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Propagation of Rayleigh Wave in Two–Temperature Dual–Phase–Lag Thermoelasticity

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The governing equations of transversely isotropic dual–phase–lag two-temperature thermoelasticity are solved for the surface wave solutions. The particular solutions in the half-space satisfy the‘ boundary conditions at a thermally insulated /isothermal stress–free surface of a half–space to obtain the frequency equation of the Rayleigh wave for the cases of coupled thermoelasticity, Lord and Shulman thermoelasticity and dual-phase-lag thermoelasticity. Some particular and special cases are obtained. The numerical values of the non-dimensional speed of the Rayleigh wave are computed and shown graphically against frequency, non-dimensional elastic constant and two–temperature parameter. The effects of frequency, two–temperature and dual–phase–lag are observed on the non-dimensional speed of Rayleigh wave.

Keywords: two–temperature, thermoelasticity, dual–phase–lag, Rayleigh wave, frequency equation.

1. Introduction

Biot [1] developed the classical dynamical coupled thermoelasticity, which consists of hyperbolic-parabolic field equations in a space–time domain. Lord and Shulman [2] and Green and Lindsay [3] extended the classical dynamical coupled theory of thermoelasticity to generalized thermoelasticity theories, which consist of hyperbolic field equations describing heat as a wave. In contrast to Biot's coupled thermoelasticity, the generalized theories of thermoelasticity predict a finite speed of heat propagation. Ignaczak and Ostoja–Starzewski [4] analyzed the above two theories in their book on "Thermoelasticity with Finite Wave Speeds". The representative theories in the range of generalized thermoelasticity are reviewed by Hetnarski and Ignaczak [5]. Tzou [6-8] developed the dual–phase–lag (DPL) thermoelastic model, which considers the interactions between phonons and electrons on the microscopic level as retarding sources causing a delayed response on the macroscopic scale. The dual–phase–lag (DPL) model is a modification of the classical thermoelastic model, where the Fourier law is replaced by an approximation to a modified Fourier law with two different time translations: a phase-lag of the heat flux τ_q and a phase–lag of temperature gradient τ_θ.

Gurtin and Williams [9, 10] proposed the second law of thermodynamics for continuous bodies in which the entropy due to heat conduction was governed by one temperature, that of the heat supply by another temperature. Based on this law, Chen and Gurtin [11] and Chen et al. [12, 13] formulated a theory of thermoelasticity which depends on two distinct temperatures, the conductive temperature Φ and the thermodynamic temperature T. The two–temperature theory involves a material parameter $a^* > 0$. The limit $a^* \to 0$ implies that $\Phi \to T$ and the classical theory can be recovered from two-temperature theory. According to Warren and Chen [14], these two temperatures can be equal in time-dependent problems under certain conditions, whereas Φ and T are generally different in particular problems involving wave propagation. Following Boley and Tolins [15], they studied the wave propagation in the two–temperature theory of coupled thermoelasticity. They showed that the two temperatures T and Φ, and the strain are represented in the form of a traveling wave plus a response, which occurs instantaneously throughout the body. Puri and Jordan [16] studied the propagation of harmonic plane waves in two temperature theory. Youssef [17] developed a theory of two–temperature generalized thermoelasticity. Kumar and Mukhopadhyay [18] extended the work of Puri and Jordan [16] in the context of the linear theory of two–temperature generalized thermoelasticity developed by Youssef [17]. Youssef [19] formulated a theory of two-temperature thermoelasticity without energy dissipation.

The possibility of a wave traveling along the free surface of an elastic half-space such that the disturbance is largely confined to the neighborhood of the boundary was considered by Rayleigh. Lockett [20] discussed the effect produced by the thermal properties of an elastic solid on the form and velocity of Rayleigh waves. Chandrasekharaiah and Srikantaiah [21] studied the temperature rate dependent thermoelastic Rayleigh waves in half-space. Wojnar [22] discussed Rayleigh waves in thermoelasticity with relaxation times. Dawn and Chakraborty [23] studied Rayleigh waves in Green–Lindsay Model of Generalized thermoelastic media. Abouelregal [24] studied Rayleigh waves in an isotropic thermoelastic solid half space using dual–phase–lag model. Singh [25] and Singh and Bala [26] studied the propagation of Rayleigh wave in a two-temperature generalized thermoelastic solid half–space.

Surface wave propagation in elastic solid with thermal effects is of interest in many geophysical, seismological and astrophysical problems. DPL model is used in investigating the micro-structural effect on the behavior of heat transfer. The two–temperature model has been widely used to predict the electron and phonon temperature distributions in ultrashort laser processing of metals. In this paper, we formulated the governing equations of transversely isotropic dual–phase–lag two–temperature thermoelasticity. These equations are solved for general surface wave solutions. A half–space with thermally insulated or isothermal stress–free surface is considered. Keeping in view of rradiation conditions, the particular solutions in the half-space are obtained which satisfy the required boundary conditions at thermally insulated/isothermal stress free surface. Frequency equations of the Rayleigh wave at thermally insulated/isothermal surface of the half-space are derived for the cases of coupled thermoelasticity, Lord and Shulman thermoelasticity and dual–phase–lag thermoelasticity. The numerical values of the non–dimensional speed of the Rayleigh wave are computed and shown graphically against frequency, non–dimensional elastic constant and two–temperature parameter.

2. Basic equations

Following Tzou [6–8] and Youssef [17], the basic equations of motion for transversely isotropic dual–phase–lag thermoelastic model in the absence of body forces and heat sources are:

- The equation of motion:

$$\rho \ddot{u}_i = \sigma_{ji,j} + \rho F_i, \tag{1}$$

- The strain-stress-temperature relation:

$$\sigma_{ij} = c_{ijkl} e_{kl} + a_{ij} \Theta, \tag{2}$$

- The displacement-strain relation:

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{3}$$

- The energy equation:

$$-q_{i,j} = \rho T_0 \dot{S}, \tag{4}$$

- The modified Fourier's law:

$$-K_{ij}(\Phi_{,j} + \tau_\theta \dot{\Phi}_{,j}) = q_i + \tau_q \dot{q}_i, \tag{5}$$

- The entropy-strain-temperature relation:

$$\rho S = \frac{\rho c_E}{T_0}\Theta - a_{ij} e_{ij}, \tag{6}$$

- The relation between two temperatures:

$$\Phi - \Theta = a^* \Phi_{,ii}, \tag{7}$$

where $\Theta = T - T_0$ is small temperature increment, T is the absolute temperature of the medium, T_0 is the reference uniform temperature of the body chosen such that $|\Theta/T_0| \ll 1$, Φ is conductive temperature, a^* is the two–temperature parameter, ρ is the mass density, q_i are components of the heat conduction vector, K_{ij} are the components of the thermal conductivity tensor, c_E is the specific heat at the constant strain, c_{ijkl} is the tensor of the elastic constants, σ_{ij} are the components of the stress tensor, u_i are the components of the displacement vector, e_{ij} are the components of the strain tensor, S is the entropy per unit mass, a_{ij} are constitutive coefficients. τ_q is the phase–lag heat flux and τ_θ is the phase–lag of the gradient of the temperature where $0 \leq \tau_\theta \leq \tau_q$. The DPL thermoelastic theory reduces to the coupled thermoelasticity (CT theory) for $\tau_\theta = 0, \tau_q = 0$ and it reduces to Lord–Shulman (LS) theory of generalized thermoelasticity when we replace $\tau_\theta = 0$ and τ_q by τ_θ.

3. Formulation of the problem and solution

We consider a homogeneous and transversely isotropic two temperature dual–phase–lag thermoelastic solid half-space of an infinite extent with Cartesian coordinates system $(x,\ y,\ z)$, which is previously at uniform temperature. The present study is restricted to the plane strain parallel to $x - z$ plane, with the displacement vector $\mathbf{u} = (u_1, 0, u_3)$. With the help of the equations (1) to (7), we obtain the following equations in x-z plane:

$$c_{11}u_{1,11} + (c_{13} + c_{44})u_{3,13} + c_{44}u_{1,33} - \beta_1\Theta_{,1} = \rho\ddot{u}_1 \tag{8}$$

$$c_{44}u_{3,11} + (c_{13} + c_{44})u_{1,13} + c_{33}u_{3,33} - \beta_3\Theta_{,3} = \rho\ddot{u}_3 \tag{9}$$

$$(1 + \tau_\theta\frac{\partial}{\partial t})[K_1\Phi_{11} + K_3\Phi_{33}] = (1 + \tau_q\frac{\partial}{\partial t})[\rho c_E\dot{\Theta} + \beta_1 T_0\dot{u}_{1,11} + \beta_3 T_0\dot{u}_{3,3}] \tag{10}$$

$$\Phi - \Theta = a^*(\Phi_{,11} - \Phi_{,33}) \tag{11}$$

For thermoelastic surface waves in the half–space propagating in x-direction, the displacement and potential functions (u_1, u_3, Φ) are taken in the following form:

$$\{u_1,\ u_3,\ \Phi\} = \{\phi_1(z), \phi_3(z), \psi(z)\}\exp \iota k(x - ct) \tag{12}$$

Using equation (11) into equations (8) to (10) and then using equation (12), we obtain the following homogenous system of three equations in ϕ_1, ϕ_3 and ψ:

$$[k^2(\zeta - a_1) + D^2]\phi_1 + \iota k(a_2 + 1)D\phi_3 - \iota k(1 + s^* - a^*D^2)\psi = 0 \tag{13}$$

$$\iota k(a_2 + 1)D\phi_1 + [k^2(\zeta - 1) + a_3D^2]\phi_3 - \bar{\beta}D(1 + s^* - a^*D^2)\psi = 0 \tag{14}$$

$$\iota k^3\epsilon\zeta\phi_1 + \bar{\beta}\zeta\epsilon k^2D\phi_3 + [k^2\{\zeta(1 + s^* - a^*D^2) - K_1{}^*\} + K_3{}^*D^2]\psi = 0 \tag{15}$$

where:

$$\epsilon = \frac{\beta_1{}^2T_0}{\rho^2c_Ec_1{}^2} \qquad \zeta = \frac{\rho c^2}{c_{44}} \qquad s^* = a^*k^2 \qquad a_1 = \frac{c_{11}}{c_{44}} \qquad a_2 = \frac{c_{13}}{c_{44}}$$

$$a_3 = \frac{c_{33}}{c_{44}} \qquad \tau^* = \frac{\tau_q + \frac{\iota}{\omega}}{1 - \iota\omega\tau_\theta} \qquad K_1{}^* = \frac{K_1}{c_Ec_{44}\tau^*} \qquad K_3{}^* = \frac{K_3}{c_Ec_{44}\tau^*} \qquad \bar{\beta} = \frac{\beta_3}{\beta_1} \tag{16}$$

The equations (13) to (15) result into following auxiliary equation:

$$(D^6 - AD^4 + BD^2 - C)(\phi_1, \phi_3, \psi) = 0 \tag{17}$$

where:

$$A=\frac{-k^2}{(a_3{K_3}^*-s^*\zeta a_3-\epsilon s^*\zeta\bar{\beta}^2)}[(\zeta-1)({K_3}^*-s^*\zeta)+(\zeta-a_1)(a_3{K_3}^*-s^*\zeta a_3)$$
$$+a_3(\zeta-{K_1}^*)+(a_2+1)^2{K_3}^*+\zeta(s^*a_3+(a_2+1)^2)+\epsilon\zeta(\bar{\beta}^2+s^*\bar{\beta}^2$$
$$+\bar{\beta}(\zeta-a_1)+\bar{\beta}(2+a_2)-a_3)]$$

$$B=\frac{k^4}{(a_3{K_3}^*-s^*\zeta a_3-\epsilon s^*\zeta\bar{\beta}^2)}[(\zeta-a_1)(\zeta-1)({K_3}^*-\zeta)+a_3(\zeta-a_1)(\zeta-{K_1}^*)$$
$$+(\zeta-1)(\zeta-{K_1}^*)+(a_2+1)^2(\zeta-{K_1}^*)+\epsilon(\bar{\beta}(\zeta-a_1)+\bar{\beta}\zeta+\bar{\zeta}(a_2+1)$$
$$+\zeta a_3s^*+s^*\zeta(\zeta-1))]$$

$$C=\frac{-k^6}{(a_3{K_3}^*-s^*\zeta a_3-\epsilon s^*\zeta\bar{\beta}^2)}[(\zeta-a_1)(\zeta-1)(\zeta-{K_1}^*)-\epsilon\zeta(\zeta-1)]$$

The general solutions of equation (17) are:

$$u_1(z)=\left[\sum_{i=1}^{3}A_i\exp(-m_iz)+\sum_{i=1}^{3}{A_i}^{'}\exp(m_iz)\right]\exp ik(x-ct) \tag{18}$$

$$u_3(z)=\left[\sum_{i=1}^{3}B_i\exp(-m_iz)+\sum_{i=1}^{3}{B_i}^{'}\exp(m_iz)\right]\exp ik(x-ct) \tag{19}$$

$$\Phi(z)=\left[\sum_{i=1}^{3}C_i\exp(-m_iz)+\sum_{i=1}^{3}{C_i}^{'}\exp(m_iz)\right]\exp ik(x-ct) \tag{20}$$

where $A_i, B_i, C_i, {A_i}^{'}, {B_i}^{'}, {C_i}^{'}$ are arbitrary constants and m_i are the roots of the equation:

$$m^6-Am^4+Bm^2-C=0 \tag{21}$$

The equation (21) is cubic in m^2 and its roots ${m_1}^2$, ${m_2}^2$, ${m_3}^2$ are related as:

$${m_1}^2+{m_2}^2+{m_3}^2=A \tag{22}$$

$${m_1}^2{m_2}^2+{m_2}^2{m_3}^2+{m_3}^2{m_1}^2=B \tag{23}$$

$${m_1}^2{m_2}^2{m_3}^2=C \tag{24}$$

In general, the roots m_i, (i = 1, 2, 3) are complex and here we are considering surface waves, without loss of generality, we can assume $Re(m_i)>0$. We choose only that form of m_i which satisfies the radiation condition:

$$u_1(z), u_3(z), \Phi(z)\rightarrow\ 0 \text{ as } z\rightarrow\infty \tag{25}$$

With the help of condition (25), the solutions (18) to (20) reduce to particular solutions in half–space $z\geq 0$ as:

$$u_1(z)=\sum_{i=1}^{3}A_i\exp(-m_iz)\exp ik(x-ct) \tag{26}$$

$$u_3(z) = \sum_{i=1}^{3} F_i A_i \exp(-m_i z) \exp ik(x - ct) \tag{27}$$

$$\Phi(z) = \sum_{i=1}^{3} F_i{}^* A_i \exp(-m_i z) \exp ik(x - ct) \tag{28}$$

where $B_i = F_i A_i$, $C_i = F_i{}^* A_i$ and:

$$F_i = [\frac{\frac{m_i}{k}[\bar{\beta}(\rho c^2 - c_{11} + \frac{m_i{}^2}{k^2} c_{44}) + (c_{13} + c_{44})]}{\iota[\bar{\beta}\frac{m_i{}^2}{k^2}(c_{13} + c_{44}) - ((\rho c^2 - c_{44}) + \frac{m_i{}^2}{k^2} c_{33})]}] \quad (i = 1, 2, 3)$$

$$\frac{F_i{}^*}{k} = -\frac{\iota\epsilon c_{44}[\bar{\beta}((\rho c^2 - c_{11}) + c_{44}\frac{m_i{}^2}{k^2}) + (c_{13} + c_{44})]}{\beta_1[(1 + s^* - a^* D^2)[\epsilon\bar{\beta} + (c_{13} + c_{44})] + (c_{13} + c_{44})(-\frac{K_1{}^*}{\zeta} + \frac{K_1{}^*}{\zeta}\frac{m_i{}^2}{k^2})]}] \quad (i = 1, 2, 3)$$

4. Derivation of frequency equation

The mechanical and thermal conditions at the free surface $z = 0$ are:

- Vanishing of the normal stress component:

$$\sigma_{zz} = o, \tag{29}$$

- Vanishing of the tangential stress component:

$$\sigma_{zx} = 0, \tag{30}$$

- Vanishing of the normal heat flux or temperature component:

$$\frac{\partial \Theta}{\partial z} + h\Theta = 0, \tag{31}$$

where $h \to 0$ corresponds to the thermally insulated surface, $h \to \infty$ corresponds to the isothermal surface, and:

$$\sigma_{33} = c_{33} u_{3,3} + c_{13} u_{1,1} - \beta_3[\Phi - a^*(\Phi_{11} + \Phi_{33})]$$

$$\sigma_{31} = c_{44}(u_{1,3} + u_{3,1})$$

$$\Theta = \Phi - a^* \Phi_{,ii}$$

Making use of solutions (26) to (28) in boundary conditions (29) to (31), we obtain the following homogeneous system of three equations in A_1, A_2 and A_3:

$$\sum_{i=1}^{3} [c_{33} m_i F_i - \iota k c_{13} + \beta_3(1 + a^* k^2 - a^* m_i{}^2) F_i{}^*] A_i = 0 \tag{32}$$

$$\sum_{i=1}^{3} (m_i - \iota k F_i) A_i = 0 \tag{33}$$

$$\sum_{i=1}^{3}[(1+a^*k^2-a^*{m_i}^2)m_i-h]{F_i}^*A_i=0 \tag{34}$$

For non–trivial solution of equations (32) to (34), the determinant of the coefficients must vanish, i.e.:

$$\sum_{i=1}^{3}[(1+a^*k^2+{m_i}^2)m_i-h]X_i=0 \tag{35}$$

where:

$$\begin{aligned}
X_1 &= {F_1}^*[\{c_{33}m_2F_2-\iota kc_{13}+\beta_3(1+a^*k^2+{m_3}^2){F_2}^*\}(m_3-\iota kF_3)\\
&\quad -\{c_{33}m_3F_3-\iota kc_{13}+\beta_3(1+a^*k^2+{m_2}^2){F_3}^*\}(m_2-\iota kF_2)],\\
X_2 &= {F_2}^*[\{c_{33}m_3F_3-\iota kc_{13}+\beta_3(1+a^*k^2+{m_1}^2){F_3}^*\}(m_1-\iota kF_1)\\
&\quad -\{c_{33}m_1F_1-\iota kc_{13}+\beta_3(1+a^*k^2+{m_3}^2){F_1}^*\}(m_3-\iota kF_3)],\\
X_3 &= {F_3}^*[\{c_{33}m_1F_1-\iota kc_{13}+\beta_3(1+a^*k^2+{m_2}^2){F_1}^*\}(m_2-\iota kF_2)\\
&\quad -\{c_{33}m_2F_2-\iota kc_{13}+\beta_3(1+a^*k^2+{m_3}^2){F_2}^*\}(m_1-\iota kF_1)].
\end{aligned}$$

The equation (35) is the frequency equation of Rayleigh wave in two–temperature dual–phase–lag transversely isotropic thermoelastic model.

5. Particular cases

Case (I): Thermally insulated case ($h \to 0$)
The frequency equation (35) reduces to:

$$\sum_{i=1}^{3}[(1-a^*(-k^2+{m_i}^2))m_i]X_i=0. \tag{36}$$

Case (II): Isothermal case ($h \to \infty$)
The frequency equation (35) reduces to:

$$X_1+X_2+X_3=0. \tag{37}$$

Case (III): Dual–phase–lag transversely isotropic thermoelastic case
In absence of two temperature i.e $a^*=0$, the frequency equations (36) and (37) reduce to:

$$m_1{X_1}^*+m_2{X_2}^*+m_3{X_3}^*=0 \tag{38}$$

$${X_1}^*+{X_2}^*+{X_3}^*=0 \tag{39}$$

where:

$${X_1}^*={F_1}^*(c_{33}m_2F_2-\iota kc_{13}+\beta_3{F_2}^*)(m_3-\iota kF_3)-(c_{33}m_3F_3-\iota kc_{13}+\beta_3{F_3}^*)(m_2-\iota kF_2)$$

$${X_2}^*={F_2}^*(c_{33}m_3F_3-\iota kc_{13}+\beta_3{F_3}^*)(m_1-\iota kF_1)-(c_{33}m_1F_1-\iota kc_{13}+\beta_3{F_1}^*)(m_3-\iota kF_3)$$

$${X_3}^*={F_3}^*(c_{33}m_2F_2-\iota kc_{13}+\beta_3{F_2}^*)(m_3-\iota kF_3)-(c_{33}m_3F_3-\iota kc_{13}+\beta_3{F_3}^*)(m_2-\iota kF_2)$$

6. Special case

In the absence of thermal effect, i.e., when $\beta_3 = 0,\ \beta_1 = 0,\ K_1 = K_3 = 0$, then the frequency equation (35) reduces to:

$$(m_1 - h)X_1{}^{**} + (m_2 - h)X_2{}^{**} + (m_3 - h)X_3{}^{**} = 0 \tag{40}$$

where:

$$X_1{}^{**} = F_1{}^*(c_{33}m_2F_2 - \iota kc_{13})(m_3 - \iota kF_3) - (c_{33}m_3F_3 - \iota kc_{13})(m_2 - \iota kF_2)$$

$$X_2{}^{**} = F_2{}^*(c_{33}m_3F_3 - \iota kc_{13})(m_1 - \iota kF_1) - (c_{33}m_1F_1 - \iota kc_{13})(m_3 - \iota kF_3)$$

$$X_3{}^{**} = F_3{}^*(c_{33}m_2F_2 - \iota kc_{13})(m_3 - \iota kF_3) - (c_{33}m_3F_3 - \iota kc_{13})(m_2 - \iota kF_2)$$

Further, if we put $c_{11} = \lambda + 2\mu,\ c_{13} = \lambda,\ c_{44} = \mu$ in equation (40), the equation (40) reduces to:

$$(2 - \frac{c^2}{c_2{}^2})^2 = 4(1 - \frac{c^2}{c_1{}^2})^{1/2}(1 - \frac{c^2}{c_2{}^2})^{1/2}, \tag{41}$$

which is frequency equation of Rayleigh wave at free surface of an elastic solid half–space.

7. Numerical example

For most of materials, ϵ is small at normal temperature. For $\epsilon \ll 1$ and using the equations (22) to (24), we obtain the following approximated relations:

$$m_1{}^2 + m_2{}^2 + m_3{}^2 = \frac{-k^2}{(a_3K_3{}^* - s^*\zeta a_3)}[(\zeta - 1)(K_3{}^* - s^*\zeta) + (\zeta - a_1)(a_3K_3{}^* - s^*\zeta a_3)$$

$$+a_3(\zeta - K_1{}^*) + (a_2 + 1)^2K_3{}^* + \zeta(s^*a_3 + (a_2 + 1)^2)]$$

$$m_1{}^2m_2{}^2 + m_2{}^2m_3{}^2 + m_3{}^2m_1{}^2 = \frac{k^4}{(a_3K_3{}^* - s^*\zeta a_3)}[(\zeta - a_1)(\zeta - 1)(K_3{}^* - \zeta)$$

$$+a_3(\zeta - a_1)(\zeta - K_1{}^*) + (\zeta - 1)(\zeta - K_1{}^*) + (a_2 + 1)^2(\zeta - K_1{}^*)]$$

$$m_1{}^2m_2{}^2m_3{}^2 = \frac{-k^6}{(a_3K_3{}^* - s^*\zeta a_3)}[(\zeta - a_1)(\zeta - 1)(\zeta - K_1{}^*)]$$

and the approximated roots as:

$$\frac{(m_1)^2}{k^2} \equiv -(\zeta - a_1) = \frac{c_{11} - \rho c^2}{c_{44}} \tag{42}$$

$$\frac{(m_2)^2}{k^2} \equiv -\frac{(\zeta - 1)}{a_3} = \frac{c_{44} - \rho c^2}{c_{33}} \tag{43}$$

$$\frac{(m_3)^2}{k^2} \equiv -\frac{(\zeta - K_1{}^*)}{K_3{}^*} = \frac{K_1 - \rho c^2 c_E \tau^*}{K_3 - a^* k^2 c_E \tau^*} \tag{44}$$

For numerical computation of the non–dimensional speed of propagation of Rayleigh wave, we restricted to the case of thermally insulated surface only. Therefore, the frequency equation (36) for thermally insulated case is approximated with the help of equations (42) to (44) and is solved numerically to obtain the speeds of propagation of Rayleigh wave for certain ranges of frequency and non-dimensional constants.

Following physical constants of single crystal of zinc are considered (Chadwick and Seet [27]):

$$c_{11} = 1.628 \times 10^{11}\, Nm^{-2}, \quad c_{33} = 1.562 \times 10^{11}\, Nm^{-2},$$

$$c_{13} = 0.508 \times 10^{11}\, Nm^{-2}, \quad c_{44} = 0.385 \times 10^{11}\, Nm^{-2}$$

$$\beta_1 = 5.75 \times 10^{6}\, Nm^{-2}deg^{-1}, \quad \beta_3 = 5.17 \times 10^{6}\, Nm^{-2}deg^{-1},$$

$$K_1 = 1.24 \times 10^{2}\, Wm^{-1}deg^{-1}, \quad K_3 = 1.34 \times 10^{2}\, Wm^{-1}deg^{-1},$$

$$C_e = 3.9 \times 10^{2}\, JKg^{-1}deg^{-1}, \quad \rho = 7.14 \times 10^{3}\, Kgm^{-3},$$

$$T_0 = 296\, K, \quad \tau_q = 0.005\, s, \quad \tau_\theta = 0.0005\, s.$$

For the above physical constants, the non-dimensional speed of propagation $\frac{\rho c^2}{c_{44}}$ is computed and plotted for the range 5 Hz $\leq \omega \leq$ 30 Hz of frequency, when $a^* = 0.5$. For the range 5 Hz $\leq \omega \leq$ 10 Hz, the speed decreases very slowly and thereafter, it increases sharply for the remaining range. The comparison of solid curve (DPL theory) and dashed curves (CT theory and LS theory) in Fig. 1, shows the effect dual phase lag on the speed at various values of frequency.

The non–dimensional speed of propagation $\frac{\rho c^2}{c_{44}}$ is computed for the range $1 \leq \frac{c_{11}}{c_{44}} \leq 2$ of non-dimensional elastic constant, when $a^* = 0.5$. The speed decreases sharply with the increase in value of non-dimensional constant $\frac{c_{11}}{c_{44}}$. The comparison of solid and dashed curves in Fig. 2, shows the effect of dual phase lag on speed at various values of non–dimensional elastic constant.

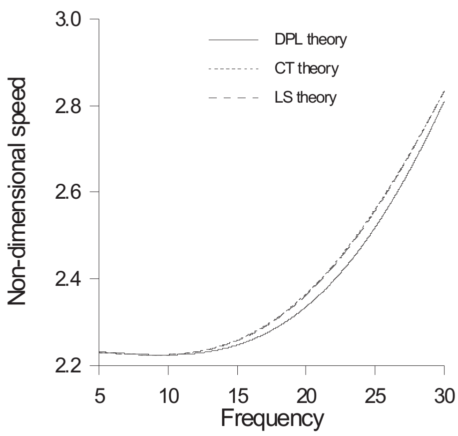

Figure 1

The non–dimensional speed of propagation $\frac{\rho c^2}{c_{44}}$ is also computed for the range $0 \leq a^* \leq 1$ of two–temperature parameter, when $\omega = 5`Hz$. For the range $0 \leq a^* \leq 1$, the non-dimensional speed of propagation increases and thereafter it oscillates. The comparison of solid and dashed curves in Fig. 3, shows the effect of dual phase lag on speed at various values of two–temperature parameter.

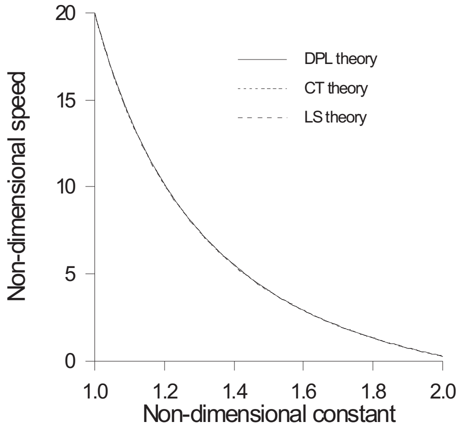

Figure 2

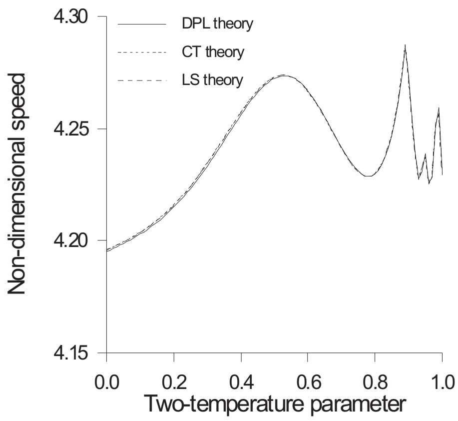

Figure 3

8. Conclusion

The general surface wave solutions of the governing equations of transversely isotropic dual phase lag two–temperature thermoelasticity are obtained. With the help of suitable radiation conditions, the general solutions are reduced to particular solutions in the half–space. The particular solutions satisfy the required boundary conditions at stress free thermally insulated or isothermal surface and we obtain the frequency equation of Rayleigh wave. Some particular cases of the frequency equation are derived. In absence of thermal and anisotropy parameters, the frequency equation reduces to the classical isotropic elastic case. For numerical purpose, the frequency equation is approximated for the case of small thermal coupling and then solved numerically for a particular model of the material. The non–dimensional speed of propagation is plotted against the frequency, non–dimensional elastic and two–temperature parameters. The numerical results indicate the effects of dual–phase–lag, frequency and two–temperature on the non–dimensional speed of propagation.

References

[1] **Biot, M. A.**: Thermoelasticity and irreversible thermodynamics, *J. Appl. Phys.*, 2, 240–253 **1956**.

[2] **Green, A. E. and Lindsay, K. A.**: Thermoelasticity, *J. Elasticity*, 2, 1–7, **1972**.

[3] **Lord, H. and Shulman, Y.**: A generalised dynamical theory of thermoelasticity, *J. Mech. Phys. Solids*, 15, 299–309, **1967**.

[4] **Ignaczak, J. and Ostoja–Starzewski, M.**: Thermoelasticity with Finite Wave Speeds, *Oxford University Press*, **2009**.

[5] **Hetnarski, R. B. and Ignaczak, J.**: Generalized thermoelasticity, *J. Thermal Stresses*, 22, 451-476 **1999**.

[6] **Tzou, D.Y.**: A unified approach for heat conduction from macro to micro–scales, *J. Heat Transfer*, 117, 8–16 **1995**.

[7] **Tzou, D.Y.**: Experimental support for the lagging behavior in heat propagation, *J. Thermophys.*, Heat Transfer, 9, 686–693, **1995**.

[8] **Tzou, D.Y.**: Macro– to microscale heat transfer: The lagging behavior, Washington, *DC: Taylor and Francis*, **1996**.

[9] **Gurtin, M. E. and Williams, W. O.**: On the Clausius–Duhem Inequality, *Zeitschrift fr Angewandte Mathematik und Physik*, 17, 626–633, **1966**.

[10] **Gurtin, M. E. and Williams, W. O.**: An Axiomatic Foundation/or Continuum Thermodynamics, *Arch. Ration. Mech. Analysis*, 26, 83–117, **1967**.

[11] **Chen, P. J. and Gurtin, M. E.**: On a theory of heat conduction involving two–temperatures, *Zeitschrift fr Angewandte Mathematik und Physik*, 19, 614–627, **1968**.

[12] **Chen, P. J., Gurtin, M. E. and Williams, W. O.**: A note on non–simple heat conduction, *Zeitschrift fr Angewandte Mathematik und Physik*, 19, 969-970 **1968**.

[13] **Chen, P. J., Gurtin, M. E. and Williams, W. O.**: On the thermodynamics of non–simple elastic materials with two temperatures, *Zeitschrift fr Angewandte Mathematik und Physik*, 20, 107–112 **1969**.

[14] **Boley, B. A. and Tolins, I. S.**: Transient coupled thermoplastic boundary value problems in the half–space, *J. Appl. Mech.*, 29, 637-646 **1962**.

[15] **Warren, W. E. and Chen, P. J.**: Wave propagation in the two-temperature theory of thermoelasticity, *Acta Mech.*, 16, 21–33, **1973**.

[16] **Puri, P. and Jordan, P. M.**: On the propagation of harmonic plane waves under the two-temperature theory, *Int. J. Engng. Sci.*, 44, 1113–1126, **2006**.

[17] **Youssef, H. M.**: Theory of two–temperature generalized thermoelasticity, *IMA J. Appl. Math.*, 71, 383–390, **2006**.

[18] **Kumar, R. and Mukhopadhyay, S.**: Effects of thermal relaxation time on plane wave propagation under two–temperature thermoelasticity, *Int. J. Engng. Sci.*, 48, 128–139, **2010**.

[19] **Youssef, H. M.**: Theory of two–temperature thermoelasticity without energy dissipation, *J. Thermal Stresses*, 34, 138–146, **2011**.

[20] **Lockett, F. J.**: Effect of the thermal properties of a solid on the velocity of Rayleigh waves, *J. Mech. Phys. Solids*, 7, 71–75, **1958**.

[21] **Chandrasekharaiah, D. S. and Srikantaiah, K. R.**: On temperature rate dependent thermoelastic Rayleigh waves in half–space, *Gerlands Beitrage Zur Geophysik*, 93, 133–141 **1984**.

[22] **Wojnar, R.**: Rayleigh waves in thermoelasticity with relaxation times, *In International Conference on Surface Waves in Plasma and Solids*, Yugoslavia, World Scientific, Singapore, **1985**.

[23] **Dawn, N. C. and Chakraborty, S. K.**: On Rayleigh waves in Green-Lindsay model of generalized thermoelastic media, *Indian J. Pure Appl. Math.*, 20, 276–283 **1988**.

[24] **Abouelregal, A. E.**: Rayleigh waves in a thermoelastic solid half space using dual–phase–lag model, *Int. J. Engng. Sci.*, 49, 781–791, **2011**.

[25] **Singh, B.**: Propagation of Rayleigh wave in a two-temperature generalized thermoelastic solid half–space, *ISRN Geophysics*, 2013, ID 857937, doi:10.1155/2013/857937, **2013**.

[26] **Singh, B. and Bala, K.**: On Rayleigh Wave in two-temperature generalized thermoelastic medium without energy dissipation, *Applied Mathematics*, 4, 107–112, **2013**.

[27] **Chadwick, P. and Seet, L. T. C.**: Wave propagation in a transversely isotropic heat conducting elastic material, *Mathematica*, 17, 255–274, **1970**.