Mechanics and Mechanical Engineering
Vol. 21, No. 3 (2017) 591–601

Propagation of Cracks in Reinforced Concrete Beams Cracked and Repaired by Composite Materials

Djamila Benarbia
Mohamed Benguediab
Faculty of Technology
Djillali Liabes University of Sidi Bel Abbes
Sidi Bel Abbes City, Ben Mhidi. B.P. 89, Algeria
d_benarbia@yahoo.com
benguediab_m@yahoo.fr

Received (29 June 2017)
Revised (6 July 2017)
Accepted (11 August 2017)

Strengthening and repairing existing reinforced concrete structures is often more economical and sustainable than rebuilding them. Today the most commonly used techniques based on reparation by externally bonded Carbon Fiber Reinforced Polymers (CFRP). However, bonding concrete beams, particularly damaged beams, suffer from the pre-existing of open cracks at the bottom face of the beams. This paper presents an investigation by finite element method using the general purpose FE software Abaqus to study the flexural behavior of initially damaged concrete beams repaired with FRP plates. In this study, it is aimed to simulate the phenomenon of propagations of cracks where the beam is initially loaded to introduce damage, then, after bonding the FRP plates. The linear elastic fracture mechanics (LEFM) approach is adopted to pursue the stress intensity factor's evolution in 3-points bending before and after reparation of RC beams. Many parameters were taken account, such us the thickness of the adhesive layer and reinforcing plate, the stiffness, and young's modulus. Results were identified and discussed.

Keywords: propagations of cracks, Carbon Fiber Reinforced Polymers (CFRP), Linear Elastic Fracture Mechanics (LEFM).

1. Introduction

Structures can be damaged due to over-loading, earthquakes, fire, blast loading, mistakes in design calculations, corrosion of reinforcement and improper concrete mix design. Damage can be defined as the change in structural performance, which can be identified in terms of discrete cracks or a weak zone formation. Undetected and not repaired damage may lead to structural failure demanding costly repair and huge loss of lives. It is important to study the behaviour of damaged RC members, since it involves huge expenditure to demolish and reconstruct them. Therefore it is necessary to increase the service life and load carrying capacity of damaged original structures. Extensive experimental studies on the behaviour of initially damaged reinforced concrete repaired with FRP plates can be found in the literature [1–13]. The general objective of these works is to understand the complex behaviour of bonded beams with FRP plates and their failure mechanism. Furthermore, several works of theoretical nature have been proposed in the literature to study some specific problems such as the concrete-FRP interface stress behaviour [14–20]. These studies are based on analytical solutions which can provide practical equations and information that are useful for design purposes. However, due to the complex behaviour of the strengthened RC beams, the analytical procedures are generally based on elastic analysis and simplified hypothesis that make them incapable to simulate the real behaviour of such structures. Unlike analytical methods, the numerical ones (particularly finite element-based strategies) have been successfully used to simulate this kind of complex structures. In the literature, one can find a lot of finite element (FE) models including different constitutive material laws for studying the behaviour of strengthened concrete beams plates [21–23]. However, simulations of the full loading history of initially damaged concrete beams repaired with composite plates are limited, and contented by study the concrete damage before application of repairing system [24] and the FRP concrete interface behaviour [25]. The present work is aimed to study the propagation of cracks in concrete structures reinforced before and after repair by FRP plates. Many parameters were taken account, such us the thickness of the adhesive layer and reinforcing plate, the stiffness, and young's modulus.

2. Linear elastic fracture mechanic

Application of the fracture mechanics to concrete [26] has been intensively studied. It was recently realized that the fracture process zone was created ahead of the crack in concrete. As a result, the non-linear fracture mechanics [27] was introduced instead of the linear elastic fracture mechanics (LEFM). Although most efforts have been lately devoted to study on fracture damage behaviour of concrete [28, 29], linear elastic fracture mechanics seems to be still useful for studying the failure of concrete.

The theory of linear-elastic fracture mechanics (LEFM) is integrated using an analytical approach occurring that solid bodies containing cracks can be characterized by defining a state of stress near a crack tip and the energy balance coupled with fracture. Introducing the Westergaard's and Airy's complex function and will allow the development a significant stress analysis at the crack tip (Fig. 1).

The Airy's complex function and Westergaard's complex function are, respectively:

$$\Phi = Re\overline{Z'} + yIm\overline{Z} \tag{1}$$

$$Z\left(z\right) = ReZ + iImZ \tag{2}$$

where:
$Re\overline{Z'}$ – real part,
$Im\overline{Z}$ – imaginary part,
Z – analytic stress function,
$i = \sqrt{-1} \qquad i^2 = -1.$

For instance, Irwin treated the singular stress field by introducing a quantity [30] known as the stress intensity factor, which is used as the controlling parameter for evaluating the critical state of a crack.

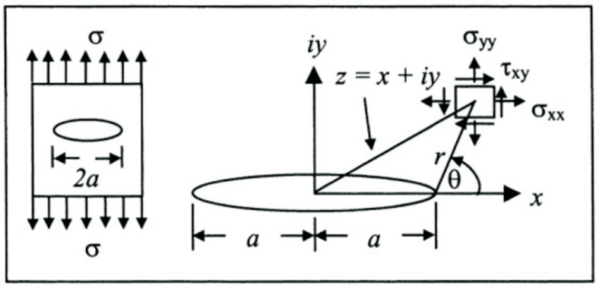

Figure 1 The crack tip stress field in complex coordinates

3. Stress intensity factor

The mechanical behaviour of a solid containing a crack of a specific geometry and size can be predicted by evaluating the stress intensity factors (KI, KII, and KIII) shown in Fig. 2.

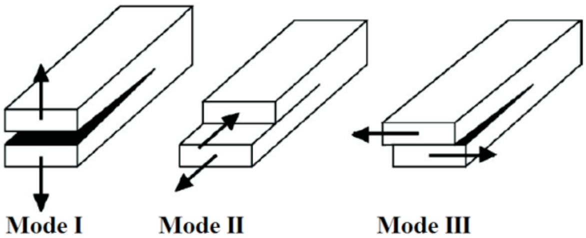

Figure 2 Three basic modes of fracture propagation

If crack growth occurs along the crack plane perpendicular to the direction of the applied external loading mode, then the stress intensity factors are defined according to the American Society for Testing Materials (ASTM) E399 Standard Test Method as:

$$K_I = \lim_{r \to 0} \left(\sigma_{yy} + \sqrt{2\pi r} \right) f_I\left(\theta\right) \tag{3}$$

$$K_{II} = \lim_{r \to 0} \left(\tau_{xy} + \sqrt{2\pi r} \right) f_{II}\left(\theta\right) \tag{4}$$

$$K_{III} = \lim_{r \to 0} \left(\tau_{yz} + \sqrt{2\pi r} \right) f_{III}\left(\theta\right) \tag{5}$$

Here: $f_I(\theta)$, $f_{II}(\theta)$ and $f_{III}(\theta)$ are trigonometric functions to be derived analytically.

4. Application

The basic assumptions are as follows:
– The studied material follows a stress-deformation of elastic linear type; non-linear areas that can develop at the tip of the crack are neglected before the size of the crack and the structure being studied;
– The cracked body is homogeneous and isotropic.

4.1. *Model studied*

The model studied in this document is a reinforced concrete beam with length of 3000 mm where a crack initiated in its tension fiber (bottom), and submitted on 3-point bending that the load is applied in the middle of the beam.

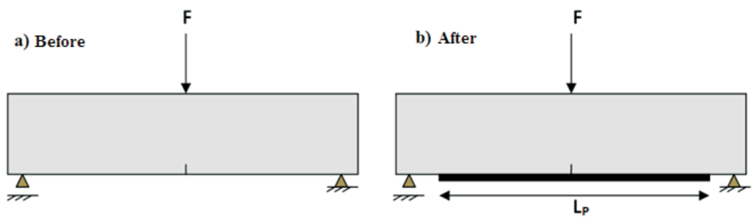

Figure 3 Cracked beam loaded in bending 3-points

Geometrical and mechanical characteristics of materials used are given in the table below:

Table 1 Geometrics and mechanicals properties of materials

Element	*Width (mm)*	*Thickness (mm)*	*Young's modulus (GPa)*	*Poisson's ratio*	*Shear modulus (GPa)*
(RC) Beam	200	300	30	0,18	-
Adhesive	200	2	3	0,35	-
CFRP	200	4	140	0,28	5
GFRP	200	4	50	0,28	5
Steel	200	4	200	0,3	-

The analysis is done using the ABAQUS software in a three-dimensional medium and get to have satisfactory results were used to finite element quadrilateral isoparametric at 8 nodes with a fine mesh in the area of cracking and in areas of high stress and a mesh coarse and medium in the other regions.

5. Discussion and results

The principle of this study is to follow the evolution of the stress intensity factors in the case of a notched beam bending 3-points before and after repair by

CFRP bonding (comparative study), the influence of the various parameters such as the thickness of the adhesive layer and reinforcing plate are taken into account (parametric study).

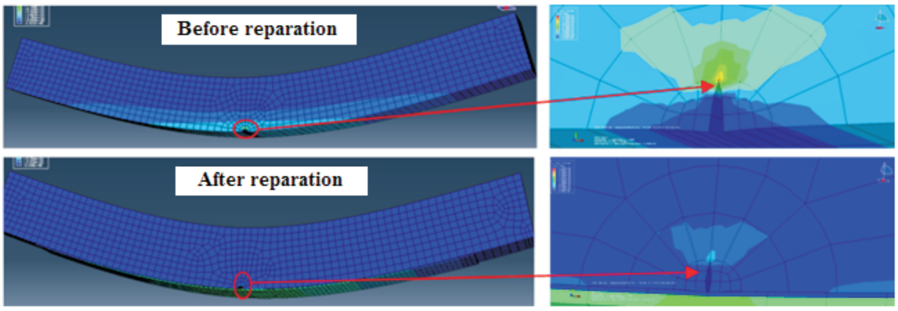

Figure 4 Effect of CFRP on the behaviour of cracked beams

5.1. *Comparative study*

Figure 5 shows a comparison chart of the variation of the values of the intensity factors constraints K_I and K_{II} depending on the evolution of the load applied in a beam notched and reinforced by composite materials and that hacked and not enhanced.

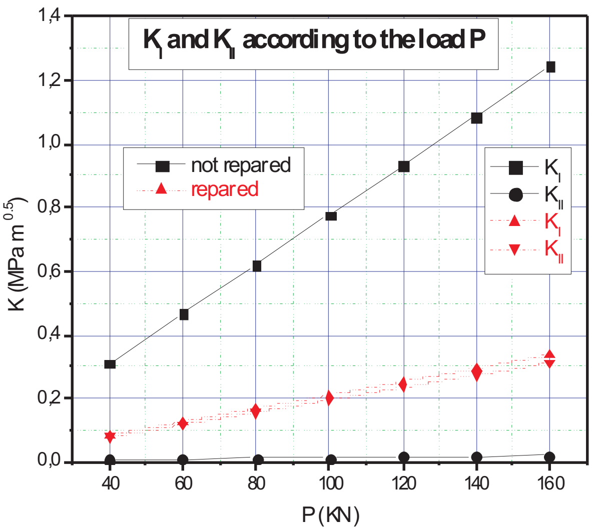

Figure 5 Propagation of cracks in notched beam before and after reparation

5.1.1. *Case 1: Before bonding CFRP plates*

We note that the value of the stress intensity factor KI in the mode I increases with the load. However the SIF for the mixed mode KII values are not influenced by the increase in the workload. The increase of the load as well as its current position (positioned in the Center) foster the development of a time bending maximal in the middle of the beam, allowing to be in tension high around the crack which makes easier the crack propagation with KI values that exceed 1.2 MPa$\sqrt{\text{m}}$. However according to mixed mode crack propagation is almost zero in this case since shear stresses in the vicinity of the notch are low due to the eccentricity of load (d = 0).

5.1.2. *Case 2: After bonding CFRP plates*

The presence of the plate of CFRP in the vicinity of the crack minimizes the evolution of the values of the SIF, which leads to infer the effectiveness of the crack repair process. Can be also observed in figure 5 increased light and little remarkable KII values which is due to the presence of the of interfacial stresses that are caused by the effect of the contact of the entire concrete-adhesive-adhering.

5.2. *Parametric study*

5.2.1. *Mode I*

a) Effect of the thickness of the CFRP The evolution of the stress intensity factors, in mode I, according to the thickness t2 of the CFRP bonding plate, under different values of the load that length, is shown in Fig. 6. Increasing thickness permit to decrease SIF values.

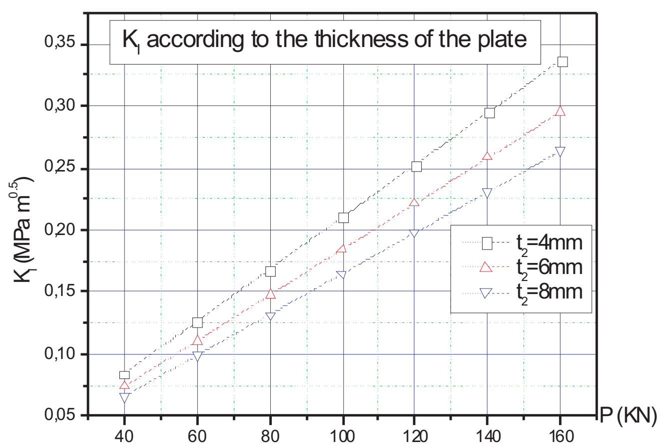

Figure 6 Stress Intensity factor K_I in repaired beams according to the thickness of the plate for a length of strengthening Lp = 2400 mm

b) Effect of the stiffness of the plate: Figure 7 represents the variation of the stress intensity factor K_I values based on the stiffness of the bonding material.
Note that glass fiber composite plates allow to have values of SIF lower than those obtained in the two other materials. Can be observed also that the maximum value of the SIF in the case of steel plate exceeds 2 times that obtained in the case of the GFRP plates, and this may contribute to confirm the restriction of the use of steel in construction sites.

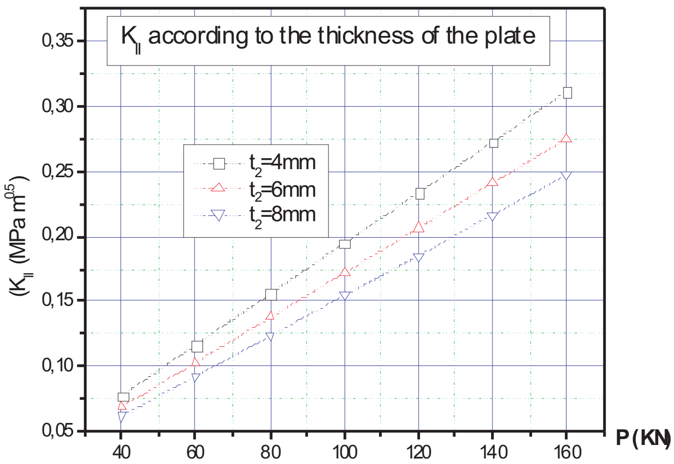

Figure 7 Stress Intensity factor K_I in repaired beams based on the stiffness of the plate for a length plate Lp = 2400 mm, thickness = 4 mm

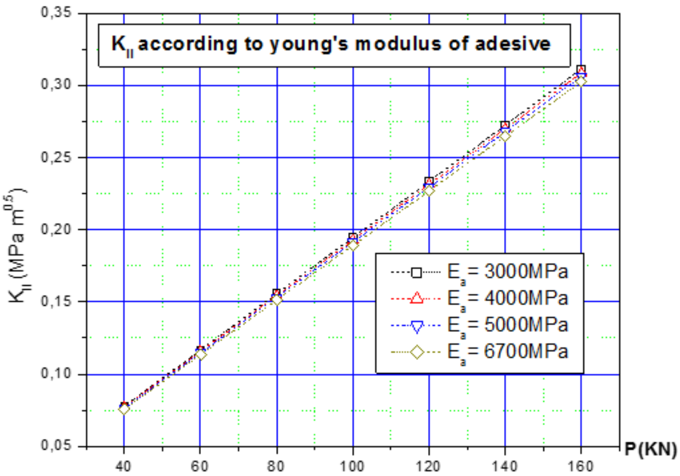

Figure 8 Stress Intensity factor K_I in repaired beams based on the stiffness of the plate for a length plate Lp = 2400 mm , thickness = 4 mm

c) Effect of young's modulus of adhesive: Figure 8 shows the variation of the values of stress intensity factor in mode I, based on the variation of young's modulus of bond and depending on the load. Note that an adhesive with high modulus allows having more low K_I. consequently we can say that the modulus of longitudinal elasticity of adhesive influence on crack propagation in mode I.

5.2.2. *Mode II:*

a) Effect of the thickness of the CFRP: The variation of the values of stress intensity factors in mode II of fracture, according to thickness bonded FRP plate t_2 with the variation of the load is shown in Figure 9. It notes that the lowest values SIF in mode II are those obtained in the thickest plate. It is noted that the thickness of the plate pasted on the beam influences the spread of cracks also in mixed mode.

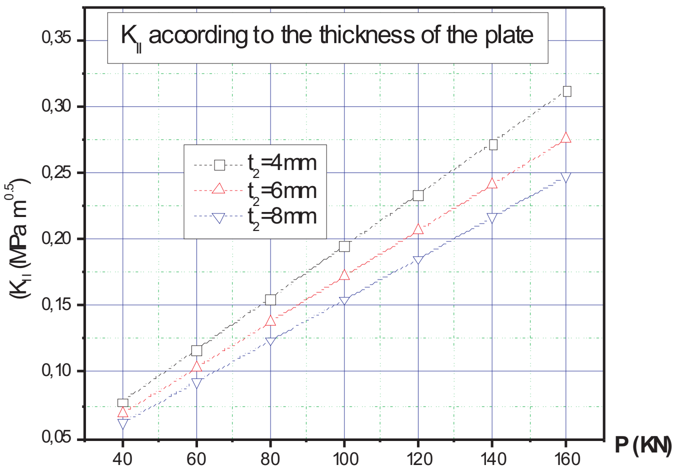

Figure 9 Stress Intensity factor K_{II}in repaired beams according to the thickness of the plate for a length of strengthening Lp = 2400 mm

b) Effect of the stiffness of the plate: Figure 10 represents the variation of the values of the mixed-mode stress intensity factor based on the stiffness of the glued plate. Note that glass fiber composite plates allow having minimum values of SIF. It notes that the type of the reinforcement remains a sensitive step during the repair of degraded structures.

c) Effect of young's modulus of adhesive: Figure 11 illustrates the variation in the values of the mixed-mode stress intensity factor based on the variation of young's modulus of bond and depending on the load. There is the allure of the K_{II} is almost similar. The modulus of longitudinal elasticity of the adhesive does not have a great influence on the crack propagation in mixed mode.

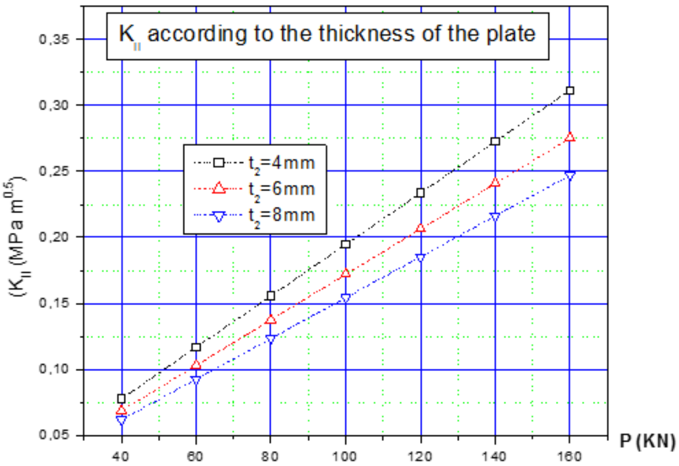

Figure 10 Stress Intensity factor K_{II} in repaired beams based on the stiffness of the plate for a length plate Lp = 2400 mm, thickness = 4 mm

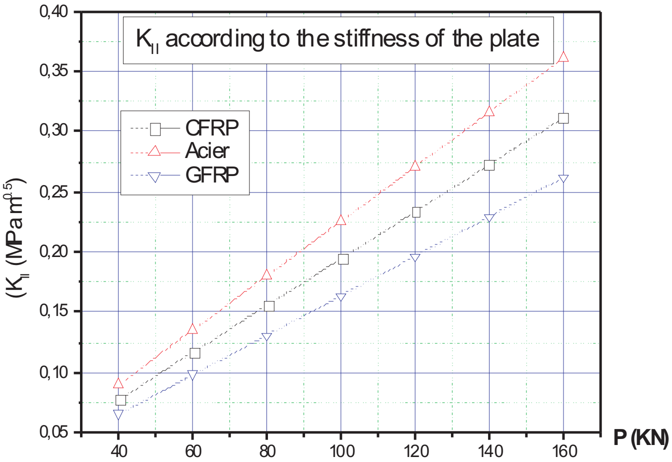

Figure 11 Stress Intensity factor K_{II} in repaired beam based on the stiffness of the plate (where Lp = 2400 mm, thickness t = 4 mm)

6. Conclusions

This research comes to confirm and supplement certain points previously conducted on the subject of the strengthening of the reinforced concrete structures and highlight the performance of the collage technique of composite materials for the repair of cracks in reinforced concrete and prestressed concrete structures. Bonding plates of composite materials on the strained surfaces or side surfaces significantly enhances the ultimate strength and structural stiffness of beams strengthened by decreasing the spread of cracks, particularly in the case of a low beam or moderately armed or part tense.

References

[1] **Jian-he, X. and Ruo-lin, H.**: Experimental study on rehabilitation of corrosion damaged reinforced concrete beams with carbon fiber reinforced polymer, *Constr Build Mater*, 38, 708–16, **2013**.

[2] **Kim, Y. J. and Brunell, G.**: Interaction between CFRP-repair and initial damage of wide flange steel beams subjected to three-point bending, *Compos Struct*, 93, 8, 1986–96, **2011**.

[3] **Kim, Y. J. and Harries, K. A.**: Fatigue behaviour of damaged steel beams repaired with CFRP strips, *Eng. Struct.*, 33, 5, 1491–502, **2011**.

[4] **Al-Saidy, A. H. and Al-Jabri, K. S.**: Effect of damaged concrete cover on the behaviour of corroded concrete beams repaired with CFRP sheets, *Compos. Struct.*, 93, 7, 1775–86, **2011**.

[5] **Dan, D.**: Experimental tests on seismically damaged composite steel concrete walls retrofitted with CFRP composites, *Eng. Struct.*, 45, 338–48, **2012**.

[6] **Thanasis, C. T.**: Shear strengthening of reinforced concrete beams using epoxy bonded FRP composites, *ACI Struct. J.*, 95, 2, 107–15, **1998**.

[7] **Si Larbi, A., Agbossou, A. and Hamelin, A. P.**: Experimental and numerical investigations about textile-reinforced concrete and hybrid solutions for repairing and/or strengthening reinforced concrete beams, *Compos. Struct.*, 99, 152–62, **2013**.

[8] **Si Larbi, A., Contamine, R. and Hamelin, P.**: TRC and hybrid solutions for repairing and/or strengthening reinforced concrete beams, *Eng. Struct.*, 45, 12–20, **2012**.

[9] **Haddad, R. H., Al-Rousan, R. Z. and Al-Sedyiri, B. Kh.**: Repair of shear-deficient and sulfate-damaged reinforced concrete beams using FRP composites, *Eng. Struct.*, 56, 228–38, **2013**.

[10] **Bonacci, J. F. and Maalej, M.**: Externally bonded fiber-reinforced polymer for rehabilitation of corrosion damaged concrete beams, *ACI Struct. J.*, 97, 5, 703–11, **2000**.

[11] **Buyukozturk, O. and Hearing, B.**: Failure behaviour of precracked concrete beams retrofitted with FRP, *J. Compos. Constr.*, 23, 138–44, **1998**.

[12] **Sharif, A., Al-Sulaimani, G. J., Basunbul, I. A., Baluch, M. H. and Ghaleb, B. N.**: Strengthening of initially loaded reinforced concrete beams using FRP plates, *ACI Struct. J.*, 91, 2, 160–8, **1994**.

[13] **Benjeddou, O., Ben-Ouezdou, M. and Bedday, A.**: Damaged RC beams repaired by bonding of CFRP laminates, *Constr. Build. Mater.*, 21, 1301–10, **2007**.

[14] **Tounsi, A.**: Improved theoretical solution for interfacial stresses in concrete beams strengthened with FRP plate, *Int. J. Solids Struct.*, 43, 4154–74, **2006**.

[15] **Tounsi, A., Hassaine Daouadji, T., Benyoucef, S. and Adda Bedia, E. A.**: Interfacial stresses in FRP-plated RC beams: effect of adherend shear deformations, *Int. J. Adhes. and Adhes.*, 29, 4, 343–51, **2009**.

[16] **Tounsi, A. and Benyoucef, S.**: Interfacial stresses in externally plated concrete beams, *Int. J. Adhes. and Adhes.*, 27, 207–15, **2007**.

[17] **Belakhdar, K., Tounsi, A., Benyoucef, S., Adda Bedia, E. A. and El Hassar, S. M.**: On the reduction of the interfacial stresses in a repaired beam with an adhesively bonded FRP plate, *Compos. Interface*, 17, 1–14, **2010**.

[18] **Guenaneche, B., Krour, B., Tounsi, A., Fekrar, A., Benyoucef, S. and Adda Bedia, E. A.**: Elastic analysis of interfacial stresses for the design of a strengthened FRP plate bonded to an RC beam, *Int. J. Adhes. and Adhes.*, 30, 636–42, **2010**.

[19] **Benachour, A., Benyoucef, S., Tounsi, A. and Adda Bedia, E. A.**: Interfacial stress analysis of steel beams reinforced with bonded prestressed FRP plate, *Eng. Struct.*, 30, 3305–15, **2008**.

[20] **Krour, B., Bernard, F. and Tounsi, A.**: Fibers orientation optimization for concrete beam strengthened with a CFRP bonded plate: a coupled analytical–numerical investigation, *Eng. Struct.*, 56, 218–27, **2013**.

[21] **Neale, K. W., Godat, A., Abdel Baky, H. M., Elsayed, W. E. and Ebead, U. A.**: Approaches for finite element simulations of FRP-strengthened concrete beams and slabs, *Arch. Civ. Eng. Environ.*, 4, 59–72, **2011**.

[22] **Kim, Y. J. and Harries, K. A.**: Modeling of timber beams strengthened with various CFRP composites, *Eng. Struct.*, 32, 10, 3225–34, **2010**.

[23] **Obaidat, Y. T., Heyden, S. and Dahlblom, O.**: The effect of CFRP and CFRP/concrete interface models when modelling retrofitted RC beams with FEM, *Compos. Struct.*, 92, 1391–8, **2010**.

[24] **Benarbia, D., Benguediab, M.**: Determination of stress intensity factor in concrete material under Brazilian disc and three-point bending tests using finite element method, *Periodica Polytechnica Mechanical Engineering*, 59, 4, 199–203, **2015**.

[25] **Buyle-Bodin, F., David, E. and Ragneau, E.**: Finite element modelling of flexural behaviour of externally bonded CFRP reinforced concrete structure, *Eng. Struct.*, 24 11, 1423–9, **2002**.

[26] **Le Bellego, C., Dube, J. F., Pijaudier-Cabot, G. and Gerard, B.**: Calibration of nonlocal damage model from size effect tests, *Eur. J. Mech. A/Solids*, 46, 22–33, **2003**.

[27] **Kumar, S. and Barai, S. V.**: Concrete fracture models and applications, *Berlin Heidelberg: Springer-Verlag*, **2011**.

[28] **De-cheng, F., Lin, T. and Peng, C.**: Study of longitudinal cracking during settlement of soil based oextended finite element method [J], *Engineering Mechanics*, 28, 5, 149–154, **2011**.

[29] **Peng, C., De-cheng, F., Lin, T. and Ru-xin, J.**: Based on elastic-plastic damage mechanics to research cracking evolution of Cement stabilized base course during maintaining period [J], *Engineering Mechanics*, 28, (S1): 99, 102–109, **2011**.

[30] **Perez, N.**: Fracture Mechanics, *Department of Mechanical Engineering University of Puerto Rico, Kluwer Academic Publishers*, ISBN 1-4020–7861-7, http://ebooks.kluweronline.com, Boston, **2004**.