Mechanics and Mechanical Engineering
Vol. 21, No. 1 (2017) 171–180
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H_∞ Active Control of a Vehicle Suspension System Exited by Harmonic and Random Roads

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Received (20 October 2016)
Revised (26 November 2016)
Accepted (18 December 2016)

This paper proposes a controller based upon H_∞ control approach in order to improve the vehicle performance under two different road profiles. In this study, there are two control targets that they are car body travel and suspension deflection. In fact, H_∞ controller is responsible for minimizing the infinity norm of two subsystems. The first one is from car body travel to road disturbance and second is from suspension deflection to road disturbance. These two control targets must be improved by a logical control input that is determined by H_∞ control approach. In order to improve the performance of the quarter–car, weighting functions are also defined. Disturbance that is the system input is considered as two types of road profiles, harmonic and random. The results show that the H_∞ controller is able to improve the quarter–car performance for both roads. In addition, the sensitivity analysis is done to show that the active suspension system is able to work when sprung mass changes as may be occurred when passengers added.

Keywords: active suspension, H_∞ control approach, road profile, quarter car model.

1. Introduction

There are three main types of vehicle suspensions. Passive suspension is the common case which includes normal spring and damper. Although most companies use this type, it is not capable to operate in a broad range of frequencies. The second one is the semi-active suspension which controls the vehicle suspension in low frequency very well; however, it has some limitations in high frequencies. The third one is active suspension which can improve the ride comfort of the vehicles properly. Although there is a contradiction between ride comfort and handling (road holding), this type can improve ride comfort and road holding simultaneously. Active suspension can improve the performance of vehicles very well although it needs more effort. As well, the active suspension works over a wide range of frequencies.

Researchers have proposed different approaches in order to control vehicle suspension. Active suspension can improve suspension performances, such as ride comfort and handling which can be used in suspension design [1–3]. Both vehicle dynamics and suspension equations were considered to investigate the effect of active suspension on ride comfort and handling [4]. Fuzzy logic controller was also used to improve vehicle rid–comfort [5]. Some optimal preview controllers for the active suspension with integrated constraint or nonlinear magneto-rheological damper have been also reported [6-8]. Also, a fuzzy adaptive sliding mode controller proposed for an air spring active suspension [9]. There are also some papers which proposed various control methods to isolate the vibration in a vehicle suspension system with nonlinear parameters. A hybrid fuzzy logic approach which combines fuzzy logic and PID controllers is designed for reducing the vibration levels of passenger seat and vehicle body [10]. Considering the hysteretic nonlinear stiffness and the square damping of a half-car vehicle model and using an optimal control to obtain better performances is done by researchers [11]. H_∞ or μ synthesis controller has been proposed in order to improve the performance of a quarter-car vehicle with semi-active suspension [12-15]. The influence of tire damping on mixed H_2/H_∞ sysnthesis of an active suspension has been investigated [16]. Also, a robust H_∞ controller for an active suspension under non-stationary running can also be used [17].

In the previous work, the effect of road profile type has not been studied in the presence of H_∞ controller. In this paper, a controller based on H_∞ approach is designated to improve vehicle performance for a two degree–of–freedom car model. Two road profiles are considered here; harmonic and random functions Weighting functions are also determined that has significant influence on the system performance. In addition, the capability of designated H_∞ controller is investigated where the number of passengers changes.

2. The quarter–car vehicle model

In this study, a simple quarter–car suspension model which consists of one–fourth of the body mass, suspension components and one wheel are depicted in Fig. 1. This model is applied extensively in many researches and has many essential characteristics of a real suspension system.

The equations of motion for this model which consists of unsprung and sprung masses are given by:

$$m_s\ddot{z}_s(t) + c_s[\dot{z}_s(t) - \dot{z}_{us}(t)] + k_s[z_s(t) - z_{us}(t)] = u(t) \tag{1}$$

$$\begin{aligned} &m_{us}\ddot{z}_{us}(t) + c_s[\dot{z}_{us}(t) - \dot{z}_s(t)] + k_s[z_{us}(t) - z_s(t)] \\ &+k_t[z_{us}(t) - z_r(t)] = -u(t) \end{aligned} \tag{2}$$

where m_s denotes the sprung mass and represents the car chassis; m_{us} denotes the unsprung mass, which represents the wheel assembly; c_s and k_s are damping and stiffness of the before using controller, respectively; k_t plays the role of pneumatic tire; $z_s(t)$ and $z_{us}(t)$ are the displacements of the sprung and unsprung masses, respectively, z_r is the road disturbance, *u(t)* denotes the external input force of the suspension system.

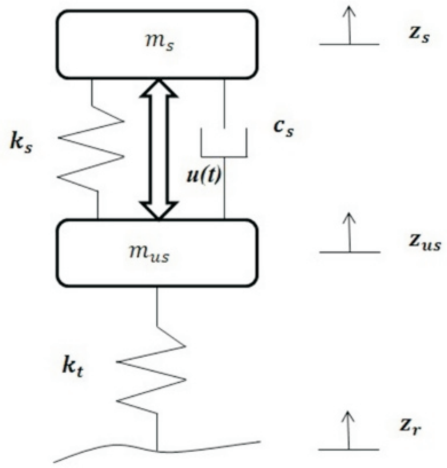

Figure 1 Quarter–car suspension model

By choosing the state variables properly as:

$$x_1(t) = z_s(t) \qquad x_2(t) = \dot{z}_s(t) \qquad x_3(t) = z_{us}(t) \qquad x_4(t) = \dot{z}_{us}(t) \tag{3}$$

where x_1*(t)* represents sprung mass displacement, x_2*(t)* represent the sprung mass speed, x_3*(t)* is unsprung mass displacement, x_4*(t)* represents the unsprung mass speed, and defining:

$$x_p(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^T \qquad w(t) = \dot{z}_r(t) \tag{4}$$

The state space form of the system can be written as:

$$\begin{aligned} \dot{x}_p(t) &= Ax(t) + Bu(t) + B_w w(t) \\ y_p(t) &= Cx(t) + Du(t) \end{aligned} \tag{5}$$

where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & \frac{k_s}{m_s} & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{c_s}{m_{us}} & \frac{c_s}{m_{us}} & -\frac{(k_s+k_t)}{m_{us}} & -\frac{c_s}{m_{us}} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_{us}} \end{bmatrix} \qquad B_w = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{m_{us}} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

are constant matrices, Y_p is quarter–car outputs that are car body travel and suspension deflection, *u(t)* is an actuator force which can be generated in many ways as using hydraulic actuator placed between two masses.

3. Road profile

Road profiles have a great effect on the suspension response. If road profiles are defined properly, the results will be more exact. Two types of road profiles are used here as inputs to the car model. The first one will be shaped by harmonic functions, while the second one is designed based on random input.

3.1. *Harmonic road profile*

Sub- Equation (6) shows the harmonic road profile:

$$w(t) = 0.05\cos(2\pi t)\sin(0.6\pi t) \tag{6}$$

Harmonic road profile may be usually used in the simulation to verify the stability and capability of the designed control system, besides the system response status.

3.2. *Random road profile*

Random road profile is also considered here as may be closer to irregular bumpiness with respect to harmonic profile. Fig. 2 illustrates the random road file which has been used here as input.

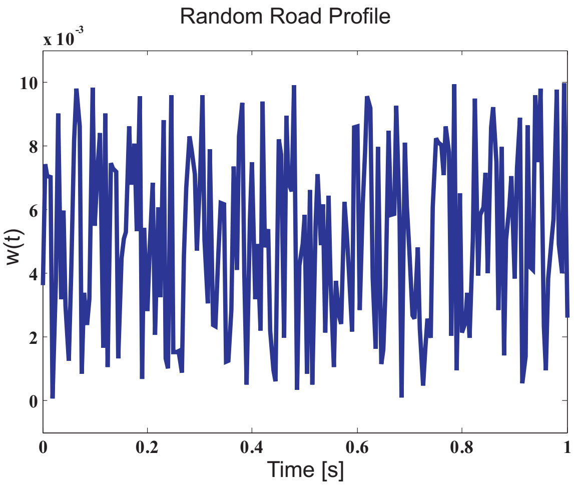

Figure 2 Random road profile

4. Weighting functions

It is required that in the H_∞ framework to use weighting functions to compromise different performance objectives. The performance objectives of a feedback system can be usually determined in terms of requirements on the sensitivity functions and/or complementary sensitivity functions or in terms of some other closed loop transfer functions [18]. The advantage of applying weighted performance in multivariable system design is firstly, some components of a vector signal are usually more important than others, secondly, measuring each signal will not be in the same unit. For instance, some components of the output error signal may be measured in terms of length, and others may be measured in terms of voltage. Therefore, weighting functions play an essential role to make these components comparable. In this study, weighting functions are:

a) W_u is a rational transform function in order to limit the input force above 50 rad/s:

$$W_u = 8\frac{s+50}{s+500}$$

b) W_{x1} is a rational transform function that keep the car deflection small over the desired frequency ranges. In fact, it guarantees car ride comfort:

$$W_{x_1} = 10\frac{10\pi}{s+10\pi}$$

c) W_{x1-x3} is a rational transform function that keeps the suspension deflection small over the desired frequency ranges:

$$W_{x_1-x_3} = 30\frac{1}{0.1s+1}$$

d) W_{ref} is also used as a static gain in order to reduce road disturbance effect:

$$W_{ref} = 0.01$$

It is necessary to rewrite all weighting functions in state space form as can be determined in time domain. It is assumed that W_u, W_{x1} and W_{x1-x3} have following state space realization:

$$W_u = \begin{bmatrix} A_u & B_u \\ C_u & D_u \end{bmatrix} \qquad W_{x_1} = \begin{bmatrix} A_{x_1} & B_{x_1} \\ C_{x_1} & 0 \end{bmatrix} \qquad W_{x_1-x_3} = \begin{bmatrix} A_{x_1-x_3} & B_{x_1-x_3} \\ C_{x_1-x_3} & 0 \end{bmatrix}$$

where the values are considered here as:

$A_u = -500$ $B_u = 1$ $C_u = -3600$ $D_u = 8$
$A_{x_1} = -31.42$ $B_{x_1} = 1$ $C_{\mathrm{x}_1} = 314.16$
$A_{x_1-x_3} = -10$ $B_{x_1-x_3} = 1$ $C_{x_1-x_3} = 300$

5. Controller

H_∞ controller is designed here in order to improve the performance of quarter–car vehicle. Accordingly, the system is shown in block diagram form as Fig. 3.

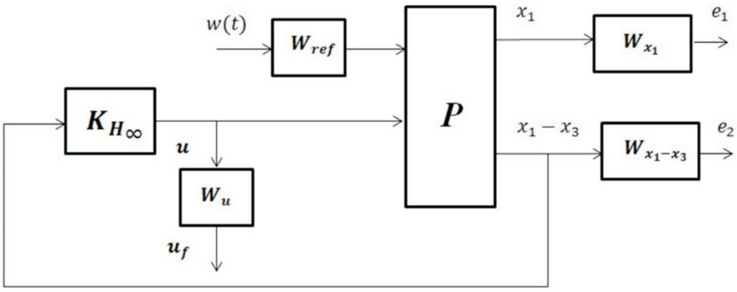

Figure 3 Control for vehicle suspension system

In Fig. 3 P represents the quarter car suspension model, K_{H_∞} is a controller which is designed by H_∞ approach. e_1 and e_2 are first and third outputs after influencing weighting functions. In practice, the suspension deflection can be measured by acoustic or radar transmitter/receiver; while the velocity is typically obtained by integrating the acceleration which is measured using accelerometer. Moreover, u_f is the control input that is limited around 50 rad/s.

The earlier work on H_∞ control approach has been formulized the robust control problem [19–23]. These researches show that an optimal H_∞ controller can be calculated both numerically and theoretically, then the controller will be achieved by suboptimal process. The suboptimal H_∞control problem can be arranged by following inequality:

$$\|F_l(G, K_{H_\infty})\|_\infty \langle \gamma \tag{7}$$

In Equation (8) G is the system by considering all weighting functions. F_1 (G, K$_{H_\infty}$) is a lower Linear Fractional Transformation (LFT) of K around G. γ also denotes a real positive number. The algorithms to solve this suboptimal problem can be noted in some references [24].

6. Results

The performance of the quarter–car vehicle with passive suspension and with active suspension under two types of road excitation is evaluated here via simulation. The values of quarter–car suspension model have been used as the following, which are derived from a medium sedan vehicle:

$$m_s = 290 \text{ kg} \qquad m_{us} = 59 \text{ kg}$$
$$k_s = 16182 \text{ N/m} \qquad k_t = 190000 \text{ N/m} \qquad c_s = 1000 \text{ N.s/m}$$

By using weighting functions defined before, the control input u and the outputs limited in a proper frequency range. The controller designed in Section 5 is applied in the simulation. It is noted that the results of active systems have been compared to passive which has no control force and shown in figures.

6.1. *Responses to Harmonic Road Profile*

The responses of the system under harmonic road profile are shown in Figure 4. The responses of the car body travel, suspension deflection and control input are depicted in Figure 4 (a)–(c), respectively. As it is illustrated in these Figures active suspension is able to stabilize and reduce the maximum value of car body travel and suspension deflection. In addition, force input has acceptable values and is stabilized in a good manner:

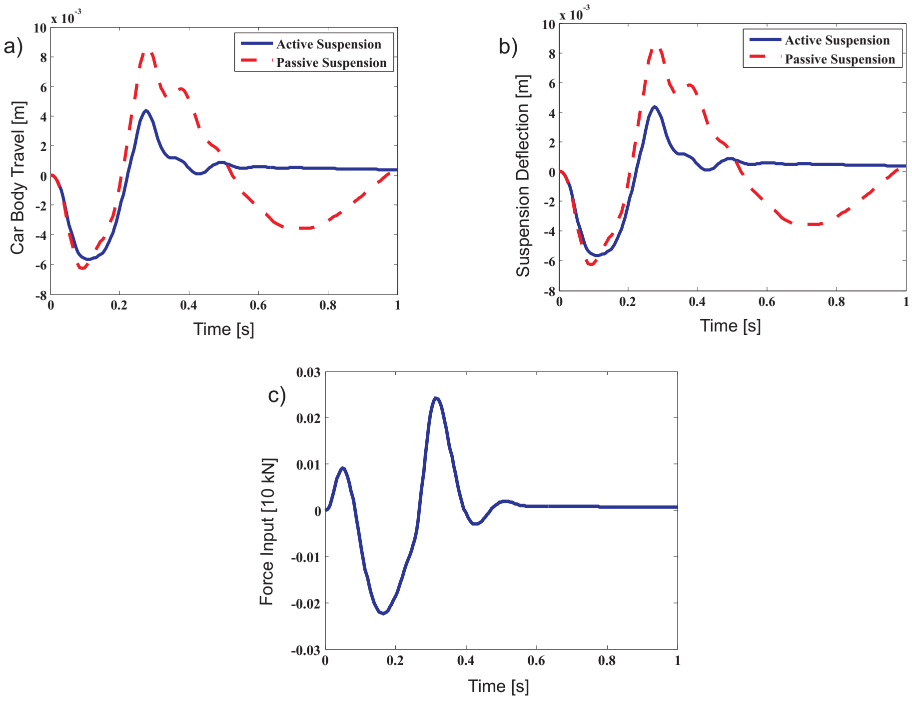

Figure 4 Quarter-car response to harmonic road profile

6.2. *Responses to Random Road Profile*

The responses of system under random input are depicted in Figure 5(a)–(c). The behavior of irregular road profiles are used in many road simulations by researchers. As may be observed, the car body travel and suspension deflection are improved well by using H_∞ control approach. In addition, the controller is able to both stabilize and guarantee performance simultaneously. Furthermore, the force input has been calculated and illustrated in Figure 5(c). The comparison shows that the required values of control force of random profile is less than that of harmonic road profile.

7. Sensitivity analysis

H_∞ controller has also capability to isolate road profiles vibration with respect to changes of sprung mass according to calculation. Changes in the value of sprung mass can be occurred with adding the number of passengers. In this study, the mass of each passenger is considered here as 75 Kg.

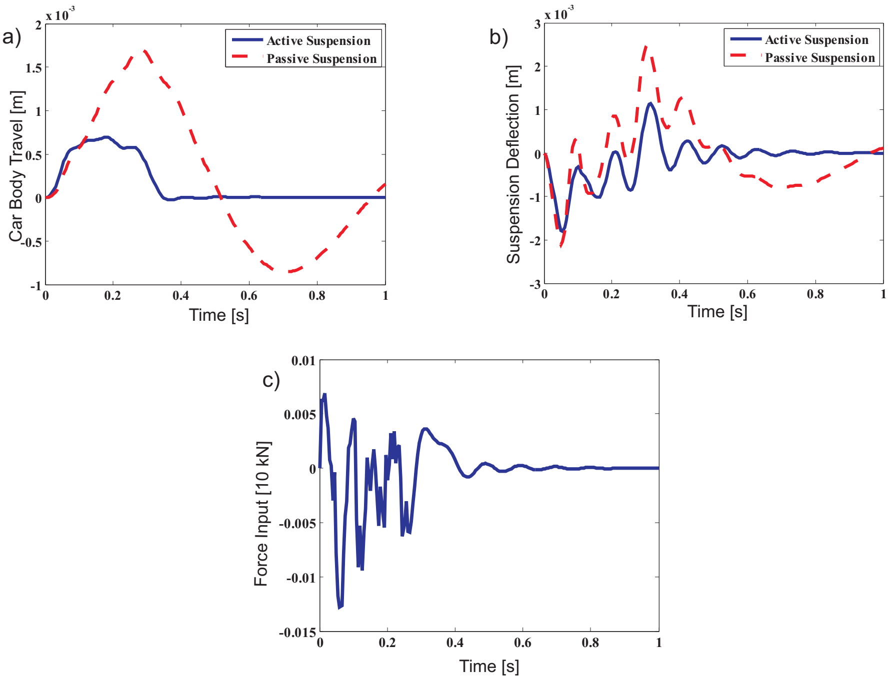

Figure 5 Quarter–car response to random road profile

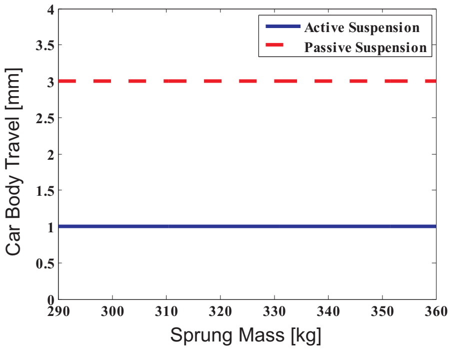

Figure 6 Car body travel versus sprung mass (Harmonic road profile)

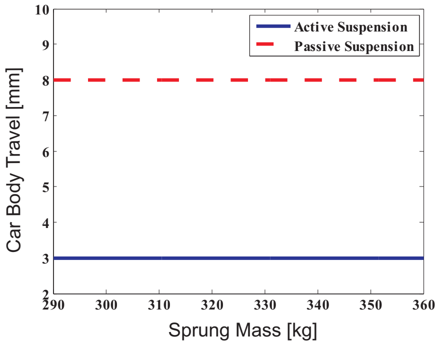

Figure 7 Car body travel versus sprung mass (Random road profile)

Figs 7 and 8 show the car body travel versus mass of passengers. As can be seen from these figures, the car body travel is much less than passive suspension when H_∞ controller is applied.

8. Conclusion

In this paper, an H_∞controller is proposed in order to isolate the two types of road disturbances for a quarter-car vehicle to improve the performance of suspension system. The quarter-car suspension is excited by two different types of road profiles that are harmonic road profile and random one. The control targets car body travel and suspension deflection are stabilized in short time in comparison with passive suspension. In addition, the control inputs not only have proper values, but also stabilized shortly. The result show that the H_∞controller creates about 3 times less travel of car body for one to four passengers which creates better comfort.

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