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Accuracy and Repeatability Tests on 6D Measurement Arm

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We present kinematic structure of measurement arm along with its construction restrains originating from using only accelerometers for determining relative positions of links. Method of calculating position of arm's end in relation to its base basing on orientation of individual links is presented. Accuracy and repeatability tests were performed using custom made test stand. The description of test stand, measurements procedures for last link changing its position in Cartesian directions and orientation in vertical axis is also included in the paper. These changes were justified in context of links orientation in relations to global coordinate system. Obtained tests results are presented and analyzed.

Keywords: 6D measurement arm, MEMS accelerometers.

1. Introduction

In the presented paper a 6D measurement arm equipped with FXLS8471Q accelerometers is described. These 3-axis sensors are characterised by low power consumption and small size (4 mm x 4 mm x 1,5 mm). The communication is made by I2C protocol, a commonly known communication interface used by microprocessor systems and peripheral devices. This communication method features are: simplicity and low cost of data transmission with high frequency. It makes these sensor suitable to be used in small sizes measurement arm.

It's been proven that it's possible to conduct measurements of position changes using MEMS accelerometers [1-4]. The presented paper is an extension of works conducted in this field. According to project assumptions, the main restrain of the construction is to maintain small size while preserving high measurement accuracy. The angular displacement in relation to gravity vector of the measurement arm can be determined using accelerometers placed on each link. The paper shows a method of calculating position of arm's end basing solely on data from accelerometers. It also shows results of accuracy and repeatability tests that were made.

2. Measurement arm kinematics description

The designed arm is characterized by serial kinematic structure that allows an effective measurement of orientation of individual links in relation to the gravity vector. During static measurements, an (fixed) accelerometer shows components of gravity vector in sensor's coordinate system. For two links connected with joint which axis is tilted from vertical direction, the tilt angle can be determined basing on accelerometer data. In case where multiple links are serially connected using such joints and all links are equipped with 3-axis accelerometers, it is possible to determine tilt angles of all joints. Obviously, the necessary condition is to tilt axis of all joints from vertical direction. Such kinematic chain enables creation of measurement arm with 6 degrees of freedom with the possibility to determine position and orientation of the last link in coordinate system of the base link. Figure 1 presents a kinematic structure and an overlook of arm consisting of 7 links and 6 rotary joints. All links were created using 3D print technique. In MATLAB software, a point cloud was created that was used to determine an area in which the specified orientation of last link is preserved. An optimal reference point placed in the work area that was used as a beginning point for accuracy and repeatability tests was determined using this method.

Planes limiting areas with the highest density of points showing position of arm's end with specified orientation were determined. They were used to determine the area providing an optimal range of last link's displacements while maintaining its desired orientation. It resulted in the possibility to perform another functional tests of the arm in the optimal range.

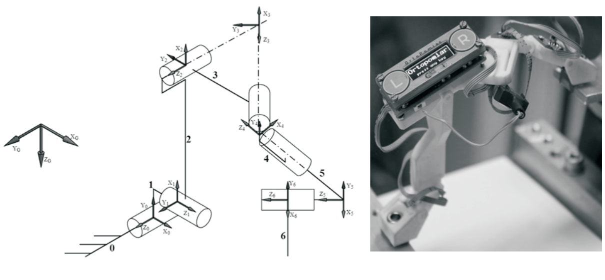

Figure 1 Kinematics of measurement arm and an overview of 3D print

3. Last link position calculation method description

The base and reference point for the measurement is a coordinate system of the first link of measurement arm (0) (Fig. 1). The idea behind the device is to perform precise measurements with changing orientation of whole arm. It enforces the necessity to transfer from global coordinate system to the system defined by first link during the measurement of orientation of another links.

Table 1 Denawit-Hartenberg parameters [6] describing measurement arm according to Fig. 1

Link no.	θ_i [^{o}]	d_i [mm]	a_i [mm]	α_i [^{o}]
1	90	0	10	90
2	0	13	50	-90
3	0	-30	0	45
4	-45	53.93	0	-90
5	-90	-34.13	0	45
6	0	16	0	0

Input value for the calculations is an vector of accelerometer data i:

$$Y_{S_i} = \begin{bmatrix} a_{x_i} \\ a_{y_i} \\ a_{z_i} \end{bmatrix} \quad (1)$$

The result of multiplication (2) of accelerometer data i (1) and raw calibration matrix K_{Si} is an acceleration vector in the coordinate system of accelerometer i after calibration (3):

$$C_i = K_{Si} \begin{bmatrix} Y_i \\ 1 \end{bmatrix} \quad (2)$$

$$C_i = \begin{bmatrix} c_{x_i} \\ c_{y_i} \\ c_{z_i} \end{bmatrix} \quad (3)$$

The acceleration vector C_i after being multiplied by (4) the rotation matrix $R_{CZ\ i}^{DHi}$ (from sensor system i to Denawit-Hartenberg system of link i) is used to determine Earth acceleration vector in in Denawit-Hartenberg system of link i (5).

$$E_i = R_{CZ\ i}^{DHi}\ K_{Si} \begin{bmatrix} Y_i \\ 1 \end{bmatrix} = \ K_i \begin{bmatrix} Y_i \\ 1 \end{bmatrix} \quad (4)$$

where: $K_i = R_{CZ\ i}^{mDHi} K_{Si}$ – calibration matrix in Denawit-Hartenberg system:

$$E_i = \begin{bmatrix} e_{x_i} \\ e_{y_i} \\ e_{z_i} \end{bmatrix} \quad (5)$$

the determination procedure of angles θ_i between individual links of arm uses vectors E_i and E_{i-1} and their relations (6, 7):

$$E_{i-1} = R_{i-1}^{i} E_i \quad (6)$$

$$E_i = \left(R_{i-1}^i\right)^T E_{i-1} \tag{7}$$

Matrix R_{i-1}^i (8) is a rotation matrix (accordant to Denawit-Hartenberg notation) between two following links:

$$R_{i-1}^i = \begin{bmatrix} cos\theta_i & -sin\theta_i c\alpha_i & sin\theta_i sin\alpha_i \\ sin\theta_i & cos\theta_i c\alpha_i & -cos\theta_i sin\alpha_i \\ 0 & sin\alpha_i & cos\alpha_i \end{bmatrix} = \left(R_i^{i-1}\right)^T \tag{8}$$

It results in equations (9, 10):

$$e_{x_{i-1}} = cos\theta_i e_{x_i} + sin\theta_i(-cos\alpha_i e_{y_i} + sin\alpha_i e_{z_i}) \tag{9}$$

$$e_{y_{i-1}} = sin\theta_i e_{x_i} - cos\theta_i(-cos\alpha_i e_{y_i} + sin\alpha_i e_{z_i}) \tag{10}$$

From the equation system (9, 10) by using determinant method (11, 12, 13) values $\sin\theta_i$ and $\cos\theta_i$ are being derived (14).

$$W = e_{x_i}^2 + (-\cos\alpha_i e_{y_i} + \sin\alpha_i e_{z_i})^2 \tag{11}$$

$$W_s = e_{y_{i-1}} e_{x_i} + e_{x_{i-1}} (-\cos\alpha_i e_{y_i} + \sin\alpha_i e_{z_i}) \tag{12}$$

$$W_c = e_{x_{i-1}} e_{x_i} - e_{y_{i-1}} (-\cos\alpha_i e_{y_i} + \sin\alpha_i e_{z_i}) \tag{13}$$

$$\sin\theta_i = \frac{W_s}{W}; \cos\theta_i = \frac{W_c}{W} \tag{14}$$

In next step angle θ_i is being determined using $a\tan 2$ function (15).

$$\theta_i = a\tan 2\left(\sin\theta_i, \cos\theta_i\right) = a\tan 2(W_s, W_c) \tag{15}$$

Basing on kinematics and information on the orientation of individual links of the arm [7] provided by the accelerometers and by using product of six transformations of homogenous transformation matrix and Denawit-Hartenberg rotation matrix T_O^i R_O^i from base system to i along with homogenous transformation matrix T_G^i R_G^i and Denawit-Hartenberg rotation matrix from Earth to i the position of last link of the arm is being calculated.

The only condition to perform accurate measurement is to fix first link of the arm to measured object, to keep the arm stable and to tilt its joints axis from vertical direction.

4. Test on accuracy and repeatability

Accuracy and repeatability tests were made using microscope MWDC equipped with cross measurement table with 150 mm x 50 mm motion range (Fig. 2) and the possibility to move in two perpendicular axis (1) and (2), X and Z in this case with resolution of 0.01 mm. The cross table supports a precise rotary table. It enables the rotation around vertical Y axis (4) with resolution 0,05 deg. Additionally, to the microscope rotary table a miniaturised precise table MX100-SS GMT with micrometre screw (3) was added. It enables the movement along vertical axis Y with resolution of 0.01 mm.

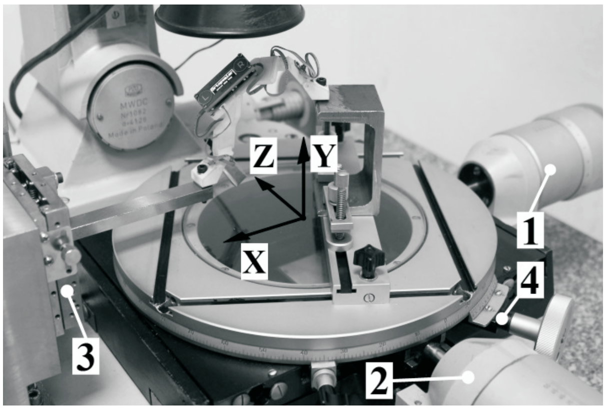

Figure 2 The test stand: adjustment lever 1) for X direction, 2) for Z direction, 3) for Y direction, 4) for angle around Y axis

There were two stages of the measurements: accuracy measurement and repeatability of position and displacement measurement. In the first stage, the arm was placed in the base position with fixed and defined orientation of arm's base and last link in reference position. The reference position enables the maximum displacement in all axis while maintaining the defined orientation of last link, identical as orientation of base link. During the measurement, last link of the arm was displaced in the range obtained during the test on work range. For each given position, the measurement was repeated 21 times in order to obtain actual information on the repeatability of the tested arm.

Table 2 The results of repeatability test of base position

Base position ,,0"	Reference measurement	Mean value of measurements	Mean absolute error of measurement $\overline{\Delta}$	Standard deviation σ
X [mm]	40,78 mm	43,61 mm	0,17 mm	0,006 mm
Y [mm]	75,00 mm	74,81 mm	0,19 mm	0,002 mm
Z [mm]	-6,40 mm	-5,71 mm	0,68 mm	0,016 mm
α [^{o}]	-29,55 ^{o}	-29,67 ^{o}	0,12 ^{o}	0,029 ^{o}

The presented results (Tab. 2) show that the obtained position repeatability results are very good for all the directions. Standard deviation doesn't exceed 0.02 mm and 0,003^o which shows the high accuracy of the FXLS8471Q sensors and also their characteristics stability. On this basis it can be assumed that it's possible to obtain much better results.

In the second stage the accuracy of positioning and displacement of an arm was tested. The measurement was done by displacing last link from the base position along all axis (X, Y, Z) in sequence while maintaining the constant change of dis-

placement value. Next, the measurements were made with changing orientation of the last link that was achieved by rotating around vertical axis Y (it's marked as an α angle). After placing an arm in desired position, 2000 samples were collected and averaged using a dedicated software. For all specified positions, the measurement was repeated 3 times in order to obtain information on its repeatability.
During whole measurement process the positions of joints angles was verified to make sure that they were not vertical as it could have corrupted the results.

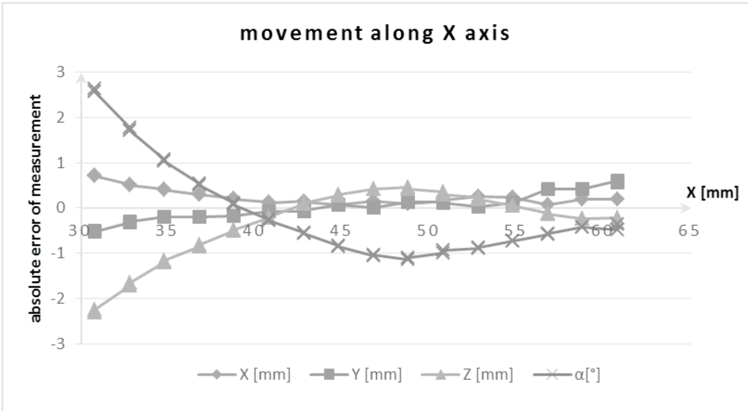

Figure 3 Graph of absolute error of the position for the movement along X axis

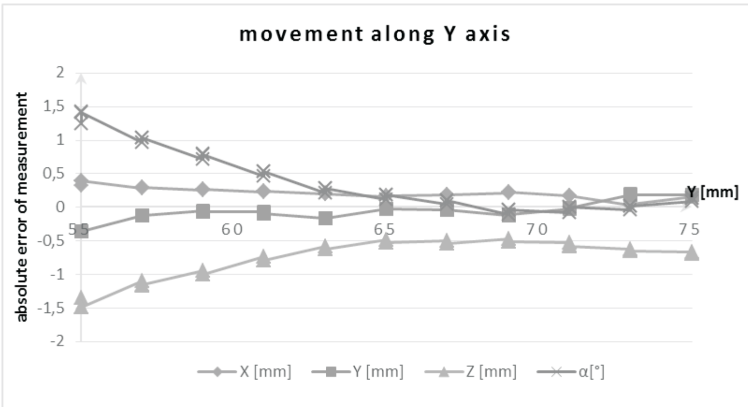

Figure 4 Graph of absolute error of the position for the movement along Y axis

Table 3 Accuracy of position measurement for the movement along X axis

Reference measurement [mm]	Mean value of position measurement [mm]	Mean absolute error of position measurement $\overline{\Delta}$ [mm]	Standard deviation σ [mm]
30,78	30,07	0,72	0,006
32,78	32,26	0,52	0,010
34,78	34,37	0,41	0,004
36,78	36,49	0,30	0,009
38,78	38,58	0,21	0,004
40,78	40,66	0,12	0,003
42,78	42,63	0,15	0,005
44,78	44,71	0,07	0,003
46,78	46,63	0,15	0,001
48,78	48,69	0,09	0,005
50,78	50,62	0,16	0,006
52,78	52,53	0,25	0,003
54,78	54,54	0,24	0,001
56,77	56,71	0,06	0,001
58,77	58,58	0,19	0,000
60,77	60,57	0,20	0,004

Table 4 The results of repeatability test of base position

Given displacement along X axis [mm]	Mean value of displacement measurement [mm]	Mean absolute error of displacement measurement $\overline{\Delta}$ [mm]	Standard deviation σ [mm]
0,00	0,00	-	-
2,00	2,20	-0,20	0,010
4,00	4,30	-0,30	0,004
6,00	6,42	-0,42	0,009
8,00	8,51	-0,51	0,004
10,00	10,59	-0,60	0,003
12,00	12,57	-0,57	0,005
13,99	14,64	-0,65	0,003
15,99	16,56	-0,57	0,001
17,99	18,62	-0,63	0,005
19,99	20,55	-0,56	0,006
21,99	22,46	-0,47	0,003
23,99	24,47	-0,48	0,001
25,99	26,64	-0,65	0,001
27,99	28,52	-0,53	0,000
29,99	30,50	-0,52	0,004

Table 5 Accuracy of position measurement for the movement along Y axis

Reference measurement [mm]	Mean value of position measurement [mm]	Mean absolute error of position measur. $\overline{\Delta}$t [mm]	Standard deviation σ [mm]
55,00	55,36	-0,36	0,004
57,00	57,12	-0,12	0,004
59,00	59,05	-0,05	0,006
61,00	61,06	-0,06	0,016
63,00	63,16	-0,16	0,002
65,00	65,03	-0,03	0,002
67,00	67,04	-0,04	0,001
69,00	69,12	-0,12	0,004
71,00	71,02	-0,02	0,006
73,00	72,81	0,19	0,004
75,00	74,81	0,19	0,002

Table 6 Accuracy of displacement measurement for the movement along Y axis

Given displacement along Y axis [mm]	Mean value of displacement measurement [mm]	Mean absolute error of displacement measurement $\overline{\Delta}$ [mm]	Standard deviation σ [mm]
0,00	0,00	-	-
2,00	1,76	0,24	0,004
4,00	3,70	0,30	0,006
6,00	5,71	0,29	0,016
8,00	7,80	0,20	0,002
10,00	9,67	0,33	0,002
12,00	11,68	0,32	0,001
14,00	13,76	0,24	0,004
16,00	15,66	0,34	0,006
18,00	17,46	0,54	0,004
20,00	19,46	0,54	0,002

It can be seen that the lowest accuracy of position (Figs. 3, 4, 5, 6) occurs along Z axis and for rotation by α angle. It differs greatly from other two directions. In X and Y directions the mean value of position error is under 1mm (Tabs. 3, 4 and 5, 6). Yet, the maximum absolute error is significant which shows great differences in obtained position values with relative value of standard deviation σ remaining under 0.04 mm. It leads to a conclusion that for some arm positions there are kinematics nonlinearities originating from faulty cooperation of individual

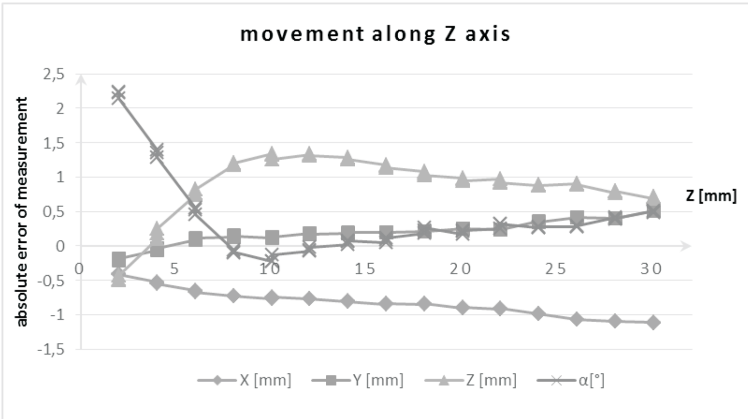

Figure 5 Graph of absolute error of the position for the movement along Z axis

Table 7 Accuracy of position measurement for the movement along Z axis

Reference measurement [mm]	Mean value of position measurement [mm]	mean absolute error of position measurement $\overline{\Delta}$ [mm]	Standard deviation σ [mm]
2,00	2,50	-0,50	0,037
4,00	3,90	0,10	0,075
6,00	5,26	0,74	0,042
8,00	6,83	1,17	0,019
10,00	8,66	1,34	0,043
12,00	10,67	1,33	0,015
14,00	12,72	1,28	0,011
16,00	14,82	1,18	0,025
18,00	16,91	1,09	0,032
20,00	19,01	0,99	0,021
22,00	21,03	0,97	0,027
24,00	23,12	0,88	0,006
26,00	25,10	0,90	0,003
28,00	27,23	0,77	0,016
30,00	29,31	0,69	0,014

Table 8 Accuracy of displacement measurement for the movement along Z axis

Given displacement along Z axis [mm]	Mean value of displacement measurement [mm]	Mean absolute error of displacement measurement $\overline{\Delta}$ [mm]	Standard deviation σ [mm]
0,00	0,00	-	-
2,00	1,40	0,60	0,075
4,00	2,76	1,24	0,042
6,00	4,33	1,67	0,019
8,00	6,16	1,84	0,043
10,00	8,17	1,83	0,015
12,00	10,22	1,78	0,011
14,00	12,32	1,68	0,025
16,00	14,42	1,58	0,032
18,00	16,51	1,49	0,021
20,00	18,53	1,47	0,027
22,00	20,62	1,38	0,006
24,00	22,60	1,40	0,003
26,00	24,73	1,27	0,016
28,00	26,82	1,18	0,014

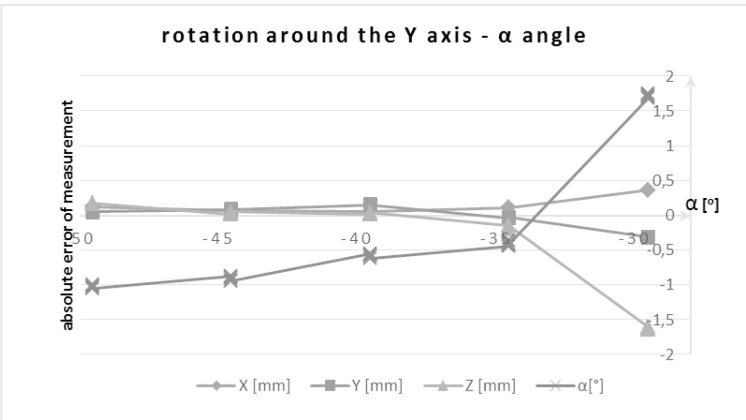

Figure 6 Graph of absolute error of the position for the rotation around Y axis by α angle

Table 9 Accuracy of position and displacement measurement for the rotation around Y axis by α angle

Given rotation around Y angle by α angle [rad]	Mean value of displacement measurement [rad]	Mean absolute error of displacement measurement $\overline{\Delta}$ [rad]	Standard deviation σ [rad]
0,00	0,00	-	-
-5,00	-3,79	-1,21	0,108
-10,00	-8,82	-1,18	0,032
-15,00	-13,47	-1,53	0,043
-20,00	-18,37	-1,63	0,030

links which results from their manufacturing technology limitations.

In turn, for position changes along Z axis and rotation around Y axis by α angle, the error values are much bigger (Tabs. 5, 6 and 9, 10). Such discrepancy may originate from arm's kinematics which enforces much bigger angle displacements between individual links with relatively low linear displacement of arm's end.

5. Summary

The obtained data can be used to prove the proper functioning of an arm with 6 degrees of freedom basing only on the accelerometers. The tests results show high repeatability of position measurement which confirms the necessary quality of used sensors. The differences in the actual and measured accuracy of position and displacement result from the discrepancy of mathematical model of the arm and actual sizes of individual links and from their deformations during the work. Additionally, the print quality and assembly accuracy of individual links has a great influence. All inaccuracies originating from assembly method and from fixing of an arm in base slot summarise with every link. It results in bigger deviation from theoretical position. As for the last link, the error might reach even several mm.
In order to make accuracy better, the corrections need to be done in the mathematical model that would take into account the differences between defined and actual values and also the quality of manufacturing need to be improved. After completing these tasks it will be possible to obtain the accuracy at a level of tenths of millimetre with the same sensors.

An actual construction differs greatly from the theoretical model which is a result of manufacturing technology. Calculation method that bases on theoretical parameters, shows biggest inaccuracies for measurements along Z axis and for rotations around vertical Y axis. It results from backlash friction forces in the joints that along with high susceptibility of the links result in great deformation of whole measurement arm. In further works, the dependencies between Denawit-Hartenberg parameters of the arm and obtained measurement accuracy will be determined. The construction of an arm will be modified in order to lower the negative friction and links susceptibility influence on the measurements. Also, the shape of the arm will be optimised in order to maximise the accuracy of measurements while extending the workspace.

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