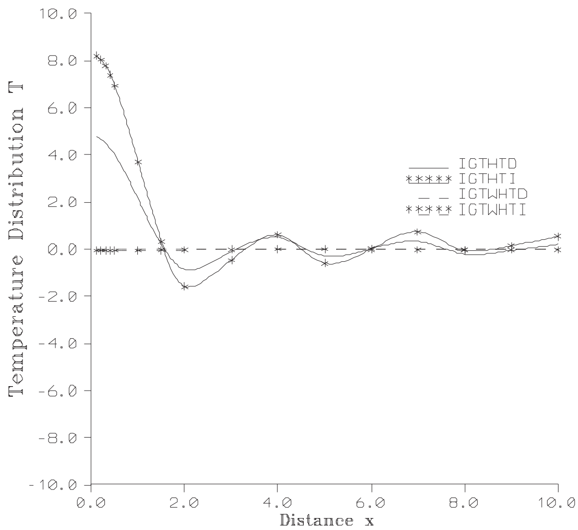

Figure 3 Variations of temparature distribution T with distance x

8. Conclusion

In this paper, we have investigated the effect of hydrostatic initial stress in an infinite isotropic generalized thermoelastic medium of temperature-dependent materials with internal heat source. The problem has been solved numerically using a normal mode analysis. The difference of the field quantities predicted by LS theory are remarkable in the presence and absence of hydrostatic initial stress and temperature-dependent materials. A parameters of hydrostatic initial stress and temperature-dependent materials have a great effect on the distribution of field quantities. The results obtained in this article may offer a theoretical basis and used in engineering, seismology and geophysics.

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Nomenclature

λ, μ: Lame's constants
ρ: density
C^*: specific heat at constant strain
$\overleftarrow{u}$: Displacement vector
t_{ij}: stress tensor
τ_0: relaxation time
t: time
T: absolute temperature
K^*: thermal conductivity
T_0: reference temperature chosen so that $|(T-T_0)/T_0| < 1$
α_t: coefficient of linear thermal expansion

APPENDIX A:

$$\Delta=G_1(D_1-D_2) \quad \Delta_1=G_1G_2, \quad \Delta_2=G_1G_3 \quad \Delta_3=G_1G_4$$
$$\Delta_4=F_1G_2 \quad \Delta_5=F_2G_2 \quad \Delta_6=F_3G_2$$

where:

$$D_1=\xi_1(\beta_2k_3-iaL_1)-\xi_2(\beta_1k_3-iaL_1)$$
$$D_2=\xi_1(iaL_1-\beta_2k_3)-\xi_2(iaL_1-\beta_1k_3)$$
$$G_1=k_1a_1(ia\gamma_2-L_3k_2)+k_2a_2(ia\gamma_1+L_3k_1)$$
$$G_2=ia(f_1\xi_2-f_2)-k_3(N_1\xi_2+\beta_2f_2)$$
$$G_3=\xi_1(N_1k_3+iag_1L_1)+f_2(\beta_1k_3-iaL_1)$$
$$G_4=\beta_1ia(f_2-f_1\xi_2)+\beta_2ia(f_2-f_1\xi_1)+iaN_1(\xi_2-\xi_1)$$
$$F_1=f_2(iaL_1-\beta_2k_3)-\xi_2(N_1k_3+iaf_1L_1)$$
$$F_2=\xi_1(N_1k_3+iaf_1L_1)-f_2(iaL_1-\beta_1k_3)$$
$$F_3=ia\beta_1(f_2-\xi_2f_1)+ia\beta_2(f_2-\xi_1f_1)+iaN_1(\xi_2-\xi_1)$$
$$N_1=-P_1+L_2e^{-(\omega t+iax)}$$