Mechanics and Mechanical Engineering
Vol. 21, No. 3 (2017) 679–701
© Lodz University of Technology

Effect of Thermal Loading Due to Laser Pulse on 3-D Problem of Micropolar Thermoelastic Solid with Energy Dissipation

Mohamed I. A. Othman
Department of Mathematics
Faculty of Science, Zagazig University
P.O. Box 44519, Egypt
Taif University
888, Taif, Saudi Arabia
m_i_a_othman@yahoo.com

Sarhan Y. Atwa
Department of Engineering Mathematics and Physics
Higher Institute of Engineering
Shorouk Academy, Cairo, Egypt
srhan_1@yahoo.com

Ahmed W. Elwan
Department of Mathematics
Faculty of Science, King Khalid University
9004, Abha, Saudi Arabia
ahmedelwan@yahoo.com

Received (25 April 2017)
Revised (26 May 2017)
Accepted (11 June 2017)

The aim of this paper is to introduce the Green-Naghdi (G-N) theory of type III (with energy dissipation) to study the effect of thermal loading due to laser pulse on generalized micropolar thermoelastic homogeneous isotropic medium in three dimensions. The normal mode analysis technique is used to solve the resulting non-dimensional equations of the problem. Numerical results for the displacement, thermal stress, strain, temperature, couple stresses and micro-rotation distributions are represented graphically to display the effect of the laser pulse on the resulting quantities. Comparisons are made within the theory in the presence and absence of the laser pulse.

Keywords: generalized thermoelasticity, three-dimensional modeling, laser pulse, micropolar, Green–Naghdi theory, normal mode analysis.

1. Introduction

The generalized theories of thermoelasticity, which admit the finite speed of thermal signal, were the center of interest of active research during last three decades.

Biot [1] introduced the theory of coupled thermoelasticity to overcome the first shortcoming in the classical uncoupled theory of thermoelasticity where it predicts two phenomena not compatible with physical observations. The generalized thermoelastic theories were introduced by Lord and Shulman [2] (L-S) and Green and Lindsay [3] (G-L) in 1960's. The (L-S) theory postulated a wave type heat conduction law to replace the classical Fourier's law. This law is the same as that suggested by Cattaneo [4]. It contains the heat flux vector as well as its time derivative and also contains a new constant that acts as a relaxation time. Othman [5] depicted the dependence of the modulus of elasticity on reference temperature in the theory of generalized thermoelastic diffusion with one relaxation time. In the context of the (L-S) theory, the generalized thermoelastic problem with temperature dependent properties was studied by He et al. [6]. The (G-L) theory modified the energy equation, and allows two relaxation times. Othman and Song [7] studied the reflection of magneto-thermoelastic waves with two relaxation times and temperature dependent elastic moduli.

Three new thermoelastic theories based on the entropy equality rather than the usual entropy inequality introduced by Green and Naghdi [8-10]. The constitutive assumptions for the heat flux vector are different in each theory. Thus, they obtained three theories that they called thermoelasticity of type I, of type II and of type III. When the theory of type I is linearized, one can obtain the classical system of thermoelasticity. The theory of type II (a limiting case of type III) does not admit energy dissipation. In the context of the Green-Naghdi theory Othman and Atwa [11, 12] discussed some problems with and without energy dissipation.

The theory of micropolar elasticity was introduced and developed by Eringen [13]. The theory of micropolar continuum mechanics gives consideration to the microstructure. Micropolar theory is useful in structure materials with a fibrous, lattice, or granular micropolar structural. The main difference of micropolar elastic material from the classical elastic material is that each point has extra rotational degrees of freedom independent of translation and the material can transmit couple stress as well as usual force stress. The linear theory of micropolar thermoelasticity has been developed by extending the theory of micropolar continua to include thermal effect and comprehensive review work on the subject was given by Eringen [14, 15] and Nowacki [16]. Dost and Taborrok [17] presented the generalized thermoelasticity by using (G-L) theory. Chandrasekharaiah [18] developed a heat flux dependent micropolar thermoelasticity. Boschi and Iesan [19] presented a generalized theory of micropolar thermoelasticity that permits the transmission of heat as thermal waves at finite speed. Othman et al. [20] studied the effect of initial stress and the gravity field on micropolar thermoelastic solid with microtemperatures. Othman and Song [21] investigated the effect of thermal relaxation and magnetic field on generalized micropolar thermoelastic medium. Kumar and Choudhary [22, 23] have discussed various problems in orthotropic micropolar continua. Othman and Atwa [24] studied the deformation of micropolar thermoelastic solid with voids considering the influence of various sources acting on the plane surface.

Recently, Othman et al. [25] discussed the effect of rotation and thermal loading due to laser pulse on micropolar generalized thermoelastic solid. Othman and Tantawi [26] investigated the effect of laser pulse and gravity field on thermoelastic medium under Green-Naghdi theory. Kumar and Kaur [27] studied the effect of two temperatures and stiffness on waves propagating at the interface of two micropolar thermoelastic media.

The so-called ultra-short lasers are those with pulse duration ranging from nanoseconds to femtoseconds. In the case of ultra-short-pulsed laser heating, the high-intensity energy flux and ultra-short duration laser beam have introduced situations where very large thermal gradients or an ultra-high heating rate may exist on the boundaries in Al-Qahtani and Datta [28] and Sun et al. [29]. Wang and Xu [30, 31] have studied the stress wave induced by nano-, pico-, and femtosecond laser pulses in a semi-infinite metal by expressing the laser pulse energy as a Fourier series. Youssef and Al-Felali [32] generalized thermoelasticity problem of material subjected to thermal loading due to laser pulse.

Recently, Ronghou et al. [33] studied a generalized thermoelastic coupled problem for the semi-infinite plane induced by pulsed laser heating locally by adopting (L-S) generalized thermoelasticity. Othman et al. [34] discussed the effect of thermal loading due to laser pulse on generalized thermoelastic medium with different theories. Kumar et al. [35] discussed the deformation of micropolar generalized thermoelastic solid subjected to thermo-mechanical loading due to thermal laser pulse.

The present investigation is to study the effect of thermal loading due to laser pulse on generalized micropolar thermoelastic homogeneous isotropic medium in three dimensions in the context of (G-N) theory of type III (with energy dissipation) without any body forces or heat sources. The problem has been solved numerically using a normal mode analysis. Numerical results for the displacement, thermal stress, strain, temperature, couple stresses and microrotation distributions, with and without laser pulses, are represented graphically.

2. Governing equations and formulation of the problem

The governing equations of an isotropic and homogeneous elastic medium with generalized thermoelastic micropolar in the context of (G-N) theory of type III in the absence of body forces are:

The equation of motion:

$$\sigma_{il,l} = \rho \ddot{u}_i \tag{1}$$

$$\varepsilon_{ilr}\sigma_{li} + m_{li,l} = \rho j \frac{\partial^2 \phi_i}{\partial t^2} \tag{2}$$

The constitutive relations:

$$\sigma_{il} = \lambda\, u_{r,r}\delta_{il} + (\mu + k)\ u_{l,i} + \mu\, u_{i,l} - k\varepsilon_{ilr}\phi_r - \hat{\gamma} T \delta_{il} \tag{3}$$

$$m_{il} = \alpha\phi_{r,r}\ \delta_{il} + \beta\phi_{i,l} + \gamma\phi_{l,i} \tag{4}$$

The Heat conduction equation:

$$KT_{,ii} + K^*\dot{T}_{,ii} = \rho\, C_E \ddot{T} + \ \hat{\gamma} T_0 \ddot{u}_{r,r} - \rho\dot{Q} \tag{5}$$

The strain-displacement relation:

$$e_{il} = \frac{1}{2}(u_{i,l} + u_{l,i}) \tag{6}$$

In the preceding equations, λ and μ are Lame' constant, ρ is the density, σ_{il} are the components of the stress tensor, $u_i = (u, v, w)$ are the components of the displacement, t is the time variable, $\phi_i = (0, \phi, 0)$ are the microrotation components, j is the microinertia moment, k, α, β, γ are the micropolar constants, m_{il} is the couple stress tensor, ε_{irl} is the alternate tensor, $\hat{\gamma}$ is a material constant given by $\hat{\gamma} = (3\lambda + 2\mu + \kappa)\,\alpha_T$, where α_T is the coefficient of linear thermal expansion, K is the thermal conductivity, K^* is the material characteristic of the theory, C_E is the specific heat at constant strain, T is the absolute temperature, and T_0 is the temperature of the medium in its natural state, assumed to be such that $|(T - T_0)/T_0| << 1$, Q is the heat input of the laser pulse.

We will consider that the surface is illuminated by a laser pulse given by the heat input as Al-Qahtani and Datta [28] and Tang and Araki [36]:

$$Q = I_0 f(t) g(y) h(x) \tag{7}$$

where I_0 is the energy absorbed, the temporal profile $f(t)$is represented as:

$$f(t) = \frac{t}{t_0^2}\exp(-\frac{t}{t_0}) \tag{8}$$

where t_0 is the pulse rise time. The pulse is also assumed to have a Gaussian spatial profile in y:

$$g(y) = \frac{1}{2\pi r^2}\exp(-\frac{y^2}{r^2}), \tag{9}$$

where r is the beam radius, and as a function of the depth x the heat deposition due to the laser pulse is assumed to decay exponentially within the solid:

$$h(x) = \eta e^{-\eta x} \tag{10}$$

From Eqs. (6–8) in Eq. (5) we get:

$$Q = \frac{I_0 \eta t}{2\pi r^2 t_0^2}\exp(-\frac{y^2}{r^2} - \frac{t}{t_0})\exp(-\eta x) \tag{11}$$

We can rewrite the equation of motion as:

$$\rho\frac{\partial^2 u}{\partial t^2} = (\lambda + \mu)\frac{\partial e}{\partial x} + (\mu + k)\nabla^2 u - k\frac{\partial \phi}{\partial z} - \hat{\gamma}\frac{\partial T}{\partial x} \tag{12}$$

$$\rho\frac{\partial^2 v}{\partial t^2} = (\lambda + \mu)\frac{\partial e}{\partial y} + (\mu + k)\nabla^2 y - \hat{\gamma}\frac{\partial T}{\partial y} \tag{13}$$

$$\rho\frac{\partial^2 w}{\partial t^2} = (\lambda + \mu)\frac{\partial e}{\partial z} + (\mu + k)\nabla^2 w + k\frac{\partial \phi}{\partial x} - \hat{\gamma}\frac{\partial T}{\partial z} \tag{14}$$

$$(\alpha + \beta + \gamma)\frac{\partial^2 \phi}{\partial y^2} + \gamma(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2}) + k(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) - 2k\phi = \rho j\frac{\partial^2 \phi}{\partial t^2} \tag{15}$$

and the conduction equation takes the form:

$$K\nabla^2 T + K^*\nabla^2\frac{\partial T}{\partial t} = \rho C_E\frac{\partial^2 T}{\partial t^2} + \hat{\gamma} T_0\frac{\partial^2 e}{\partial t^2} - \rho\dot{Q} \tag{16}$$

The constitutive equations can be written as:

$$\sigma_{xx} = \lambda\, e + (2\mu + k)\frac{\partial u}{\partial x} - \hat{\gamma}\, T \tag{17}$$

$$\sigma_{yy} = \lambda\, e + (2\mu + k)\frac{\partial v}{\partial y} - \hat{\gamma}\, T \tag{18}$$

$$\sigma_{zz} = \lambda\, e + (2\mu + k)\frac{\partial w}{\partial z} - \hat{\gamma}\, T \tag{19}$$

$$\sigma_{xy} = (\mu + k)\frac{\partial v}{\partial x} + \mu\frac{\partial u}{\partial y} - k\phi \tag{20}$$

$$\sigma_{yx} = (\mu + k)\frac{\partial u}{\partial y} + \mu\frac{\partial v}{\partial x} + k\phi \tag{21}$$

$$\sigma_{yz} = (\mu + k)\frac{\partial w}{\partial y} + \mu\frac{\partial v}{\partial z} - k\phi \tag{22}$$

$$\sigma_{zy} = (\mu + k)\frac{\partial v}{\partial z} + \mu\frac{\partial w}{\partial y} + k\phi \tag{23}$$

$$\sigma_{zx} = (\mu + k)\frac{\partial u}{\partial z} + \mu\frac{\partial w}{\partial x} - k\phi \tag{24}$$

$$\sigma_{xz} = (\mu + k)\frac{\partial w}{\partial x} + \mu\frac{\partial u}{\partial z} + k\phi \tag{25}$$

$$m_{xy} = \gamma\frac{\partial \phi}{\partial x} \tag{26}$$

$$m_{yx} = \beta\frac{\partial \phi}{\partial x} \tag{27}$$

$$m_{zy} = \gamma\frac{\partial \phi}{\partial z} \tag{28}$$

$$m_{yz} = \beta\frac{\partial \phi}{\partial z} \tag{29}$$

where:

$$e = (e_{xx} + e_{yy} + e_{zz}) = (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}) \tag{30}$$

For simplifications we will use the following non-dimensional variables:

$$(x', y', z') = \frac{\varpi}{C_1}(x, y, z) \qquad (u', v', w') = \frac{\rho C_1 \varpi}{\hat{\gamma} T_0}(u, v, w) \qquad T' = \frac{T}{T_0}$$

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\hat{\gamma} T_0} \qquad t = \varpi t \qquad m'_{il} = \frac{\varpi}{C_1 \hat{\gamma} T_0} m_{il} \qquad \phi' = \frac{\rho C_1^2}{\hat{\gamma} T_0}\phi \tag{31}$$

$$Q' = \frac{Q}{\varpi T_0 C_E} \qquad \varpi = \frac{\rho C_E C_1^2}{K} \qquad C_1^2 = \frac{(\lambda + 2\mu)}{\rho}$$

Eqs. (12–19) in the non-dimensional forms (after suppressing the primes) reduce to:

$$\frac{\partial^2 u}{\partial t^2} = \beta_1\, \nabla^2 u + (1 - \beta_1)\frac{\partial e}{\partial x} - \beta_2\frac{\partial \phi}{\partial z} - \frac{\partial T}{\partial x} \tag{32}$$

$$\frac{\partial^2 v}{\partial t^2} = \beta_1 \nabla^2 v + (1-\beta_1)\frac{\partial e}{\partial y} - \frac{\partial T}{\partial y} \tag{33}$$

$$\frac{\partial^2 w}{\partial t^2} = \beta_1 \nabla^2 w + (1-\beta_1)\frac{\partial e}{\partial z} + \beta_2\frac{\partial \phi}{\partial x} - \frac{\partial T}{\partial z} \tag{34}$$

$$a_1\frac{\partial^2 \phi}{\partial y^2} + \gamma(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2}) + a_2(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) - 2a_2\phi = a_3\frac{\partial^2 \phi}{\partial t^2} \tag{35}$$

$$a_4\nabla^2 T + a_5\nabla^2\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial t^2} + \varepsilon_T\frac{\partial^2 e}{\partial t^2} - \frac{\partial Q}{\partial t} \tag{36}$$

$$\sigma_{xx} = \beta_3\frac{\partial u}{\partial x} + (1-\beta_3)\, e - T \tag{37}$$

$$\sigma_{yy} = \beta_3\frac{\partial v}{\partial y} + (1-\beta_3)\, e - T \tag{38}$$

$$\sigma_{zz} = \beta_3\frac{\partial w}{\partial z} + (1-\beta_3)\, e - T \tag{39}$$

where:
$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ $\quad \beta_1 = \frac{\mu+k}{\lambda+2\mu+k}$ $\quad \beta_2 = \frac{k}{\lambda+2\mu+k}$ $\quad \beta_3 = \frac{2\mu+k}{\lambda+2\mu+k}$
$\varepsilon_T = \frac{\hat{\gamma}^2 T_0}{\rho K \varpi}$ $\quad a_1 = \alpha+\beta+\gamma$ $\quad a_2 = kC_1^2/\varpi^2$ $\quad a_3 = \rho j C_1^2$
$a_4 = 1/\varpi$ $\quad a_5 = K^*/K$
From Eqs. (37–39) by addition, we get:

$$\sigma = \alpha_1 e - T \tag{40}$$

where:
$\sigma = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$
$\alpha_1 = (3 - 2\beta_3)/3$
From Eqs. (32–34) after using Eq. (30) we can get:

$$\frac{\partial^2 e}{\partial t^2} = \nabla^2 e - \nabla^2 T \tag{41}$$

Eliminating e from Eqs. (36) and (41) by using Eq. (40), we obtain:

$$\nabla^2\sigma + a_6\nabla^2 T = \frac{\partial^2 T}{\partial t^2} + \frac{\partial^2 \sigma}{\partial t^2} \tag{42}$$

$$a_4\nabla^2 T + a_5\nabla^2\frac{\partial T}{\partial t} = a_7\frac{\partial^2 T}{\partial t^2} + a_8\frac{\partial^2 \sigma}{\partial t^2} - \frac{\partial Q}{\partial t} \tag{43}$$

where: $a_6 = 1 - \alpha_1$, $\quad a_7 = 1 + \frac{\varepsilon_T}{\alpha_1}$, $\quad a_8 = \frac{\varepsilon_T}{\alpha_1}$

3. The solution of the problem

The solution of the considered physical variables can be decomposed in terms of normal modes as in the following form:

$$\begin{aligned}&[u, v, w, e, T, \sigma_{il}, m_{il}, \phi](x, y, z, t)\\ &= [u^*, v^*, w^*, e^*, T^*, \sigma^*_{il}, m^*_{il}, \phi^*](x)\ \exp[\omega t - i(ay + bz)]\end{aligned} \tag{44}$$

where: $i = \sqrt{-1}$, ω is the angular frequency and a, b are the wave numbers in the y and z-directions respectively.
Using Eq. (44) into Eqs. (42) and (43), we can obtain the following equations:

$$(\mathrm{D}^2 - A_1)\sigma^* + a_6(\mathrm{D}^2 - A_2)T^* = 0 \tag{45}$$

$$A_3(\mathrm{D}^2 - A_4)T^* - A_5\sigma^* = -\ Q_0 f^*(y, z, t)\exp(-\eta x) \tag{46}$$

where:
$\mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}x}$ $\quad A_1 = a^2 + b^2 + \omega^2$ $\quad A_2 = a^2 + b^2 + \frac{\omega^2}{a_6}$ $\quad A_3 = \frac{a_4 + a_5\omega}{\omega}$
$A_4 = a^2 + b^2 + \frac{a_7\omega^2}{a_4 + a_5\omega}$ $\quad A_5 = a_8\omega$ $\quad Q_0 = \frac{I_0\eta}{2\pi r^2 t_0^2}$
$f^*(y, z, t) = (1 - \frac{t}{t_0})\exp[-\frac{y^2}{r^2} - \omega t + i(ay + bz) - \frac{t}{t_0}]$
Eliminating T^* and σ^* between Eqs. (45) and (46), we get the following two fourth order ordinary differential equations:

$$(\mathrm{D}^4 - A_6\mathrm{D}^2 + A_7)T^*(x) = A_8 Q_0 f^*(y, z, t)\exp(-\eta x) \tag{47}$$

$$(\mathrm{D}^4 - A_6\mathrm{D}^2 + A_7)\sigma^*(x) = A_9 Q_0 f^*(y, z, t)\exp(-\eta x) \tag{48}$$

where:
$A_6 = A_1 + A_4 - \frac{a_6 A_5}{A_3}$ $\quad A_7 = A_1 A_4 - \frac{a_6 A_2 A_5}{A_3}$ $\quad A_8 = \frac{A_1 - \eta^2}{A_3}$ $\quad A_9 = \frac{a_6(\eta^2 - A_2)}{A_3}$
Eq. (47) can be factored as:

$$(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)T^*(x) = A_8 Q_0 f^*(y, z, t)\exp(-\eta x) \tag{49}$$

where k_n $(n = 1, 2)$ are the roots of the characteristic equation of Eq. (47).
We can consider the general solution of Eqs. (47) and (48) which are bound at infinity in the form:

$$T^*(x) = \sum_{n=1}^{2} M_n \exp(-k_n x) + B_1 Q_0 f^*(y, z, t)\exp(-\eta x) \tag{50}$$

$$\sigma^*(x) = \sum_{n=1}^{2} H_{1n} M_n \exp(-k_n x) + B_2 Q_0 f^*(y, z, t)\exp(-\eta x) \tag{51}$$

where:
$B_1 = \frac{A_8}{\eta^4 - A_6\eta^2 + A_7}$ $\quad B_2 = \frac{A_9}{\eta^4 - A_6\eta^2 + A_7}$ $\quad H_{1n} = \frac{A_3(k_n^2 - A_4)}{A_5}$ $\quad (n = 1, 2)$
From Eqs. (50) and (51) into Eq. (40) after using Eq. (35) we obtain:

$$e^*(x) = \sum_{n=1}^{2} H_{2n} M_n \exp(-k_n x) + B_3 Q_0 f^*(y, z, t)\exp(-\eta x) \tag{52}$$

where: $H_{2n} = \frac{1+H_{1n}}{\alpha_1}$ $B_3 = \frac{B_1+B_2}{\alpha_1}$
From Eqs. (51) and (52) together with (32), (34) and (35) after using Eq. (44) we get:

$$(\mathrm{D}^2 - A_{10})\, u^* + ibA_{11}\phi^* = -\sum_{n=1}^{2} k_n H_{3n} M_n \exp(-k_n x) - B_4 \eta Q_0 f^*(y,z,t) \exp(-\eta x) \tag{53}$$

$$(\mathrm{D}^2 - A_{10})\, w^* + A_{11}\mathrm{D}\phi^* = -\sum_{n=1}^{2} ibH_{3n} M_n \exp(-k_n x) - ibB_4 Q_0 f^*(y,z,t) \exp(-\eta x) \tag{54}$$

$$(\mathrm{D}^2 - A_{12})\phi^* - ibA_{13}u^* - A_{13}\mathrm{D}w^* = 0 \tag{55}$$

where:
$A_{10} = a^2 + b^2 + \frac{\omega^2}{\beta_1}$ $A_{11} = \frac{\beta_2}{\beta_1}$ $A_{12} = b^2 + \frac{a^2 a_1 + 2a_2 + a_3\omega^2}{\gamma}$ $A_{13} = \frac{a_2}{\gamma}$
$B_4 = \frac{B_1 - (1-\beta_1)B_3}{\beta_1}$ $H_{3n} = \frac{k_n - (1-\beta_1)k_n H_{2n}}{\beta_1}$
Eliminating ϕ^* between Eqs. (53–55), we get:

$$-ib(\mathrm{D}^2 - A_{10})w^* + (\mathrm{D}^3 - A_{10}\mathrm{D})u^* + \sum_{n=1}^{2} H_{4n} M_n \exp(-k_n x) + B_5 Q_0 f^*(y,z,t)\exp(-\eta x) = 0 \tag{56}$$

$$ibA_{14}\mathrm{D}w^* + (\mathrm{D}^4 - A_{15}\mathrm{D}^2 + A_{16})u^* + \sum_{n=1}^{2} H_{5n} M_n \exp(-k_n x) + B_6 Q_0 f^*(y,z,t)\exp(-\eta x) = 0 \tag{57}$$

where:
$A_{14} = A_{11}A_{13}$ $A_{15} = A_{10} + A_{12}$ $A_{16} = A_{10}A_{12} - b^2 A_{11}A_{13}$
$B_5 = (b^2 - \eta^2)B_4$ $B_6 = (\eta^2 - A_{12})\eta B_4$
$H_{4n} = (b^2 - k_n^2)H_{3n}$ $H_{5n} = (k_n^2 - A_{12})k_n H_{3n}$

From (56) and (57) we can obtain the following sixth order ordinary differential equations:

$$(\mathrm{D}^6 - \mathrm{A}_{17}\mathrm{D}^4 + A_{18}\mathrm{D}^2 - A_{19})u^*(x) = \sum_{n=1}^{2} H_{6n} M_n \exp(-k_n x) + B_7 Q_0 f^*(y,z,t)\exp(-\eta x) \tag{58}$$

$$(\mathrm{D}^6 - \mathrm{A}_{17}\mathrm{D}^4 + A_{18}\mathrm{D}^2 - A_{19})w^*(x) = \sum_{n=1}^{2} H_{7n} M_n \exp(-k_n x) + B_8 Q_0 f^*(y,z,t)\exp(-\eta x) \tag{59}$$

where:
$A_{17} = A_{10} + A_{15} - A_{14}$ $A_{18} = A_{16} + A_{10}A_{15} - A_{10}A_{14}$ $A_{19} = A_{10}A_{16}$
$B_7 = (A_{10} - \eta^2)B_6 + \eta B_5$ $B_8 = \frac{1}{ib}\left[(\eta^4 - A_{15}\eta^2 + A_{16})B_5 + (\eta^2 - A_{10})\eta B_6\right]$
$H_{6n} = (A_{10} - k_n^2)H_{5n} + k_n H_{4n}$

$H_{7n} = \frac{1}{ib}\left[(k_n^4 - A_{15}k_n^2 + A_{16})H_{4n} + (k_n^2 - A_{10})k_n H_{5n}\right]$

Eq. (58) can be factored as:

$$(\mathrm{D}^2 - \lambda_1^2)(\mathrm{D}^2 - \lambda_2^2)(\mathrm{D}^2 - \lambda_3^2)u^*(x) = \sum_{n=1}^{2} H_{6n}M_n \exp(-k_n x) + B_7 Q_0 f^*(y,z,t)\exp(-\eta x) \tag{60}$$

where: λ_m $(m = 1, 2, 3)$ are the roots of the characteristic equation of Eq. (58). We can consider the general solution of Eqs. (58) and (59) which are bound at infinity in the form:

$$u^*(x) = \sum_{n=1}^{2} H_{8n}\ M_n \exp(-k_n x) + E_1 Q_0 f^*(y,z,t)\exp(-\eta x) + \sum_{m=1}^{3} L_m \exp(-\lambda_m x) \tag{61}$$

$$w^*(x) = \sum_{n=1}^{2} H_{9n}\ M_n \exp(-k_n x) + E_2 Q_0 f^*(y,z,t)\exp(-\eta x) + \sum_{m=1}^{3} R_{1m} L_m \exp(-\lambda_m x) \tag{62}$$

From (61) and (62) into (53) we get:

$$\phi^*(x) = \sum_{n=1}^{2} H_{10n}\ M_n \exp(-k_n x) + E_3 Q_0 f^*(y,z,t)\exp(-\eta x) + \sum_{m=1}^{3} R_{2m} L_m \exp(-\lambda_m x) \tag{63}$$

where:

$E_1 = \frac{B_7}{\eta^6 - A_{17}\eta^4 + A_{18}\eta^2 - A_{19}}$ $\quad E_2 = \frac{B_8}{\eta^6 - A_{17}\eta^4 + A_{18}\eta^2 - A_{19}}$ $\quad E_3 = \frac{(A_{10} - \eta^2)E_1 - \eta B_4}{ibA_{11}}$

$H_{8n} = \frac{H_{6n}}{k_n^6 - A_{17}k_n^4 + A_{18}k_n^6 - A_{19}}$ $\quad H_{9n} = \frac{H_{7n}}{k_n^6 - A_{17}k_n^4 + A_{18}k_n^6 - A_{19}}$

$H_{10n} = \frac{(A_{10} - k_n^2)H_{8n} - k_n H_{3n}}{ibk_n}$ $\quad R_{1m} = \frac{(\lambda_m^4 - A_{15}\lambda_m^2 + A_{16})}{ibA_{14}\lambda_m}$ $\quad R_{2m} = \frac{A_{10} - \lambda_m^2}{ibA_{11}}$

From Eqs. (44), (50), (52), (60) and (61) into (37) and (39) we get:

$$\sigma_{xx}^*(x) = \sum_{n=1}^{2} H_{11n} M_n \exp(-k_n x) + E_4 Q_0 f^*(y,z,t)\exp(-\eta x) - \sum_{m=1}^{3} \beta_3 \lambda_m L_m \exp(-\lambda_m x) \tag{64}$$

$$\sigma_{zz}^*(x) = \sum_{n=1}^{2} H_{12n} M_n \exp(-k_n x) + E_5 Q_0 f^*(y,z,t)\exp(-\eta x) - ib\sum_{m=1}^{3} R_{1m} L_m \exp(-\lambda_m x), \tag{65}$$

where:

$E_4 = (1-\beta_3)B_3 - \beta_3\eta E_1 - B_1 \qquad E_5 = (1-\beta_3)B_3 - ibE_2 - B_1$

$H_{11n} = (1-\beta_3)H_{2n} - \beta_3 k_n H_{8n} - 1 \qquad H_{12n} = (1-\beta_3)H_{2n} - ibH_{9n} - 1$

4. Applications

In order to complete the solution we have to know the parameters $M_n (n = 1, 2)$ and $L_m (m = 1, 2, 3)$, so we will consider the following boundary conditions at $x = 0$ and $I_0 = 0$:

- Mechanical boundary condition that the bounding plane to the surface has no traction anywhere and has no variation of microrotation, so we have

$$\begin{aligned} &\sigma(0,y,z,t) = \sigma_{xx}(0,y,z,t) = \sigma_{yy}(0,y,z,t) = \sigma_{zz}(0,y,z,t) = 0 \\ &\frac{\partial\phi}{\partial x} = 0 \end{aligned} \tag{66}$$

- The thermal boundary condition is that the surface of the half space is subjected to a thermal shock:

$$T(0,y,z,t) = s(0,y,z,t) = s^* \exp[\omega t - i(ay + bz)] \tag{67}$$

Applying the boundary conditions (66) and (67) on Eqs. (50), (51), (63–65) we obtain a system of five equations. After applying the inverse of matrix method:

$$\begin{pmatrix} M_1 \\ M_2 \\ L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ H_{11} & H_{12} & 0 & 0 & 0 \\ k_1 H_{101} & k_2 H_{102} & \lambda_1 R_{21} & \lambda_2 R_{22} & \lambda_3 R_{23} \\ H_{111} & H_{112} & -\beta_3\lambda_1 & -\beta_3\lambda_2 & -\beta_3\lambda_3 \\ H_{121} & H_{122} & -ibR_{11} & -ibR_{12} & -ibR_{13} \end{pmatrix}^{-1} \begin{pmatrix} s^* \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{68}$$

We obtain the values of the five constants $M_n$and L_m. Hence, we obtain the expressions for the displacement components (u, w), the stress, the strain, the temperature, the tangential couple stress and the microrotation distribution of the micropolar generalized thermoelastic medium.

Finally, we can obtain the displacement component v from Eqs. (50) and (52) into (33) after using (44), so we get:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} - \lambda_v^2\right) v^* = \sum_{n=1}^{2} \xi_n \exp(-k_n x) + \xi_3 Q_0 f^* \exp(-\eta x) \tag{69}$$

where:

$\lambda_v^2 = (a^2 + b^2 + \frac{\omega^2}{\beta_1}) \qquad \xi_n = ia[\frac{(1-\beta_1)H_{2n}-1}{\beta_1}]M_n \qquad \xi_3 = ia[\frac{(1-\beta_1)B_3-B_1}{\beta_1}]$

The solution of the ordinary differential Eq. (69) takes the form:

$$v^*(x) = \sum_{n=1}^{2} r_n \exp(-k_n x) + r_3 Q_0 f^* \exp(-\eta x) + r_4 \exp(-\lambda_v x), \tag{70}$$

where $r_n = \frac{\xi_n}{k_n^2-\lambda_v^2}$, $r_3 = \frac{\xi_3}{\eta^2-\lambda_v^2}$, and r_4 is a constant to be determined from the boundary conditions.
From Eqs. (40) and (44) into (38) after using the boundary conditions (66) and (67) we get:

$$r_4 = \frac{(1-\alpha_1-\beta_3)}{ia\alpha_1\beta_3}s^* - r_3 - \sum_{n=1}^{2} r_n \tag{71}$$

From Eq. (63) and (44) into (26) we can get:

$$m_{xy}^*(x) = \sum_{n=1}^{2} -\gamma k_n H_{10n} M_n \exp(-k_n x) - \gamma\eta E_3 Q_0 f^*(y,z,t)\exp(-\eta x)$$
$$-\sum_{m=1}^{3} \gamma\lambda_m R_{2m} L_m \exp(-\lambda_m x) \tag{72}$$

Similarly we can get the other components of the couple stresses.

5. Particular cases

Case 1: Without micropolar effective:
The corresponding equations for the generalized thermoelastic medium without the micropolar effect can be obtained from the above mentioned cases by taking:

$$k = \alpha = \beta = \gamma = 0.$$

Case 2: Without energy dissipation
To obtain the field variables for the micropolar thermoelastic medium without energy dissipation (the linearized G-N theory of type II), we can take $K^* = 0$.

6. Numerical results and discussions

In order to illustrate our theoretical results obtained in the preceding section, we now present some numerical results. In the calculation, we take the copper as the material subjected to mechanical thermal disturbances. Sinceω is complex, we take $\omega = \omega_0 + i\zeta$, where i is the imaginary number. The numerical constants of the problem were taken at $T_0 = 293K$ as:

$\lambda = 9.4\times10^{11}$ kg m^{-1} s^{-2}, $\mu = 4\times10^{11}$ kg m^{-1} s^{-2},
$\rho = 1.74\times10^{3}$ kg m^{-3}, $k = 1\times10^{11}$ kg m^{-1} s^{-2}, $K = 1\times10^{-4}$ Wm^{-1} k^{-1},
$K^* = 1.3\times10^{-4}$ Wm^{-1} k^{-1}, $\alpha_T = 1.78\times10^{-5}$ K^{-1}, $C_E = 1.04\times10^{3}$ kg m^{-3},
$j = 2\times10^{-20}$ m^2, $\alpha = \beta = \gamma = 0.779\times10^{-8}$ kg m^{-1} s^{-2}, $\omega_0 = -2.5$,
$\zeta = -0.1$, $a = 0.2$, $b = 1.2$, $\eta = 1$ m^{-1}, $r = 100$ μm, $t_0 = 8$ nan.s, $s^* = 10$

The numerical technique, outlined above, was used for the distribution of the real part of the displacement component u, stress σ, strain e, the temperature T, microrotation ϕ and tangential couple stress $m_{xy}$for the problem. Here, all the variables are taken in non-dimensional form.

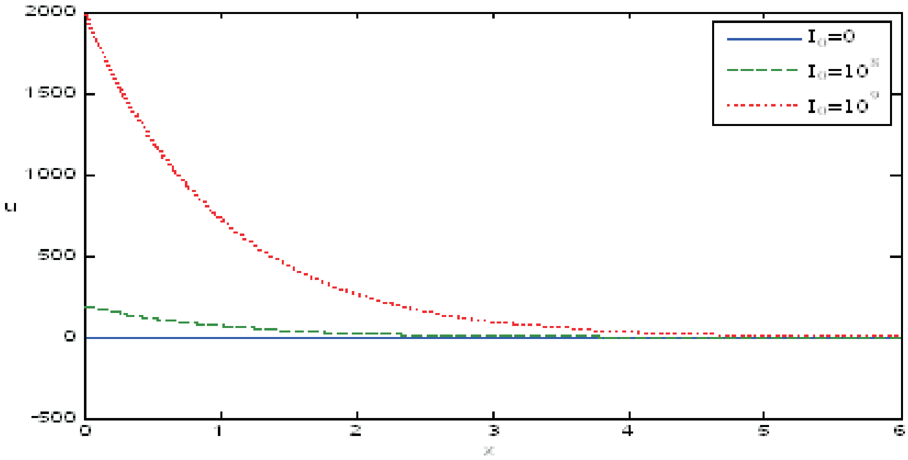

Figure 1 Displacement distribution at $y = z = 0.1$ and $t = 0.1$

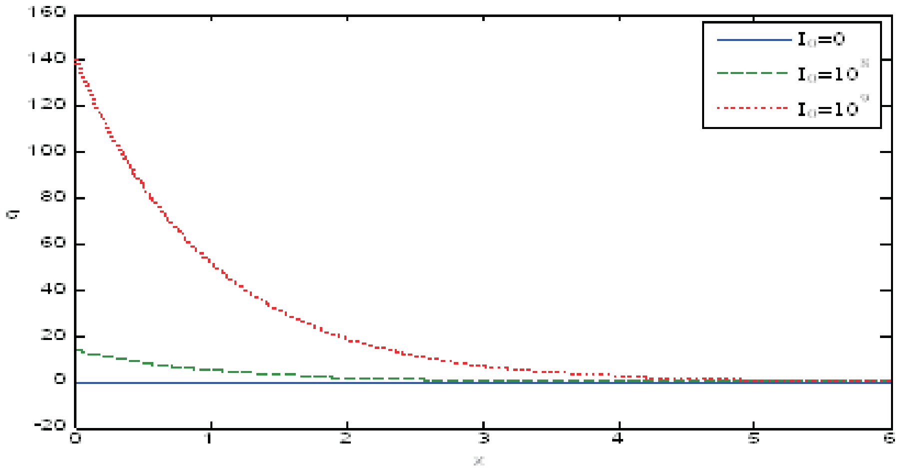

Figure 2 Stress distribution at $y = z = 0.1$ and $t = 0.1$

Figures 1-6 represented 2D curves for the distributions of the physical quantities against the distance x at $y = z = 0.1$, $t = 0.1$ in the cases of the absence and presence of laser pulse effect ($I_0 = 0,\ 10^8,\ 10^{10}$). In these figures, the solid line, dashed line and dotted line correspond for $I_0 = 0,\ 10^8,\ 10^{10}$ respectively, which is furthermore precisely explained in each figure in the legend. Figures 1, 2, 5 illustrate the variations of the displacement component u, stress σ and microrotation ϕ with a distance x. These figures show that the above mentioned quantities decrease with the increase of the distance x and finally all curves terminate at the zero value at $x > 6$ approximately.

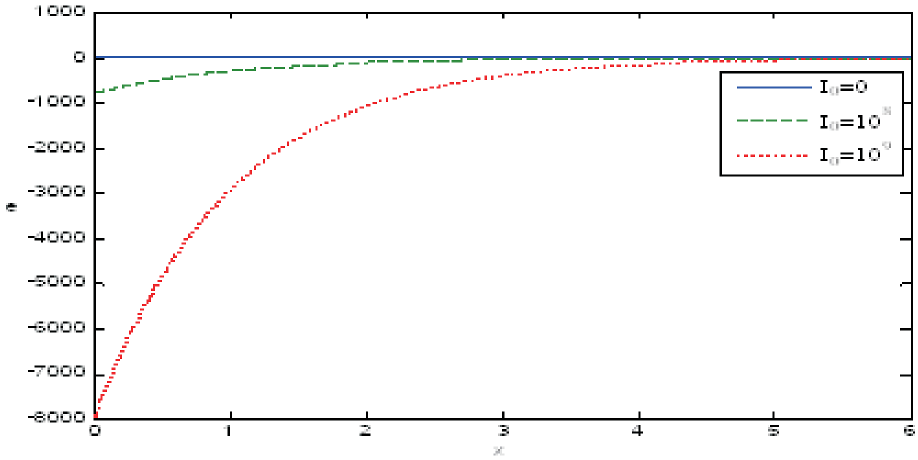

Figure 3 Strain distribution at $y = z = 0.1$ and $t = 0.1$

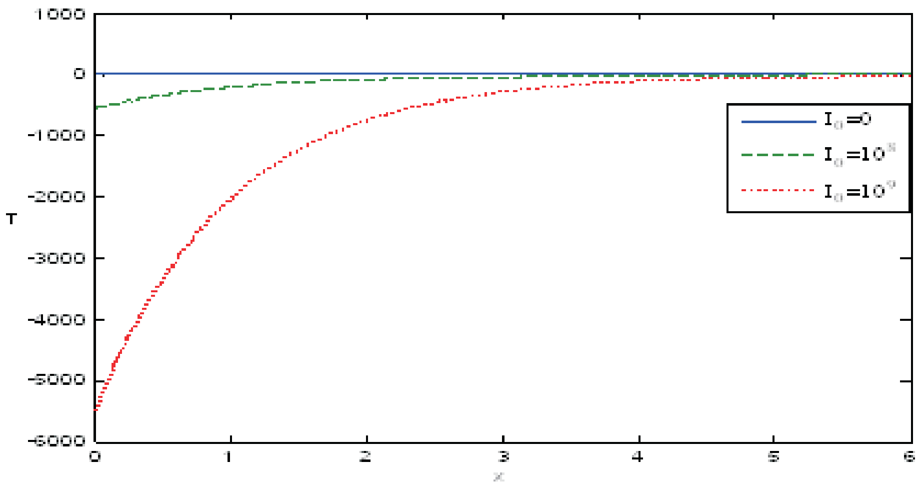

Figure 4 Temperature distribution at $y = z = 0.1$ and $t = 0.1$

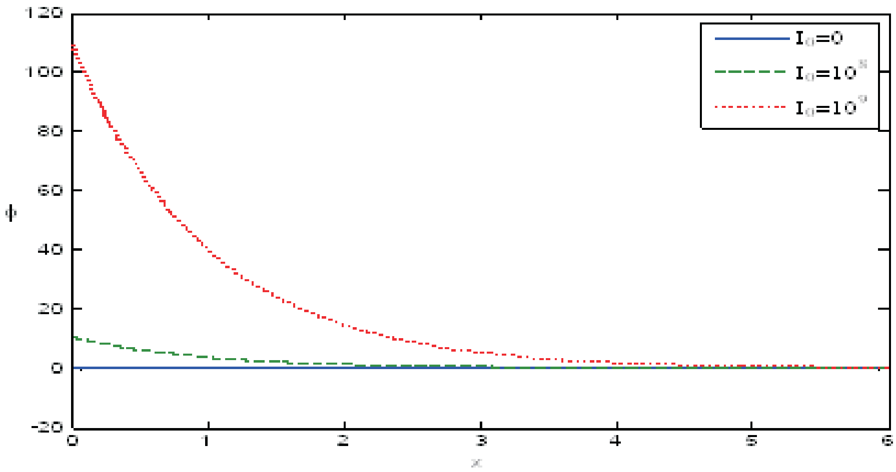

Figure 5 Microrotation distribution at $y = z = 0.1$ and $t = 0.1$

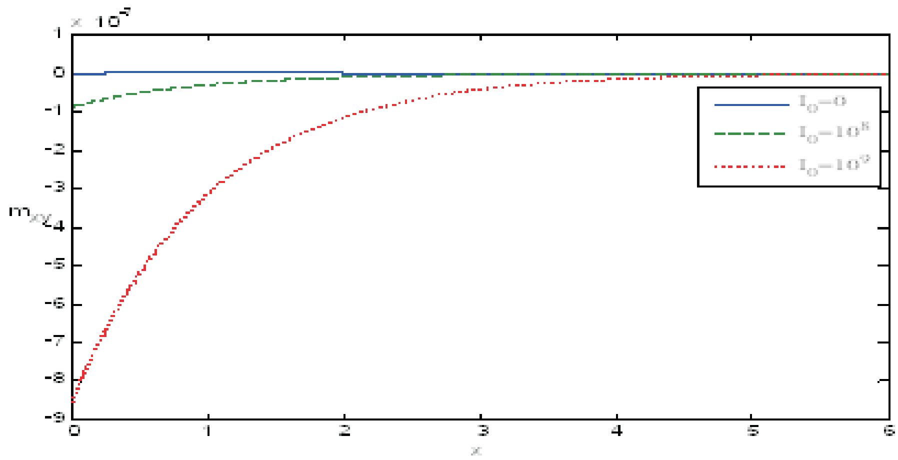

Figure 6 Tangential couple stress distribution at $y = z = 0.1$ and $t = 0.1$

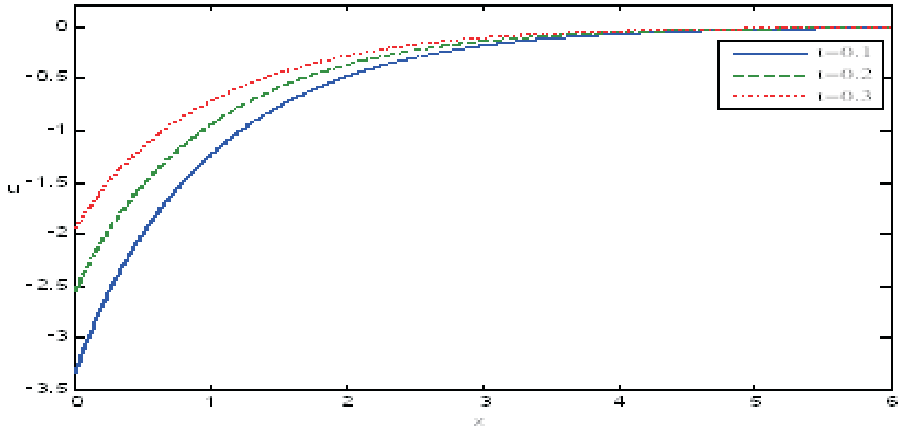

Figure 7 Displacement distribution at $y = z = 0.1$ and $I_0 = 10^5$

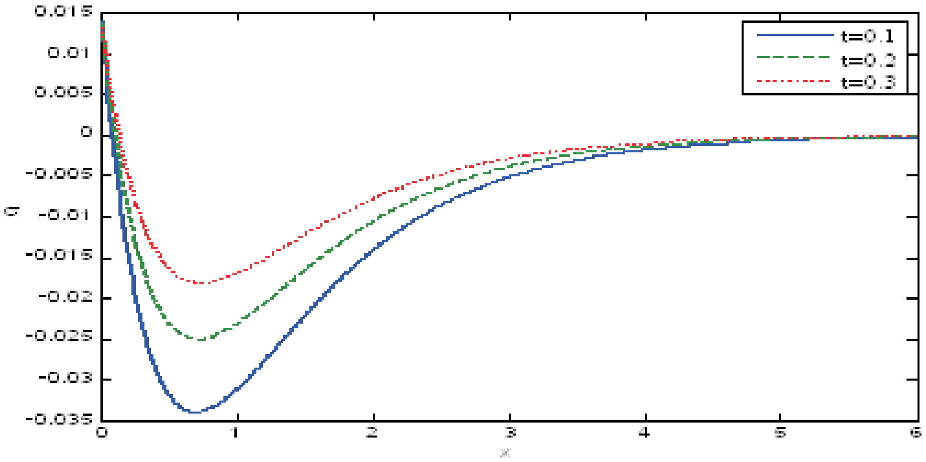

Figure 8 Stress distribution at $y = z = 0.1$ and $I_0 = 10^5$

It can be observed from these figures that the laser pulse value has an increasing effect on both the displacement component u, the stress $\sigma$and the microrotation ϕ. Figures 3, 4, 6 describe the variations of the strain e, the temperature T and the tangential couple stress $m_{xy}$with a distance x. These figures show that these quantities increase with the increase of the distance x and finally all curves converge to zero for $x > 6$ approximately. It is observed from these figures that the laser pulse value has a decreasing effect on both the strain e, the temperature T and tangential couple stress m_{xy}.

Figures 7-12 represented 2D curves for the distributions of the physical quantities against the distance x at $y = z = 0.1$ (with a fixed value of $I_0 = 10^5$) taking three values of the dimensionless time, namely $t = 0.1, 0.2, 0.3$. In these figures, the solid line, dashed line and dotted line correspond for $t = 0.1, 0.2, 0.3$ respectively, which is furthermore precisely explained in each figure in the legend.

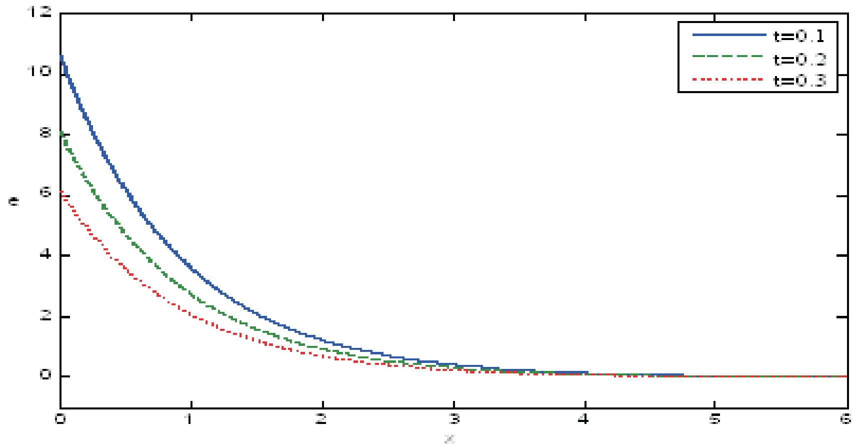

Figure 9 Strain distribution at $y = z = 0.1$ and $I_0 = 10^5$

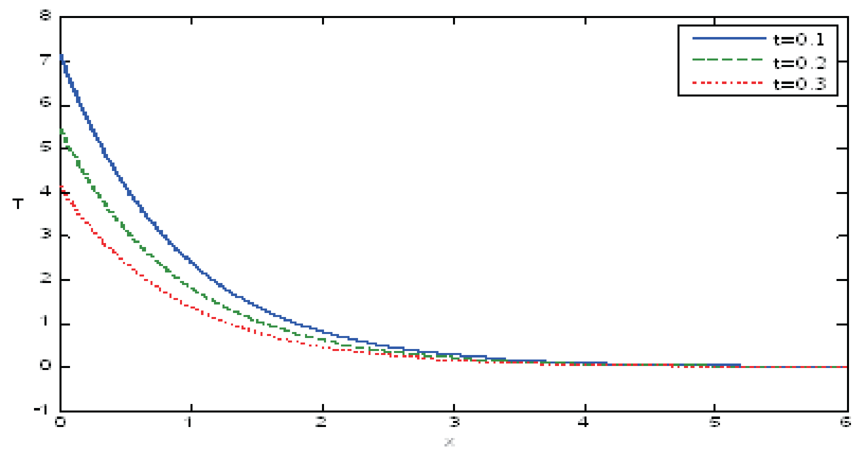

Figure 10 Temperature distribution at $y = z = 0.1$ and $I_0 = 10^5$

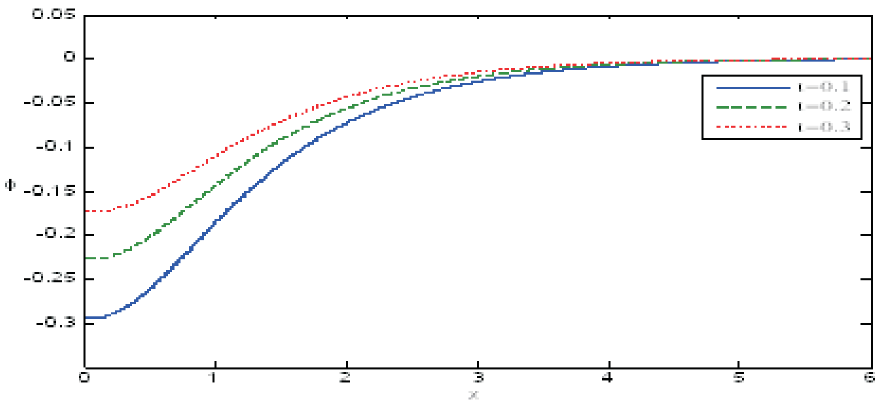

Figure 11 Microrotation distribution at $y = z = 0.1$ and $I_0 = 10^5$

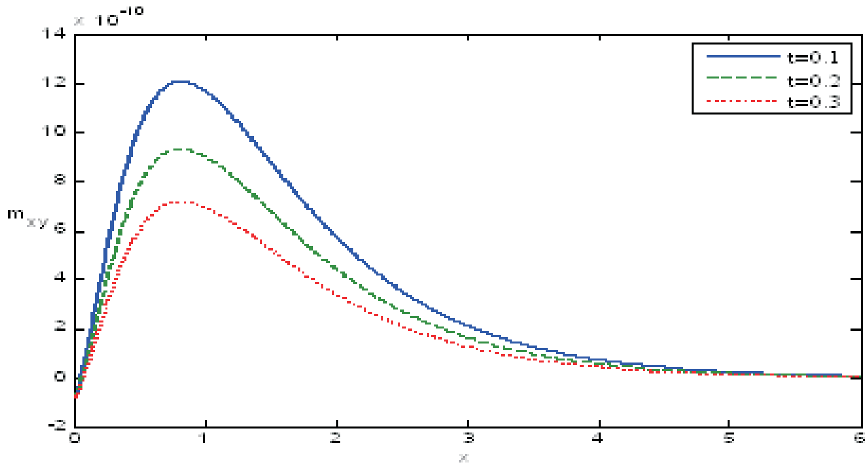

Figure 12 Tangential couple stress distribution at $y = z = 0.1$ and $I_0 = 10^5$

Figures 7, 11 illustrate the variations of the displacement component u and the microrotation ϕ with a distance x. These figures show that the displacement component u and the microrotation ϕ increase with the increase of the distance x and finally all curves tends to zero value at $x > 6$ approximately and it is observed from these figures that the dimensionless time t has an increasing effect on both the displacement component u and the microrotation ϕ.

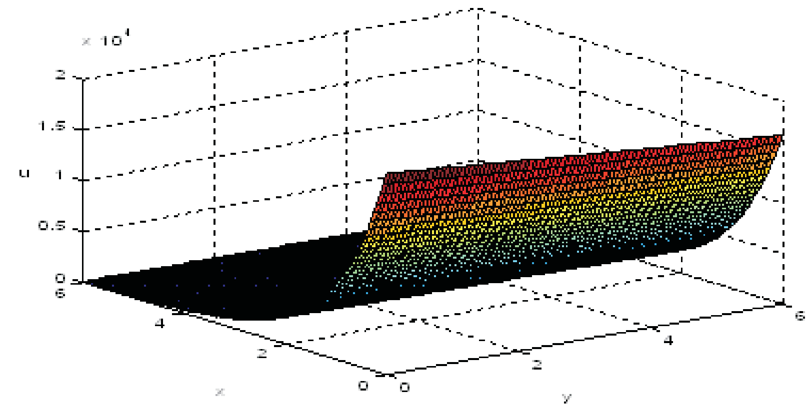

Figure 13 Displacement distribution at $z = 0.1,\ t = 0.1$ and $I_0 = 10^8$

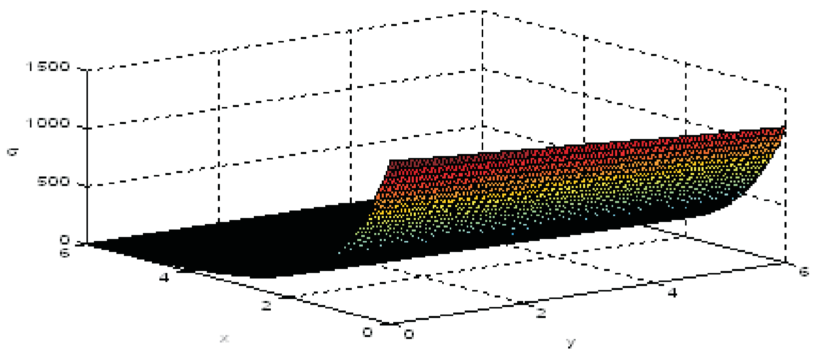

Figure 14 Stress distribution at $z = 0.1,\ t = 0.1$ and $I_0 = 10^8$

Figures 8 describes the variations of the stress σ with a distance x. We can see from this figure that stress σdecreases with the increase of the distance x and has a minimum value at $x = 0.71$ for the three values of the time t and then all curves increase tending to zero for $x > 6$. From this figure, it can be seen that the dimensionless time t has an increasing effect on the stress σ.

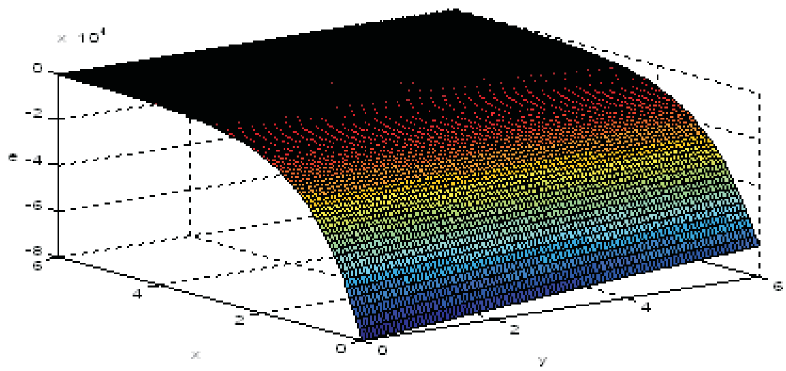

Figure 15 Strain distribution at $z = 0.1,\ t = 0.1$ and $I_0 = 10^8$

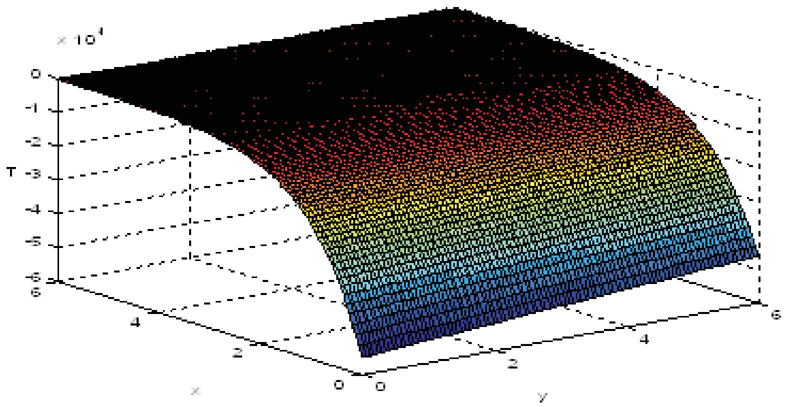

Figure 16 Temperature distribution at $z = 0.1,\ t = 0.1$and $I_0 = 10^8$

Figures 9, 10 display the variations of the strain e and the temperature T with a distance x and it is clear that the above mentioned physical quantities decrease with the increase of the distance x for all values of t and finally all curves converge to zero for $x > 6$. It is noticed from these figures that the dimensionless time t has a decreasing effect on both the strain e and the temperature T.

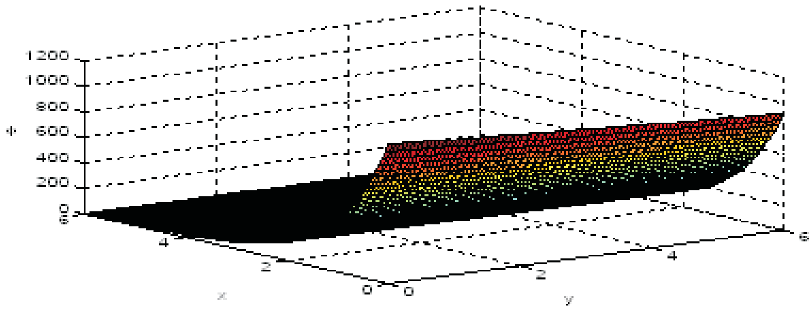

Figure 17 Microrotation distribution at $z = 0.1,\ t = 0.1$ and $I_0 = 10^8$

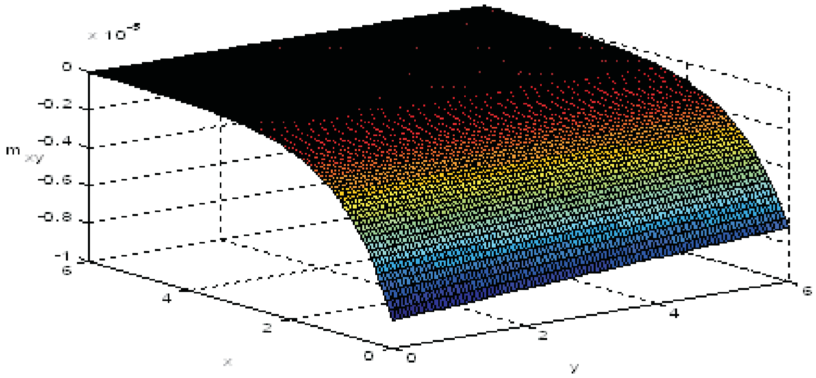

Figure 18 Tangential couple stress distribution at $z = 0.1,\ t = 0.1$ and $I_0 = 10^8$

Figure 12 explains the variations of the tangential couple stress $m_{xy}$with a distance x. We can see from this figure that the tangential couple stress increases with the increase of the distance x and has a maximum value at $x = 0.82$ for the three values of the time t and then all curves decrease tending to zero for $x > 6$. From this figure, it can be seen that the dimensionless time t has an increasing effect on the tangential couple stress m_{xy}.

Figures 13-18 represented 3D curves for the variations of the physical quantities against the distance x at $z = 0.1$, $t = 0.1$ and $I_0 = 10^8$. These figures are very important to study the dependence of the physical quantities on both components of distance x, y. It can be clearly seen that the curves obtained are highly depending on both distance components and we can see that some quantities increase on the negative direction of the distance, while some on positive direction.

7. Concluding remarks

A three-dimensional model of the generalized micropolar thermoelastic medium under the influence of thermal loading due to laser pulse was established and according to the results the following conclusions can be obtained:

1. The results indicate that the effect of the thermal loading due to laser pulse on the components of the displacement, thermal stress, strain, temperature, microrotation and tangential couple stress distributions is very pronounced.

2. The normal mode analysis, used in this article to solve the problem, is applicable to a wide range of problems in thermodynamics and thermoelasticity. This method gives exact solutions without any assumed restrictions on either the temperature or stress distributions.

3. The values of the distributions of all physical quantities converge to zero with increasing distance x and all functions are continuous. Using these results; it possible to investigate the disturbance caused by more general sources for practical applications.

4. Physical applications are found in the mechanical engineering, geophysical, and industrial sectors.

References

[1] **Boit, M.**: Thermoelasticity and Irreversible Thermodynamics, *J. Appl. Phys.*, 27, 240–251, **1956**.

[2] **Lord, H. and Shulman, Y. A.**: Generalized dynamical theory of thermoelasticity, *J. Mech. and Phys. of Solids*, 15, 299–309, **1967**.

[3] **Green, A. E. and Lindsay, K. A.**: Thermoelasticity, *J. Elasticity*, 2, 1–7, **1972**.

[4] **Catteneo, C.**: A form of heat conduction equation which eliminates the paradox of instantaneous propagation, *Compute Rendus*, 247, 431–433, **1958**.

[5] **Othman, M. I. A.**: Lord-Shulman theory under the dependence of the modulus of elasticity on the reference temperature in two dimensional generalized thermoelasticity, *J. Therm. Stress.*, 25, 1027–1045, **2002**.

[6] **He T., Rao T., Shi S., Li H. and Ma Y.**: Finite element method to generalized thermo- elastic problems with temperature-dependent properties, *J. Appl. Sci.*, 13, 2156–2160, **2013**.

[7] **Othman, M. I. A. and Song, Y.**: Reflection of magneto-thermoelastic waves with two relaxation times and temperature dependent elastic moduli, *Appl. Math. Model.*, 32, 483–500, **2008**.

[8] **Green, A. E. and Naghdi, P. M.**: A re-examination of the basic postulates of thermo-mechanics., *Proc. Roy. Soc. Lond.*, Series A, 432, 171–194, **1991**.

[9] **Green, A. E. and Naghdi, P. M.**: On undamped heat waves in an elastic solid, *J. Thermal Stresses*, 15, **1992**, 253–264.

[10] **Green, A. E. and Naghdi, P. M.**: Thermoelasticity without energy dissipation, *J. Elasticity*, 31, 189–208, **1993**.

[11] **Othman, M. I. A. and Atwa, S. Y.**: 2-D problem of a Mode-I crack for a generalized thermo-elasticity under Green-Naghdi theory, *Meccanica*, 48, 1543–1551, **2013**.

[12] **Othman, M. I. A. and Atwa, S. Y.**: Two-dimensional problem of a fiber-reinforced anisotropic thermoelastic medium. Comparison with the Green–Naghdi theory, *Computational Mathematics and Modeling*, 24, 307–325, **2013**.

[13] **Eringen, A. C.**: Linear Theory of micropolar elasticity, *J. Appl. Math. and Mech.*, 15, 909–923, **1966**.

[14] **Eringen, A. C.**: Foundations of micropolar thermoelasticity, Udline Course and Lectures 23, *International Centre for Mechanical Science, Springer*, Berlin, Germany, **1970**.

[15] **Eringen, A. C.**: Microcontinuum field theories-I, Foundations and Solids, *Springer*, Berlin, Germany, **1999**.

[16] **Nowacki, W.**: Theory of asymmetric elasticity, *Pergamon*, Oxford, UK, **1986**.

[17] **Dost, S. and Tabarrok, B.**: Generalized micropolar thermoelasticity, *Int. J. Eng. Sci.*, 16, 173–183, **1978**.

[18] **Chandrasekharaiah, D. S.**: Heat-flux dependent micropolar thermoelasticity, *Int. J. Eng. Sci.*, 24, 1389–1395, **1986**.

[19] **Boschi, E. and Iesan, D.**: A generalized theory of linear micropolar thermoelasticity, *Meccanica*, 8, 154–157, **1973**.

[20] **Othman M. I. A, Tantawi, R. S. and Hilal, M. I. M.**: Effect of initial stress and the gravity field on micropolar thermoelastic solid with microtemperatures, *J. Theor. and Appl. Mech.*, 54, 847–857, **2016**.

[21] **Othman M. I. A and Song, Y. Q.**: The effect of thermal relaxation and magnetic field on generalized micropolar thermoelastic medium, *J. Appl. Mech. and Tech. Phys.*, 57, 108–116, **2016**.

[22] **Kumar R. and Choudhary S.**: Dynamical behavior of orthotropic micropolar elastic medium, *J. Vib. and Control*, 5, 1053–1069, **2002**.

[23] **Kumar R. and Choudhary S.**: Response of orthotropic micropolar elastic medium due to time harmonic sources, *Sadhana*, 29, 83–92, **2004**.

[24] **Othman, M. I. A. and Atwa, S. Y.**: Response of micropolar thermoelastic solid with voids due to various sources under Green-Naghdi theory, *Acta Mechanica Solida Sinica*, 25, 197–209, **2012**.

[25] **Othman, M. I. A. and Hasona, W. M., Abd-Elaziz, E. M.**: The influence of thermal loading due to laser pulse on generalized micropolar thermoelastic solid with comparison of different theories, *Multidiscipline Modeling in Materials and Sstructures*, 10, 328–345, **2014**.

[26] **Othman M. I. A and Tantawi, R. S.**: The effect of laser pulse and gravity field on thermoelastic medium under Green-Naghdi theory, *Acta Mechanica*, 227, 3571-3583, **2016**.

[27] **Kumar, R. and Kaur, M.**: Effect of two temperatures and stiffness on waves propagating at the interface of two micropolar thermoelastic media, *J. Eng. Phys. and Thermophysics*, 88, 543–555, **2015**.

[28] **Al-Qahtani, H. M. and Datta, S. K.**: Laser-generalized thermoelastic waves in an anisotropic infinite plate: exact analysis, *J. Thermal Stresses*, 31, 569–583, **2008**.

[29] **Sun, Y., Fang, D., Saka, M. and Soh, A. K.**: Laser induced vibrations of micro-beams under different boundary conditions, *Int. J. Sol. and Struct.*, 45, 1993–2013, **2008**.

[30] **Wang, X. and Xu, X.**: Thermoelastic wave induced by pulsed laser heating, *Appl. Phys. A*, 73, 107–114, **2001**.

[31] **Wang, X. and Xu, X.**: Thermoelastic wave in metal induced by ultrafast laser pulses, *J. Thermal Stresses*, 25, 457–473, **2002**.

[32] **Youssef, H. M. and Al-Felali, A. S.**: Generalized thermoelasticity problem of material subjected to thermal loading due to laser pulse, *Appl. Math.*, 3, 142–146, **2012**.

[33] **Ronghou, X., Yanfeng, G. and Weiqin, L.**: Study on generalized thermoelastic problem of semi-infinite plate heated locally by the pulse laser, *International Journal of Engineering Practical Research*, 3, 95–99, **2014**.

[34] **Othman, M. I. A., Zidan, M. E. M. and Hilal, M. I. M.**: Effect of magnetic field on a rotating thermoelastic medium with voids under thermal loading due to laser pulse with energy dissipation. country-regionplace, *Can. J. Phys.*, 10, 1–13, **2014**.

[35] **Kumar, R. and Kumar, A., Singh, D.**: Interaction of laser beam with micropolar thermoelastic solid, *Advances in Physics Theories and Applications*, 40, 10–15, **2015**.

[36] **Tang, D. W. and Araki, N.**: Wavy, wavelike, diffusive thermal responses of finite rigid slabs to high-speed heating of laser-pulses, *Int. J. Heat Mass Trans.*, 42, 855–860, **1999**.