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On Generalized Magneto–Thermoelastic P-, T- and SV-Waves Propagation at the Interface between Two Magnetized Solid–Liquid Media with Initial Stress

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In this paper, we study the effects of magnetic field and initial stress on plane waves propagation. We have investigated the problem of reflection and refraction of thermoelastic waves at a magnetized solid-liquid interface in the presence of initial stress, in the context of CT (Classical theory) of thermoelasticity, the problem has been solved. The boundary conditions applied at the interface are: (i) continuity of the displacement, (ii) vanishing of the tangential displacement, (iii) continuity of normal force per unit initial area, (iv) vanishing of the tangential stress and (v) continuity of temperature. The amplitudes ratios for the incident p-, T-, and SV- waves have been obtained. The reflection and transmission coefficients for the incident waves are computed numerically, considering the initial stress and magnetic field effects and the results are represented graphically.

Keywords: initial stress, CT theory, rotating frame, reflection, refraction, thermoelasticity.

1. Introduction

Recently, attention has been focused on the theory of thermoelasticity because of its utilitarian aspects in diverse fields, especially in Structure Mechanics, Biology, Geology, Geophysics, Acoustics, Plasma Physics. The generalized thermoelasticity theories were developed to eliminate the paradox inherent in the classical theories predicting infinite speed of propagation of heat. The generalized thermoelasticity theories admit the so-called second-sound effects, predicting only finite velocity of propagation of heat. The two theories (LS and GL) ensure finite speeds of propagation for the heat wave. The theory of elasticity with nonuniform heat which was in half-space subjected of thermal shock in this context which known as the theory of uncoupled thermoelasticity and the temperature is governed by a parabolic partial differential equation in temperature term only has been discussed by Danilovskaya [1]. Wide spread attention has been given to thermoelasticity theories which consider finite speed for the propagation of thermal signal. Initial stresses develop in the medium due to various reasons, such as the difference of temperature, process of quenching shot pinning and cold working, slow process of creep, differential external forces, and gravity variations. The Earth isunder high initial stress and, therefore, it is of great interest to study the effect of these stresses on the propagation of elastic waves. Alot of systematic studies have been carried out on the propagation of elastic waves. Biot [2] showed that the acoustic propagation under initial stresses would be fundamentally different from that under stress free state. Lord and Shulman [3] reported a new theory based on a modified Fourier's law of heat conduction with one relaxation time. Later on, a more rigorous theory of thermoelasticity was formulated by Green and Lindsay [4] introducing two relaxationtimes. These non-classical theories are often regarded as the generalized dynamic theories of thermoelasticity. Various problems have been investigated and discussed in the light of these two theories and the studies reveal some interesting phenomena. Green and Naghdi [5, 6] re-examined the basic postulates of thermo-mechanics and discussed undamped heat waves in an elastic solid. Green and Naghdi [7], Chandrasekharaiah [8] discussed different problems in thermoelasticy without energy dissipation. Problems on wave propagation phenomena in coupled or generalized thermoelasticity were discussed by Sinha and Elsibai [9] and Abd-alla and Al-Dawy [10]. Abd-alla et al. [11] investigated the reflection of generalized magneto-thermo-viscoelastic waves at the boundary of a semi-infinite solid adjacent to vacuum.Sinha and Elsibai, [12] investigated the re?ection and refraction of thermoelastic waves at the interface of two semi-in?nite media with two relaxation times. The representative theories in the frame of generalized thermoelasticity are presented by Hetnarski and Ignaczak [13]. Singh [14] investigated reflection and transmission of plane harmonic waves at the interface between liquid and micropolar viscoelastic solid with stretch. Kumar and Sarathi [15] studied reflection and refraction of thermoelastic plane waves at the interface between two thermoelastic media without energy dissipation.Othman and Song [16] discussed plane waves reflection from an elastic solid half-space under hydrostatic initial stress without energy dissipation.Abd-Alla and Abo-Dahab [17] discussed the influence of the viscosity on the reflectionand transmission of plane shear elastic waves at the interface of twomagnetized semi-infinite media.The generalized magneto-thermoelasticity model with two relaxation times in an isotropic elastic medium under the effect of reference temperature on the modulus of elasticity is investigated by Othman and Song [18]. Estimation of the magnetic field effect in an elasticsolid half-space under thermoelastic diffusion is discussed by Abo-Dahab and Singh [19]. The impact of magnetic field, initialpressure, and hydrostatic initial stress on the reflection of P and SV waves considering a Green Lindsay theory is discussed by Abo-Dahab and Mohamed [20].Abo-Dahab et al. [21] studied the rotation and magnetic field effects on P wave reflection from a stress-free surface of an elastic half-space with voids under one thermal relaxation time. Reflection of P and SV waves from stress-free surface of an elastic half-space under the influence of magnetic field and hydrostatic initial stress without energy dissipation is investigated by Abo-Dahab [22]. Abo-Dahab et al. [23] studied relaxation times and magnetic field effects on the reflection of thermoelastic waves from isothermal and insulated boundaries of a half-space. Abo-Dahab and Asad [24] estimated Maxwell's stresses effect on the reflection and transmission of plane waves between two thermo-elastic media in the context of GN model. Deswal et al. [25] studied the reflection and refraction at an interface between two dissimilar, thermally conducting viscous liquid half-spaces. Chakraborty and Singh [26] studied the problem of reflection and refraction of thermo-elastic wave under normal initial stress at a solid-solid interface under perfect boundary condition. Abd-Alla et al. [27] studied the radial deformation and thecorresponding stresses in a homogeneous annular fin of an isotropic material.

Recently, Abo-Dahab and Singh [28] investigated the effects of rotation and voids on the reflection of P waves from stress-free surface of an elastic half-space under magnetic field, initial stress and without energy dissipation. Reflection and refraction of P-, SV- and thermal waves, at an initially stressed solid-liquid interface in generalized thermoelasticity has been discussed by Singh and Chakraborty [29]. Abo-Dahab and Salama [30] discussed plane thermoelastic waves re?ection andtransmission between two solid media under perfect boundary conditions and initial stresswithout and with influence of a magnetic field. Abd-Alla et al. [31] investigated the effect of rotation on the peristaltic flow of a micropolar fluid through a porous medium in the presence of an external magnetic field. Abd-Alla et al. [32] discussed the effects of rotation and initial stress on the peristaltic transport of a fourth grade fluid with heat transfer and induced magnetic field. Song, et al. [33] investigated the reflection and refraction of micropolar magneto-thermoviscoelastic waves at the interface between two micropolar viscoelastic media.

In this paper, p-, T, and SV-waves propagation is investigated under the influence of magnetic field and initial stress. The problem of reflection and refraction of thermoelastic wave at asolid-liquid interface in presence of initial stress and magnetic field considering CT theory has been solved. The boundary conditions at the interface are applied to solve the problem. The appropriate expressions to find the amplitudes ratios for the three incidence waves (P-, SV, and T-wave) have been obtained to calculate the reflection and transmission coefficients numerically. The effects of the initial stress and magnetic field are represented graphically.

2. Formulation of the problem

Let us consider the plane interface between a solid half-space of a homogeneous, isotropic and elastic material and a liquid medium with a primary temperature T_0. A magnetic field acts in the z-direction. The magnetic field effect $\overrightarrow{H}$ in both two media acts in the z-direction, but the medium M (solid) only is under initial stress P. A plane p- or SV-wave is incident in medium M on the plane interface. It is re?ected to p-wave (dilatational wave), SV-wave (rotational wave) and thermal wave (dilatational wave). The rest of the wave continues to travel in the other medium $M^{'}$ after refraction, as p-wave and thermal wave as shown in (Fig. 1).

We assume a system of orthogonal Cartesian coordinates oxyz with origin 'o' in the plane y = 0. Since the problem two-dimensional, we restrict our analysis to plane strain parallel to the oxy-plane. Hence all the ?eld variables depend only on x, y and time t.

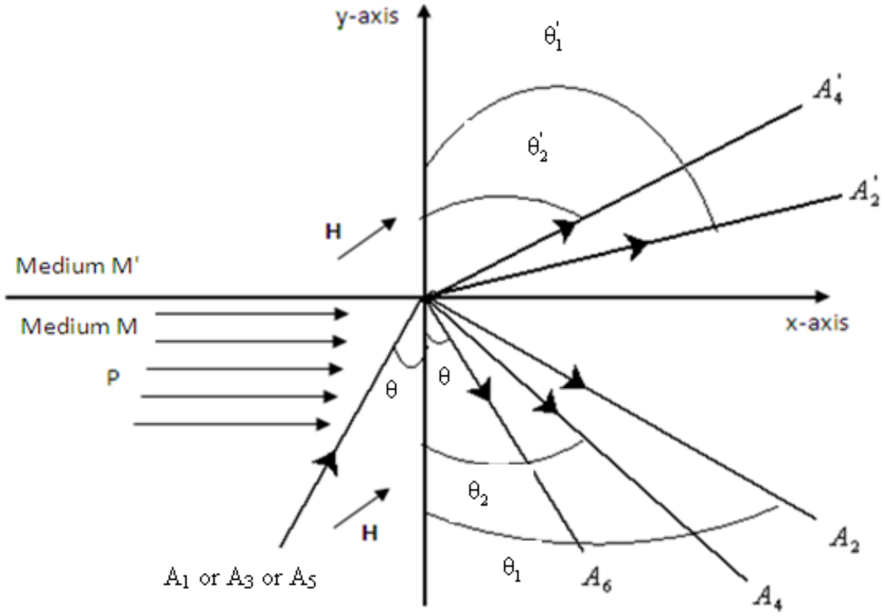

Figure 1 Geometry of the problem

where, θ is the angle of incidence for a plane waves, θ_1 and θ_2 are the angle of reflected waves, $\theta_1^{'}$ and $\theta_2^{'}$ are the angles for the transmitted waves, **H** is the magnetic field vector acting in the z-direction, A_1, A_3 and A_5 are the amplitudes of the incident waves, A_2, A_4 and A_6 are the amplitudes of reflected waves, and $A_2^{'}$and $A_4^{'}$ are the amplitudes of the transmitted T- and SV-waves, respectively (there are two transmitted waves only in medium M').
The initial stress affects medium M only, as shown in Fig. 2, $P = S_{22} - S_{11}$; S_{11} and S_{22} are the normal stresses in the x and y directions, respectively.

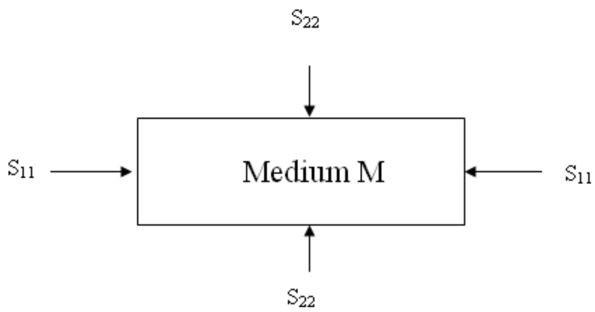

Figure 2 Components of initial stress in solid medium

3. Basic equations

1) The dynamical equations of motion the rotating frame of reference for a plane strain under initial stress in the absence of heat source, as given by Biot [2], taking into account the presence of Lorentz force are:

$$\begin{aligned} &\frac{\partial S_{11}}{\partial x}+\frac{\partial S_{21}}{\partial y}-P\frac{\partial \bar{\omega}}{\partial y}+F_1=\rho\frac{\partial^2 u}{\partial t^2}\\ &\frac{\partial S_{21}}{\partial x}+\frac{\partial S_{22}}{\partial y}-P\frac{\partial \bar{\omega}}{\partial x}+F_2=\rho\frac{\partial^2 v}{\partial t^2}\end{aligned} \tag{1}$$

where: $\bar{\omega}=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$, F_1 and $F_2$are the components of the magnetic field in x and y directions, respectively.

2) The stress-strain relations with incremental isotropy are given by Biot [2]:

$$\begin{aligned} &S_{11}=(\lambda+2\mu+P)\ e_{xx}+(\lambda+P)\ e_{yy}-\gamma\left(T+\tau_1\frac{\partial T}{\partial t}\right)\\ &S_{22}=\lambda e_{xx}+(\lambda+2\mu)\ e_{yy}-\gamma\left(T+\tau_1\frac{\partial T}{\partial t}\right)\\ &S_{12}=2\mu e_{xy}\end{aligned} \tag{2}$$

3) The incremental strain-components are given by Biot [2]:

$$e_{xx}=\frac{\partial u}{\partial x}\qquad e_{yy}=\frac{\partial v}{\partial y}\qquad e_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \tag{3}$$

4) The modified heat conduction equation is:

$$\begin{aligned} &K\nabla^2 T=\rho C_e\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)\\ &+T_0\gamma\left[\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\tau_0\delta_{ij}\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]\end{aligned} \tag{4}$$

5) Taking into account the absence of displacement current, the linearized Maxwell's equations governing the electromagnetic field for a slowly moving solid medium with perfect electrical conductivity are:

$$\begin{aligned} &curl\,\overrightarrow{h}=\overrightarrow{J}\qquad curl\,\overrightarrow{E}=-\mu_e\frac{\partial\overrightarrow{h}}{\partial t}\\ &div\,\overrightarrow{h}=0\qquad div\,\overrightarrow{E}=0\end{aligned} \tag{5}$$

where:

$$\overrightarrow{h}=curl(\overrightarrow{u}\times\overrightarrow{H_0})$$

We have used: $\overrightarrow{H} = \overrightarrow{H_0} + \overrightarrow{h}\,(x,\, z,\, t) \qquad \overrightarrow{H_0} = (0,\, 0,\, H)$

then:

$$F_x = \mu_e H^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right)$$
$$F_y = \mu_e H^2 \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} \right) \tag{6}$$

Again, Maxwell's stress is given as:

$$\tau_{ij} = \mu_e \left[H_i h_j + H_j h_i - H_k h_k \delta_{ij} \right] \tag{7}$$

which reduces to: $\tau_{11} = \tau_{22} = \mu_e H^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right),\ \tau_{12} = 0.$

4. Solution of the problem

With the help of Eqs. (2–3) and (7) in Eq. (1), one gets:

$$\begin{aligned} &\left(\lambda + 2\mu + P + \mu_e H^2\right) \frac{\partial^2 u}{\partial x^2} + \left(\lambda + \frac{P}{2} + \mu + \mu_e H^2 \right) \frac{\partial^2 v}{\partial x \partial y} \\ &+ \left(\mu + \frac{P}{2} \right) \frac{\partial^2 u}{\partial y^2} = \rho \frac{\partial^2 u}{\partial t^2} + \gamma \left(\frac{\partial T}{\partial x} + \tau_1 \frac{\partial^2 T}{\partial x \partial t} \right) \end{aligned} \tag{8}$$

$$\begin{aligned} &\left(\mu - \frac{P}{2} \right) \frac{\partial^2 v}{\partial x^2} + \left(\lambda + e\frac{P}{2} + \mu + \mu_e H^2 \right) \frac{\partial^2 u}{\partial x \partial y} + \left(2\mu + \lambda + \mu_e H^2\right) e \frac{\partial^2 v}{\partial y^2} \\ &= \rho \left(\frac{\partial^2 u}{\partial t^2} \right) + \gamma \left(\frac{\partial T}{\partial y} + \tau_1 \frac{\partial^2 T}{\partial y \partial t} \right) \end{aligned} \tag{9}$$

To separate the dilatational and rotational components of strain, we introduce displacement potentials Φ and Ψ defined by the following relations:

$$u = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial y} \qquad v = \frac{\partial \Phi}{\partial y} + \frac{\partial \Psi}{\partial x} \tag{10}$$

From Eqs. (8) and (10), the following equations are obtained:

$$\begin{aligned} \nabla^2 \Phi = &\frac{\rho}{(\lambda + 2\mu + P + \mu_e H^2)} \left(\frac{\partial^2 \Phi}{\partial t^2} \right) \\ &+ \frac{\gamma}{(\lambda + 2\mu + P + \mu_e H^2)} \left(T + \tau_1 \frac{\partial T}{\partial t} \right) \end{aligned} \tag{11}$$

$$\nabla^2 \Psi = \frac{\rho}{\left(\lambda + \frac{P}{2}\right)} \left[\frac{\partial^2 \Psi}{\partial t^2} \right] \tag{12}$$

From Eqs. (9) and (10), we get:

$$\nabla^2\Phi = \frac{\rho}{(\lambda + 2\mu + \mu_e H^2)}\left[\frac{\partial^2\Phi}{\partial t^2}\right] + \frac{\gamma}{(\lambda + 2\mu + \mu_e H^2)}\left(T + \tau_1\frac{\partial T}{\partial t}\right) \tag{13}$$

$$\nabla^2\Psi = \frac{\rho}{\left(\mu - \frac{P}{2}\right)}\left[\frac{\partial^2\Psi}{\partial t^2}\right] \tag{14}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.
Using Eq. (10) in (4), we get:

$$K\nabla^2 T = \rho C_e\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) + T_0\gamma\frac{\partial}{\partial t}\left(1 + t_0\delta_{ij}\frac{\partial}{\partial t}\right)\nabla^2\Phi \tag{15}$$

5. Solution using CT theory

In Green-Lindsay theory: $\tau_1 = \tau_0 = 0$ and $\delta_{ji} = 0$. Eqs. (11) and (14) can be rewritten as:

$$\nabla^2\Phi = \frac{1}{C_1^2\left(1 + R_H\right)}\frac{\partial^2\Phi}{\partial t^2} + \frac{\gamma}{\rho C_1^2\left(1 + R_H\right)}T \tag{16}$$

$$\nabla^2\Psi = \frac{1}{C_2^2}\left[\frac{\partial^2\Psi}{\partial t^2}\right] \tag{17}$$

where: $R_H = \frac{C_A^2}{C_1^2}$, $C_1^2 = \frac{\lambda+2\mu+P}{\rho}$, $C_2^2 = \frac{\mu-\frac{P}{2}}{\rho}$, $C_A^2 = \frac{\mu_e H^2}{\rho}$. Here R_H, C_A, C_1, C_2, R_H, C_A, C_1, C_2, represent the Alfven speed, the sensitive part of the magnetic field, the velocities of isothermal dilatational and rotational waves respectively, in medium M.
Using CT theory, Eq. (15) may be written as:

$$K\nabla^2 T = \rho C_e\frac{\partial T}{\partial t} + T_0\gamma\frac{\partial}{\partial t}\left(\nabla^2\Phi\right) \tag{18}$$

Eliminating T from Eqs. (16) and (18), we obtain a fourth order differential equation in terms of Φ as:

$$T = \left[\frac{\rho C_1^2\left(1 + R_H\right)}{\gamma}\nabla^2\Phi - \frac{\rho}{\gamma}\left(\frac{\partial^2\Phi}{\partial t^2}\right)\right] \tag{19}$$

$$C_3^2\left(1 + R_H\right)\nabla^4\Phi - \left[\left(1 + R_H + \varepsilon_T\right)\frac{\partial}{\partial t} + \frac{C_3^2}{C_1^2}\frac{\partial^2}{\partial t^2}\right]\nabla^2\Phi + \frac{1}{C_1^2}\frac{\partial^3\Phi}{\partial t^3} = 0 \tag{20}$$

where, $C_3^2 = \frac{K}{\rho C_e}$, $\varepsilon_T = \frac{T_0\gamma^2}{\rho^2 C_e C_1^2}$ (ε_T is thermoelastic coupling constant of the solid medium M).
We now assume a solution in the form:

$$\begin{aligned}\Phi &= f\left(y\right)\exp[ik\left(x - ct\right)]\\ \Psi &= g\left(y\right)\exp[ik\left(x - ct\right)]\\ T &= h\left(y\right)\exp[ik\left(x - ct\right)]\end{aligned} \tag{21}$$

where $c = \frac{\omega}{k}$.

Substituting Eq. (21) in (20), one gets:

$$(1+R_H)\frac{d^4 f}{dy^4}+\left[-2k^2(1+R_H)+ikc\frac{(1+R_H+\varepsilon_T)}{C_3^2}+\frac{k^2c^2}{C_1^2}\right]\frac{d^2 f}{dy^2}$$
$$+\left[k^4(1+R_H)-\frac{k^4c^2}{C_1^2}+\frac{ik^3c^3}{C_1^2C_1^2}\left(1-\frac{C_1^2}{C^2}(1+R_H+\varepsilon_T)\right)\right]f(y)=0 \tag{22}$$

The function Φ in Eq. (21) takes the form:

$$\Phi=[A_1\exp(ikm_1y)+A_2\exp(-ikm_1y)$$
$$+A_3\exp(ikm_2y)+A_4\exp(-ikm_2y)]\exp[ik(x-ct)] \tag{23}$$

where $m_1=\sqrt{q^2c^2-1}$, $m_2=\sqrt{p^2c^2-1}$ and:

$$p^2=\frac{1}{2c_1^2c_3^2}\left[\left\{c_3^2+\frac{i(1+R_H+\varepsilon_T)\,c_1^2}{\omega}\right\}+\sqrt{N}\right] \tag{24}$$

$$q^2=\frac{1}{2c_1^2c_3^2}\left[\left\{c_3^2+\frac{i(1+R_H+\varepsilon_T)\,c_1^2}{\omega}\right\}-\sqrt{N}\right] \tag{25}$$

$$N=\left[c_3^2+\frac{i(1+R_H+\varepsilon_T)\,c_1^2}{\omega}\right]^2-\frac{4i(1+R_H)\,c_1^2c_3^2}{\omega} \tag{26}$$

Using Eq. (21) in (17), we get:

$$\frac{d^2 g}{dy^2}+k^2\left(\frac{c^2}{c_2^2}-1\right)g=0. \tag{27}$$

Eq. (26) suggests that the solution yields two values of $g(y)$, Eq. (20) can be written as:

$$\Psi=\left[\ A_5\exp(ikm_3y)+A_6\exp(-ikm_3y)\ \right]\exp[ik(x-ct)] \tag{28}$$

where:

$$m_3=\sqrt{\frac{c^2}{c_2^2}-1}.$$

The constants $A_i(i=1,2,3,4,5,6)$ in pairs represent the amplitudes of incident and reflected thermal, P- and SV-waves respectively.
Substituting from Eqs. (24) into Eq. (11), we get the value of $h(y)$. Eq. (19) becomes:

$$T=\frac{\rho}{\gamma}\begin{bmatrix} b_1(A_1\exp(ikm_1y)+A_2\exp(-ikm_1y)) \\ b_2(A_3\exp(ikm_2y)+A_4\exp(-ikm_2y)) \end{bmatrix}\exp[ik(x-ct)] \tag{29}$$

where:

$$b_1=\omega^2\left(1-(1+R_H)\,q^2c_1^2\right)\qquad b_2=\omega^2\left(1-(1+R_H)\,p^2c_1^2\right)$$

Setting $\mu = P = 0$in Eqs. (1–4) we obtain the basic equations for a non-viscous liquid medium in the absence of body forces.Using them, we get thedisplacement equations and thetemperature field equation, valid for the liquid medium $M^{'}$. These equations read:

$$\left(\lambda^{'} + \mu_e H^2\right) \frac{\partial^2 u^{'}}{\partial x^2} + \left(\lambda^{'} + \mu_e^{'} H^2\right) \frac{\partial^2 v^{'}}{\partial x \partial y} = \rho^{'} \frac{\partial^2 u^{'}}{\partial t^2} + \gamma^{'} \frac{\partial T^{'}}{\partial x} \tag{30}$$

$$\left(\lambda^{'} + \mu_e H^2\right) \frac{\partial^2 u^{'}}{\partial x \partial y} + \left(\lambda^{'} + \mu_e^{'} H^2\right) \frac{\partial^2 v^{'}}{\partial y^2} = \rho^{'} \frac{\partial^2 v^{'}}{\partial t^2} + \gamma^{'} \frac{\partial T^{'}}{\partial y} \tag{31}$$

$$k^{'} \nabla^2 T^{'} = \rho^{'} C_e^{'} \frac{\partial T^{'}}{\partial t} + T_0^{'} \gamma^{'} \frac{\partial}{\partial t} \left(\nabla^2 \Phi^{'}\right) \tag{32}$$

The primes have been used to designate the corresponding quantities in the liquid medium $M^{'}$ as already been defined for the solid medium M.
Taking:

$$u = \frac{\partial \Phi^{'}}{\partial x}, \quad v = \frac{\partial \Phi^{'}}{\partial y} \tag{33}$$

we get:

$$\nabla^2 \Phi' = \frac{1}{C_1'^2 \left(1 + R_H'\right)} \frac{\partial^2 \Phi'}{\partial t^2} + \frac{\gamma^{'}}{\rho' C_1'^2 \left(1 + R_H'\right)} T^{'} \tag{34}$$

$$K^{'} \nabla^2 T^{'} = \rho^{'} C_e^{'} \frac{\partial T^{'}}{\partial t} + T_0^{'} \gamma^{'} \frac{\partial}{\partial t} \left(\nabla^2 \Phi^{'}\right) \tag{35}$$

where $C_1'^2 = \frac{\lambda^{'}}{\rho^{'}}$
Solving Eqs. (34) and (35) and proceeding exactly in a similar way as for the solid medium M, we get the appropriate solution for $\Phi^{'}$and $T^{'}$ as:

$$\Phi^{'} = \left[A_2^{'} \exp\left(ikm_1^{'} y\right) + +A_4^{'} \exp\left(ikm_2^{'} y\right)\right] \exp\left[ik\left(x - ct\right)\right] \tag{36}$$

$$T^{'} = \frac{\rho^{'}}{\gamma^{'}} \left[b_1^{'} A_2^{'} \exp\left(ikm_1^{'} y\right) + b_2^{'} A_4^{'} \exp\left(ikm_2^{'} y\right)\right] \exp\left[ik\left(x - ct\right)\right] \tag{37}$$

where:

$$b_1^{'} = \omega^2 \left(1 - \left(1 + R_H^{'}\right) q^{'2} C_1^{'2}\right) \qquad b_2^{'} = \omega^2 \left(1 - \left(1 + R_H^{'}\right) P^{'2} C_1^{'2}\right) \tag{38}$$

The constants $A_2^{'}$ and $A_4^{'}$ represent the amplitudes of refracted thermal and p-waves, respectively.
In CT theory, the solutions for $\Phi^{'}$ and $T^{'}$ are in the same form as in Eqs. (23), (26), (28), (36) and (37), respectively, with $m_1^{'} = \sqrt{q^{'2} c^{'2} - 1}$, $m_2^{'} = \sqrt{p^{'2} c^{'2} - 1}$ and $\tau = \tau^{'} = 1$, where:

$$q^{'2}, p^{'2} = \frac{1}{2C_1^{'2} C_3^{'2}} \left[\left\{c_3^{'2} + \frac{i\left(1 + R_H^{'} + \varepsilon_T^{'}\right) C_1^{'2}}{\omega}\right\} \pm \sqrt{N'}\right]$$

$$N^{'} = \left[C_3^{'2} + \frac{i\left(1 + R_H^{'} + \varepsilon_T^{'}\right) C_1^{'2}}{\omega}\right]^2 - \frac{4i\left(1 + R_H^{'}\right) C_1^{'2} C_3^{'2}}{\omega}$$

6. Boundary conditions

1) The normal displacement is continuous at the interface, i.e. $v = v'$. This leads to:

$$\frac{\partial \Phi}{\partial y} + \frac{\partial \Psi}{\partial x} = \frac{\partial \Phi'}{\partial y} \tag{39}$$

Using Eqs. (23), (28) and (36) in the above continuity relation, we get:

$$m_1 A_1 - m_1 A_2 + m_2 A_3 - m_2 A_4 + A_5 + A_6 - m_1^{'} A_2^{'} - m_2^{'} A_4^{'} = 0 \tag{40}$$

2) The tangential displacement must vanish at the interface i.e. $u = 0$. This leads to:

$$\frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial y} = 0$$

Using Eqs. (23) and (28) in the above boundary condition, we get:

$$A_1 + A_2 + A_3 + A_4 - m_3 A_5 + m_3 A_6 = 0 \tag{41}$$

3) The normal force per unit initial area must be continuous at the interface, i.e. $\nabla f_y = \nabla f_y^{'}$. This leads to $s_{22} + \tau_{22} = s_{22}' + \tau_{22}'$ where:

$$\tau_{ij} = \mu_e \left[H_i h_j + H_j h_i - (\overrightarrow{H} \cdot \overrightarrow{h}) \delta_{ij} \right] \qquad i, j = 1, 2, 3$$

Using Eqs. (2), (3) and (7) for medium M, the corresponding equations for medium $M^{'}$ and Eqs. (10) and (32), we obtain:

$$\begin{aligned} &\left(\lambda + \mu_e H^2\right) \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \right) + 2\mu \left(\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial \Psi}{\partial x \partial y} \right) - \gamma \left(T + \tau_1 \frac{\partial T}{\partial t} \right) \\ &= \left(\lambda^{'} + \mu_e^{'} H^{'2}\right) \left(\frac{\partial^2 \Phi^{'}}{\partial x^2} + \frac{\partial^2 \Phi^{'}}{\partial y^2} \right) - \gamma^{'} \left(T^{'} + \tau_1 \frac{\partial T'}{\partial t} \right) \end{aligned} \tag{42}$$

Substituting from Eqs. (23), (28), (29), (36) and (37) into the above equation, we get:

$$\begin{aligned} &\left[-(2+\beta) + c^2 \left(\frac{1}{C_2^2} - \beta q^2 \right) \right] (A_1 + A_2) \\ &+ \left[-(2+\beta) + c^2 \left(\frac{1}{c_2^2} - \beta p^2 \right) \right] (A_3 + A_4) \\ &(2+\beta)\, m_3 \left(A_5 - A_6\right) - \rho^* \left(1 + m_3^2\right) \left(A_2^{'} + A_4^{'} \right) = 0 \end{aligned} \tag{43}$$

where $\rho^* = \frac{\rho'}{\rho}$ and $\beta = \frac{P}{\rho C_2^2}$.

4) The tangential force per unit initial area must vanish at the interface, i.e., $\nabla f_x = 0$. This leads to $s_{12} + Pe_{xy} + \tau_{12} = 0$.

Using Eqs. (2), (3), (7), (10), (23) and (28), we obtain:

$$m_1(A_1 - A_2) + m_2(A_3 - A_4) - \frac{1}{2}\left(m_3^2 - 1\right)(A_5 + A_6) = 0 \tag{44}$$

5) Temperature must be continuous at the interface, i.e, $T = T^{'}$.
Using Eqs. (29) and (36) and simplifying, we get:

$$\begin{aligned}&\left(1-(1+R_H)\,q^2C_1^2\right)(A_1+A_2)+\left(1-(1+R_H)\,p^2C_1^2\right)(A_3+A_4)-\\&-\tfrac{\rho*}{\gamma*\tau*}\left[\left(1-\left(1+R_H^{'}\right)q^{'2}C_1^{'2}\right)A_2^{'}+\left(1-\left(1+R_H^{'}\right)p^{'2}C_1^{'2}\right)A_4^{'}\right]=0\end{aligned} \tag{45}$$

where $\gamma^* = \frac{\gamma^{'}}{\gamma}$ and $\tau^* = \frac{\tau^{'}}{\tau}$.

7. Equations for the reflection and refraction coefficients

To consider the reflection and refraction of a thermoelastic plane wave which is incident at the solid-liquid interface at $y = 0$ making an angle θ with the y–axis, we have three different cases.
Case I: For p-wave incidence, we put $c = p^{-1}\text{cosec}\,\theta$ and $A_1 = A_5 = 0$
Case II: For thermal wave incidence, we put $c = q^{-1}\text{cosec}\,\theta$ and $A_3 = A_5 = 0$.
Case III: For SV-wave incidence, we put $c = c_2\text{cosec}\,\theta$ and $A_1 = A_3 = 0$.
Generalizing, we get a system of five non-homogeneous equations for an incident thermoelastic plane wave:

$$\sum_{i=1}^{5} a_{ij} Z_j = y_i \qquad j = 1,\, 2, ...5) \tag{46}$$

where:

$$\begin{aligned}
&a_{11} = -m_1,\;\; a_{12} = -m_2,\;\; a_{13} = 1,\;\; a_{14} = -m_1^{`},\;\; a_{15} = -m_2^{`},\;\; a_{22} = a_{21} = 1\\
&a_{23} = m_3, \quad a_{24} = a_{25} = 0, \quad a_{31} = \left[-(2+\beta) + c^2\left(\frac{1}{C_2^2} - \beta q^2\right)\right]\\
&a_{32=}\left[-(2+\beta) + c^2\left(\frac{1}{C_2^2} - \beta q^2\right)\right], \quad a_{24} = a_{25} = 0\\
&a_{31} = \left[-(2+\beta) + c^2\left(\frac{1}{C_2^2} - \beta q^2\right)\right], \quad a_{32=}\left[-(2+\beta) + c^2\left(\frac{1}{C_2^2} - \beta q^2\right)\right]\\
&a_{33} = -(2+\beta)\,m_3, \quad a_{34} = a_{35} = -\rho^*\left(1+m_3^2\right), \quad a_{41} = -m_1, \quad a_{42} = -m_2\\
&a_{43} = -0.5\left(m_3^2 - 1\right), \quad a_{44} = a_{45} = 0, \quad a_{51} = \left(1-(1+R_H)\,q^2c_1^2\right)\\
&a_{52} = \left(1-(1+R_H)\,p^2c_1^2\right), \quad a_{53} = 0, \quad a_{54} = -\frac{\rho^*}{\gamma^*\tau^*}\left(1-\left(1+R_H^{'}\right)q^{'2}c_1^{'2}\right)\\
&a_{55} = -\frac{\rho^*}{\gamma^*\tau^*}\left(1-\left(1+R_H^{'}\right)p^{'2}c_1^{'2}\right)
\end{aligned} \tag{47}$$

where, $Z_j\,(j = 1,\, 2...5)$ are the ratios of amplitudes of the reflected thermal, p-, SV-waves and refracted thermal, p-waves to that of incident wave respectively.
For the three particular cases, we get:

(I) For the incident p-wave:

$$y_1 = a_{12}, \quad y_2 = -a_{22}, \quad y_3 = -a_{32}, \quad y_4 = a_{42}, \quad y_5 = -a_{52},$$
$$Z_1 = \tfrac{A_2}{A_3}, \quad Z_2 = \tfrac{A_4}{A_3}, \quad Z_3 = \tfrac{A_6}{A_3}, \quad Z_4 = \tfrac{A_2^{'}}{A_3}, \quad Z_5 = \tfrac{A_4^{'}}{A_3}$$

(II)For the incident thermal wave:

$$y_1 = a_{11}, \quad y_2 = -a_{21}, \quad y_3 = -a_{31}, \quad y_4 = a_{41}, \quad y_5 = -a_{51},$$
$$Z_1 = \tfrac{A_2}{A_1}, \quad Z_2 = \tfrac{A_4}{A_1}, \quad Z_3 = \tfrac{A_6}{A_1}, \quad Z_4 = \tfrac{A_2^{1}}{A_1}, \quad Z_5 = \tfrac{A_4^{'}}{A_1}$$

(III)For the incident SV-wave:

$$y_1 = -a_{13}, \quad y_2 = a_{23}, \quad y_3 = a_{33}, \quad y_4 = -a_{43}, \quad y_5 = a_{53},$$
$$Z_1 = \tfrac{A_2}{A_5}, \quad Z_2 = \tfrac{A_4}{A_5}, \quad Z_3 = \tfrac{A_6}{A_5}, \quad Z_4 = \tfrac{A_2^{'}}{A_5}, \quad Z_5 = \tfrac{A_4^{'}}{A_5}$$

Eq. (46) takes a matrix equation as follows: $AZ = Y$.

8. Numerical results and discussion

With a view to illustrate the numerical analysis of the expressions for the reflection and refraction coefficients, we have used the data for crust as solid medium following Choi and Gurnis [34] and water as liquid medium.

For solid medium (M Crust)

$$\lambda = \mu = 3 \times 10^{10}\,\mathrm{Nm^{-2}}, \qquad \alpha_t = 1.0667 \times 10^{-5}\,\mathrm{K^{-1}},$$
$$C_e = 1100\,\mathrm{JKg^{-1}K^{-1}}, \qquad \rho = 2900\,\mathrm{Kgm^{-3}}, \qquad K = 3\,\mathrm{Wm^{-1}K^{-1}}$$

For liquid medium (M' Water)

$$\lambda^{'} = \mu^{'} = 20.4 \times 10^{9}\,\mathrm{Nm^{-2}}, \qquad \alpha_t^{'} = 69 \times 10^{-6}\,\mathrm{K^{-1}},$$
$$C_e^{'} = 4187\,\mathrm{J\,Kg^{-1}K^{-1}}, \qquad \rho^{'} = 1000\,\mathrm{Kgm^{-3}}, \qquad K^{'} = 0.6\,\mathrm{Wm^{-1}K^{-1}}$$

Taking into consideration $R_H = R_H^{'}$, $\omega = 7.5 \times 10^{13}\,\mathrm{s^{-1}}$, $T_0 = 300\,\mathrm{K}$.[35].

Figs. 3-5, 6-8 and 9-11 show the amplitudes ratios variation with the angle of incident p-wave, T-wave and SV-wave, respectively. The solid line (—) corresponds to the case with initial stress or magnetic field, the dotted line (...) for the case without initial stress or magnetic field.

Figs. 3 and 5 show the amplitudes ratios Z_i (i = 1,2,...,5) variation with the angle of incidence of p-wave for variations of the magnetic field with and without initial stress. It appears that the amplitudes of the reflected T-wave, refracted T- and p-waves start from their maximum values and decreases to zero at$\theta = 90^o$, amplitude ratio of the reflected p-wave tends to unity, on the other hand, the reflection coefficient for the reflected SV-wave equals zero at $\theta = \{0^o, 90^o\}$, increases to its maximum value and then decreases with the increasing of the angle of incidence.

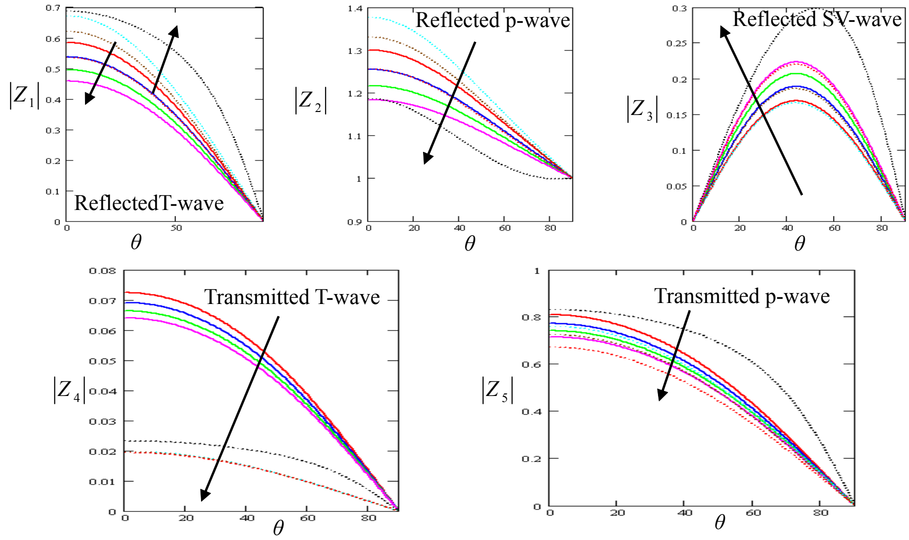

Figure 3 Variation of the amplitudes zi (i=1, 2,, 5) with the angle of incidence of p-wave for variation of magnetic field: H = 0.1, 0.2, 0.3, 0.4, P = $1.1(10)^{11}$(—), P = 0 (...)

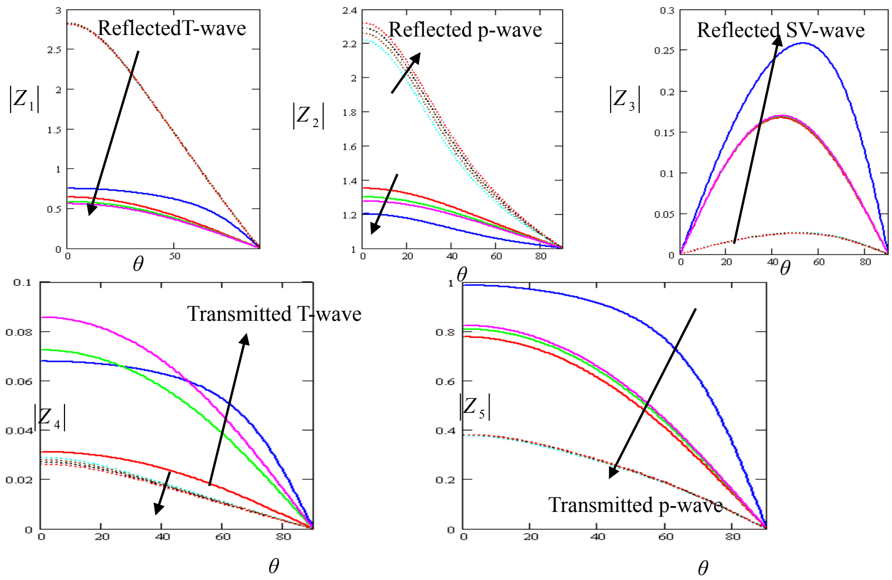

Figure 4 Variation of the amplitudes zi (i = 1, 2,, 5) with the angle of incidence of p-wave for variation of initial stress: P = (1.1, 1.2, 1.3, 1.4) $(10)^{11}$, H = 0.3 (—), H = 0 (...).

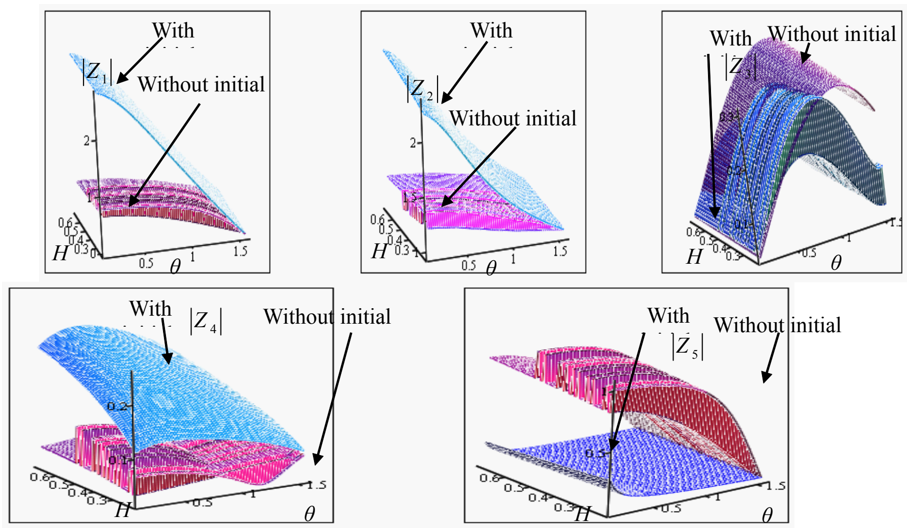

Figure 5 Variation of the amplitudes z_i (i = 1, 2,, 5) with respect to of p-wave with and without variation of initial stress

Physically, we conclude that the reflected and transmitted T- and p- waves start from their maximum values and tend to zero for reflected T-wave, refracted T- and p-waves that indicateinterruption of these waves at the maximum values of the angle of incidence but the reflected p-wave arrives to unity for the maximum angle of incidence; also, the reflected SV-wave starts from and arrives to zero at the minimum and maximum values of θ that indicate the creation of the reflection coefficient if $\theta = 0^o$ and interrupted at $\theta = 90^o$.

With the variation of the magnetic field in the presence or absence of initial stress, it is seen that $|Z_2|$, $|Z_4|$ and $|Z_5|$ decrease with an increase of the magnetic field parameter but $|Z_3|$ increases, $|Z_1|$ decreases with the increasing of the magnetic field in the presence of initial stress but increases if the initial stress is absence. It is shown that if the initial stress is absent, $|Z_1|$, $|Z_2|$, $|Z_3|$ and $|Z_5|$ take on larger values than the correspond values in the presence of initial stress, and vice versa for $|Z_4|$.

Fig. 4 displays the amplitudes ratios with the angle of incidence and variation of the initial stress in the presence or absence of the magnetic field. It is obvious that $|Z_1|$ and $|Z_5|$ decrease with the increase of the initial stress;$|Z_3|$ increases, $|Z_2|$ decreases in the presence of the magnetic field, increases in the absence of H, vice versa for $|Z_4|$. Also, we concluded that the absence of the magnetic field makes small interruption on $|Z_3|$, $|Z_4|$ and $|Z_5|$ but additional factor on $|Z_1|$ and $|Z_2|$.

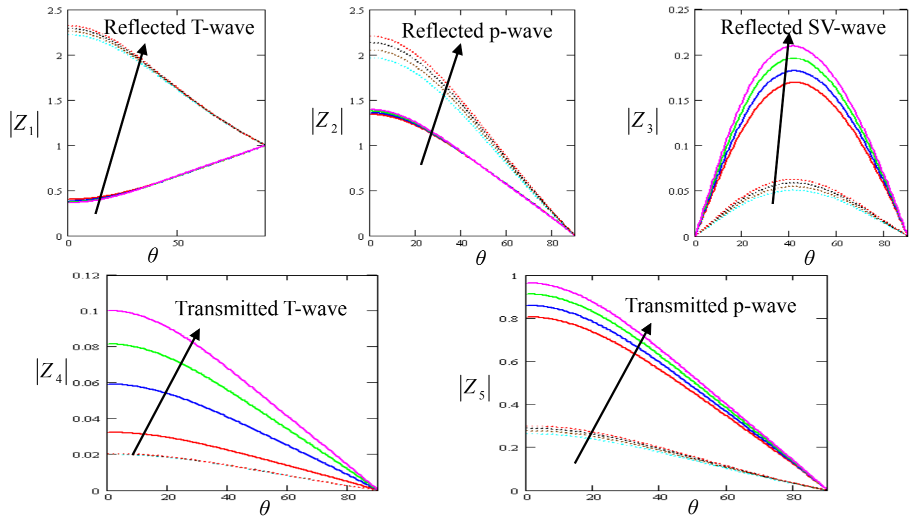

Figure 6 Variation of the amplitudes z_i (i = 1, 2,. . . , 5) with the angle of incidence of thermal-wave for variation of magnetic field: H = 0.1, 0.2, 0.3, 0.4, P = $1.1(10)^{11}$ (—), P = 0 (...)

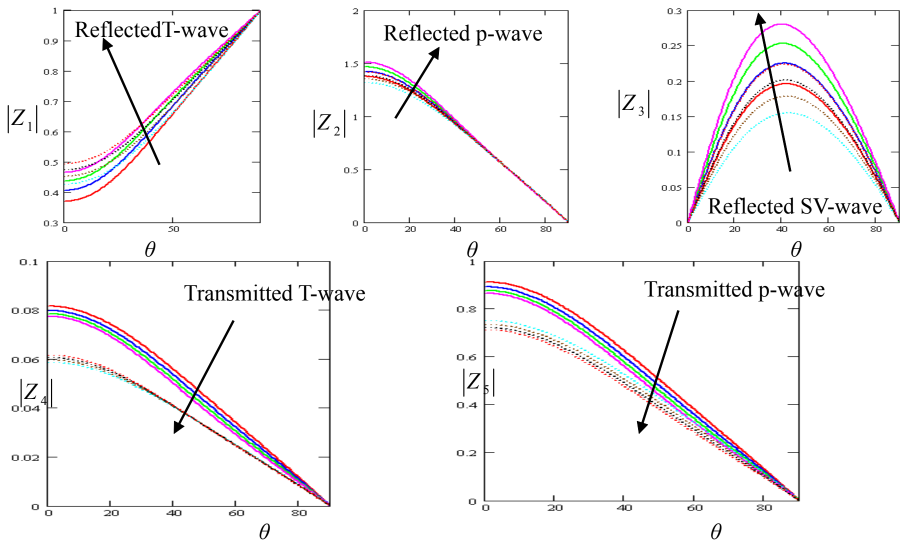

Figure 7 Variation of the amplitudes z_i (i = 1, 2,. . . , 5) with the angle of incidence of thermal-wave for variation of initial stress: P = (1.1, 1.2, 1.3, 1.4) $(10)^{11}$, H = 0.3 (—), H = 0 (...)

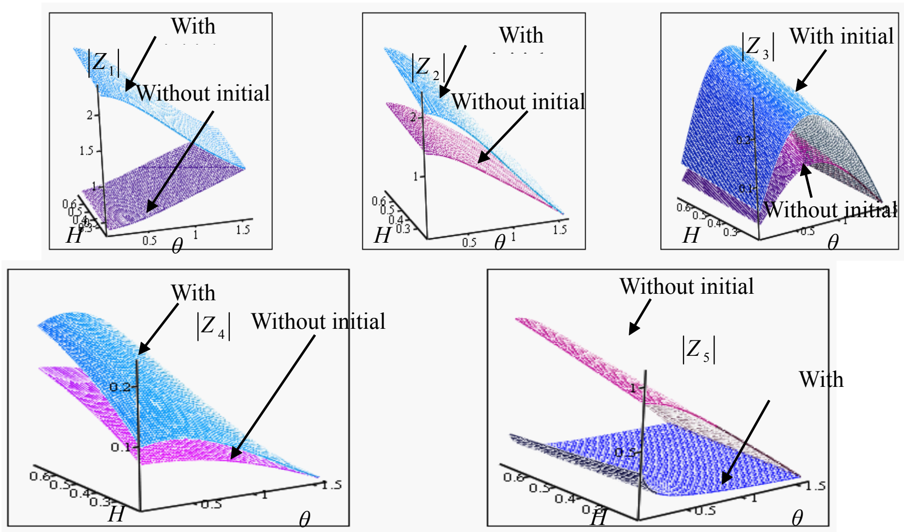

Figure 8 Variation of the amplitudes z_i (i = 1, 2,..., 5) with respect to (θ, H) of thermal-wave with and without variation of initial stress

From Figs. 6–8, it appears that the amplitudes of the reflected p-wave, refracted T- and p-waves start from their maximum values and decrease to zero at $\theta = 90^o$, the amplitude ratio of the reflected T-wave tends to the unity, on the other hand, the reflection coefficient for the reflected SV-wave equal zero at $\theta = \{0^o, 90^o\}$, increases to arrive to its maximum value and then decreases with the increasing of angle of incidence.

Figs. 6 and 8 display the variation of the amplitude ratios Z_i (i = 1,2,...,5) with the angle of incidence of T-wave for variations of the magnetic field, with or without initial stress. It is shown that all amplitudes increase with an increase of the magnetic field in the presence and absence of initial stress, Also, it is clear that $|Z_1|$ and $|Z_2|$ in the presence of initial stress take on larger values than their corresponding values in the absence of initial stress; this indicates the positive effect of the initial stress on the amplitudes ratios, but a negative effect on $|Z_3|$, $|Z_4|$ and $|Z_5|$.

Fig. 7 shows the amplitudes ratios with the angle of incidence and variation of the initial stress in the presence or absence of the magnetic field. It is appear that $|Z_1|$, $|Z_2|$ and $|Z_3|$ increase with the increased values of the initial stress but $|Z_4|$ and $|Z_5|$ decrease, also, we concluded that the absence of the initial stress makes small interruption on $|Z_2|$, $|Z_3|$, $|Z_4|$ and $|Z_5|$but an additional factor on$|Z_1|$.

Finally, for the incidence SV-wave, Figs. 9-11 display the variation of the amplitudes ratios Z_i (i = 1,2,...,5) with the angle of incidence of SV-wave for variation of magnetic field and initial stress. It is shown that $|Z_1|$, $|Z_2|$, $|Z_4|$ and $|Z_5|$start from their maximum values arriving to zero at $\theta = 90^o$ but $|Z_3|$ arrives to unity at $\theta = 90^o$and there is a slight change with variation of magnetic field or initial stress.

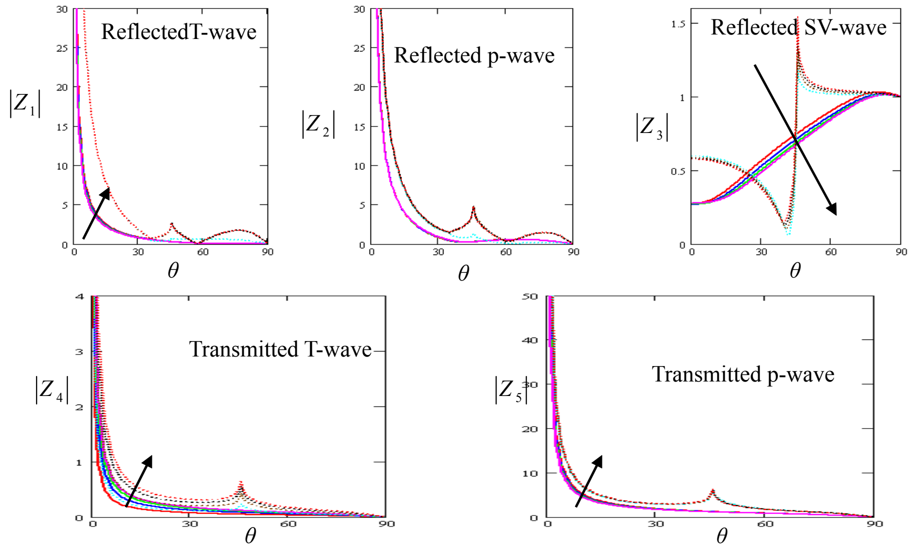

Figure 9 Variation of the amplitudes z_i (i = 1, 2,. . . , 5) with the angle of incidence of SV-wave for variation of magnetic field: H = 0.1, 0.2, 0.3, 0.4, P = 1.1(10)^{11} (—), P = 0 (...)

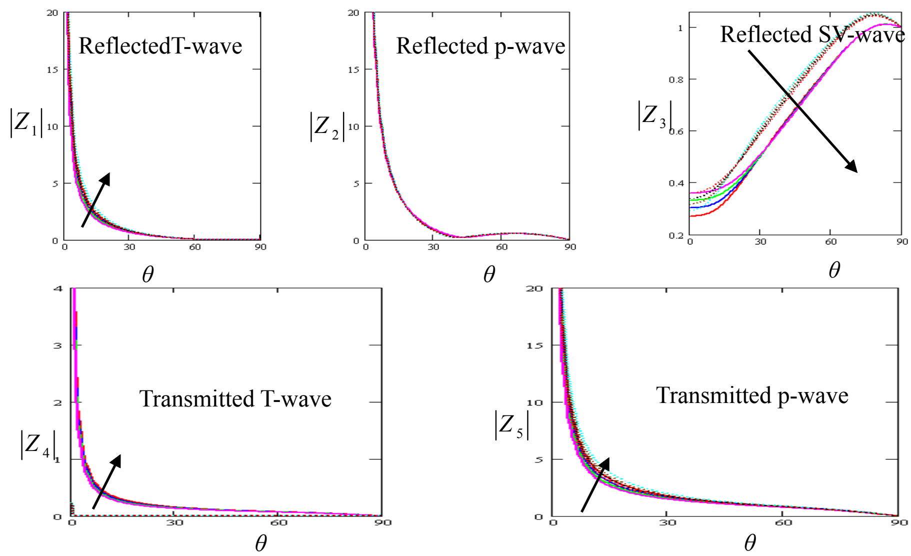

Figure 10 Variation of the amplitudes z_i (i = 1, 2,. . . , 5) with the angle of incidence of SV-wave for variation of initial stress: P = (1.1,1.2,1.3,1.4) (10)^{11}, H = 0.3 (—), H = 0 (...)

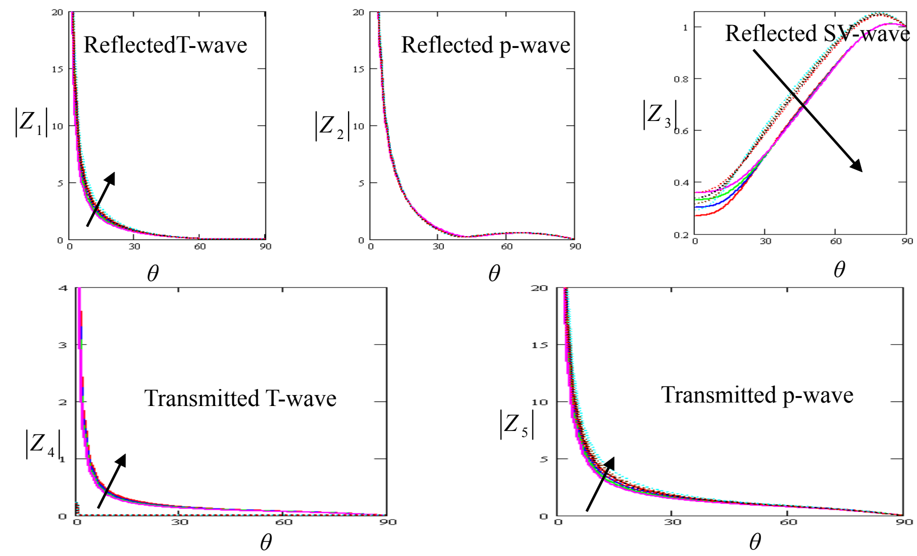

Figure 11 Variation of the amplitudes z_i (i = 1, 2,. . . ,5) with respect to (θ, H) of SV-wave with and without variation of initial stress

9. Concluding remarks

We model the effect of initial stress and magnetized on reflection and refraction of a plane waves at a solid-liquid interface under perfect boundary conditions in the context of CT theory. The waves amplitudes ratios with initial stress and magnetic field with the angle of incidence are obtained in the framework of CT theory investigated numerically and presented graphically.

The following conclusions can be made:

1. The reflected and refracted amplitudes depend on the angle of incidence, initial stress, and magnetic field, the nature of this dependence is different for different reflected waves.

2. The initial stress and magnetic field play a significant role and the effect has the inverse trend for the reflected and transmitted waves.

Finally, it is observed that the reflection and the refraction coefficients strongly appear in the phenomena that has a lot of applications, especially, in Seismic waves, Earthquakes, Volcanoes, and Acoustics.

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Nomenclature

$\vec{B}$ – Magnetic induction vector,
C_v – Specific heat per unit mass,
e_{ij} – Strain components,
$\vec{E}$ – Electric intensity vector,
$\vec{F}$ – Lorentz body forces vector,

$\vec{h}$ – Perturbed magnetic field vector,
$\vec{H}$ – Magnetic field vector,
$\vec{H}_0$ – Primary constant magnetic field vector,
$\vec{J}$ – Electric current density vector,
k – Wave number,
K – Thermal conductivity,
P – Initial stress,
S – Entropy per unit mass,
s_{11}, $s_{22,}s_{12}$Incremental stress components,
T_0 – Natural temperature of the medium,
T – Absolute temperature of the medium,
u_i – Components of the displacement vector,
α_t – Coefficient of linear thermal expansion,
μ_e – Magnetic permeability,
λ and μ – Lamé's constants,
δ_{ij} – Krönecker delta
σ_{ij} – Components of the stress tensor,
τ_{ij} – Maxwell's stress tensor,
τ_0 and τ_1 – Thermal relaxation times,
ω – Frequency,
$\bar{\omega}$ – Magnitude of local rotation,
υ – Phase speed.