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Influence of the Variability of the Elastics Properties on Plastic Zone and Fatigue Crack Growth

Tayeb Kebir
Mohamed Benguediab
Department of Mechanical Engineering-Faculty of Technology
Djillali Liabes University of Sidi Bel Abbes, Algeria
kebirtayeb@live.fr
benguediabm@gmail.com

Abdellatif Imad
Polytech'Lille
University of Lille 1, France
Abdellatif.imad@polytech-lille.fr

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Several theoretical models have been proposed to predict the fatigue crack growth range (FCGR) process using solid mechanics, based theoretical tools and basic or fundamental mechanical properties. Moreover crack growth is linked to the existence of a plastic zone at the crack tip when the formation and intensification are accompanied by dissipation of energy. The overall objective of the present research is to develop, verify, and extend the computational efficiency of the model for fatigue crack growth range (FCGR) function by elastic properties, cyclic hardening and celebrated Paris law. The influence of the variability to elastic properties (Young's modulus E, tensile strength σ_e and cyclic hardening exponent n') is a necessary analysis in this work. The predictions of the proposed model were compared with experimental data obtained by [1].

Keywords: cyclic hardening exponent, tensile strength, Young's modulus, crack tip, analytical model for fatigue crack growth, elastic properties.

1. Introduction

Fatigue crack growth resistance of a material depends upon a number of factors, such as: its composition, mechanical properties and heat treatment conditions, external loading and the environment. The understanding of the mechanisms governing fatigue crack growth has made significant advances since the Paris [2] proposed a law that relates the crack growth rate (da/dN) and the amplitude of stress intensity factor range (ΔK):

$$\frac{da}{dN} = C\left(\Delta K\right)^m \tag{1}$$

In the Paris law there is no evidence about the mechanical parameters effects. Several authors have attempted to integrate others parameters such as the stress ratio (R), critical stress intensity factor (K_C), etc. Broek and al [3] have proposed an empirical relationship by introducing stress ratio effect (Eq. 2):

$$\frac{da}{dN} = C_1\left(\frac{\Delta K}{1-R}\right)^3 \exp\left(-C_2 R\right) \tag{2}$$

Where C_1 and C_2 are the characteristic parameters of the material
For R = 0, this relationship is reduced to:

$$\frac{da}{dN} = C_1\left(\Delta K\right)^3 \tag{3}$$

Experimentally, the exponent m given by the Paris law can vary between 2 and 6 for most metals and alloys. Forman [4] has proposed a new relationship that reflects the fracture by incorporating the stress ratio and critical stress intensity factor (KIC) expressed as follow:

$$\frac{da}{dN} = \frac{C\left(\Delta K\right)^m}{\left(1-R\right)\left(K_{IC}-\Delta K\right)} \tag{4}$$

This equation has been verified many times in the case of aluminum alloys [5, 6, 7]. Richards and al [8] has proposed a relationship from modifications of Bilby's equation [9]. The fatigue crack growth rate law is given by the following equation:

$$\frac{da}{dN} = A\left[\frac{\left(\Delta K-\Delta K_{th}\right)^4}{\sigma_m^2\left(K_C^2-K_{\max}^2\right)}\right]^m \tag{5}$$

In general, the phenomenon of crack closure has been most widely accepted as a critical mechanism influencing many aspects of the behavior of fatigue cracks in metallic materials, including R-ratio effects, variable amplitude loading, crack size, microstructure, environment and the magnitude of the fatigue threshold.
Elber [10] was the first researcher to introduce the concept of crack closure in fatigue crack growth. This concept reveals the premature contact of the crack faces during the unloading portion of the loading cycle while some tensile load is still applied. The fatigue crack growth rate is in function of a new term named effective stress intensity factor range(ΔK_{eff}), defined as:

$$\frac{da}{dN} = C\left(\Delta K_{eff}\right)^m \tag{6}$$

with $\Delta K_{eff} = K_{\max}+K_{op}$, where $K_{\max}$ defines the maximum stress intensity factor and K_{op} the stress intensity factor for the crack opening.
Besides this mechanically based concept, other approaches based on energy consideration [5] or on micro mechanisms acting at the crack tip had been developed.

In these approaches certain authors attempted to express the crack growth rate by explaining the effects of different parameters by means of theoretical models based on the crack tip opening theory [11, 12, 13] and cyclic hardening, on dislocation theory [9, 14].
In the study conducted by Leiting and al [15] an approximation was proposed for (a-N) relation as well as the $da/dN - \Delta K$, in fatigue crack propagation using the Moving Least Squares (MLS) method. This approach can avoid the internal inconsistencies caused by the celebrated Paris's power law approximation of the $da/dN - \Delta K$ relation.
Other models join the macroscopic and cyclical properties of the material such as the cyclic or static consolidation coefficient of the Miner or Manson-Coffin laws, the measurement of the strain hardening exponent is proposed.
Duggan [16] focused on the deformation of a volume element located at the crack-tip. This volume element is only subjected to elastic deformations and so the effect of softening or hardening is neglected. Duggan [16] supposed that the propagation will occur when the Manson-Coffin and Miner laws are verified at the same time. Duggan [16] himself considered the boundary conditions ($\Delta K \to \Delta K_{th}$ or $K_{\max} \to K_C t$), found that:

$$\frac{da}{dN} = \frac{\pi}{4}\left(\frac{1}{\varepsilon'_f E K_C}\right)^2 \Delta K^4 \tag{7}$$

McClintock [17, 11] developed theoretical models to express crack growth based on the crack opening. Taking into account the size of the plastic zone, the expression of the crack propagation is given by:

$$\frac{da}{dN} = \frac{1}{8}\frac{\Delta K^2}{\sigma_e E} \tag{8}$$

Radon [18] proposed a crack growth model for the near threshold region (ΔK_{th}) which incorporates mechanical, cyclic and fatigue properties of the material, and a cyclic plastic strain ($\Delta\varepsilon_P$) based failure criterion. He also introduced an effective stress intensity factor range (ΔK_{eff}) which characterizes the crack tip opening displacement and the strains immediately ahead of the crack tip. The cyclic plastic strain range ($\Delta\varepsilon_P$) at the crack front is given by the expression,

$$\Delta\varepsilon_P = \frac{2\sigma_e}{E}\left[\frac{\Delta K^2}{4\pi(1+n')\sigma_e^2 a}\right]^{\frac{1}{1+n'}} \tag{9}$$

Radon [18] obtained a simple model of cyclic crack propagation in the threshold region and developed an expression for the crack growth rate, given below.

$$\frac{da}{dN} = \frac{2^{1+n'}(1-2\upsilon)^2}{4(1+n')\pi}\frac{(\Delta K_{eff}^2 - \Delta K_{th}^2)}{\sigma_e^{1-n'} E^{1+n'} \varepsilon_f^{1+n'}} \tag{10}$$

where (σ_e) is the cyclic yield stress, (n') is the cyclic strain hardening exponent, (E) is the young's modulus, (υ) is Poisson's ratio and (ε_f) is the fracture strain.
Models based on the calculation of the crack tip with hardening have been developed by many researchers [19, 1] and [21]. These models are about calculating

the plasticized monotonic and cyclical areas created by the cyclical stress; it is generally assumed that the crack propagation is proportional to the energy lost in the plasticized zones. All these models use hysteresis loops to evaluate the plastic deformation. The macroscopic behavior characterized by the hysteresis loop is introduced into these models to describe the microscopic behavior at the crack tip. Lal and al [22] developed a model that integrates some notions, such as the effective strain intensity factor defined by Elber. These authors calculate the monotonous and cyclic plasticized zones:

$$r_{pm} = \left(\frac{\Delta K}{2\sigma_e}\right)^{1+n'} a^{1-n'/2} \tag{11}$$

This relationship is obtained from the calculation performed under simple traction replacing the monotonous work hardening exponent by the cyclic hardening exponent and K by ΔK.

If we apply Elber's hypothesis, Lal and al [22] presume that a single part of a cycle is necessary to close the crack, and thus to create the plasticized zone, the other part of the cycle inducing the grounding of the crack lips. Then they use the definition of the monotonous plastic zone that was calculated previously, by replacing ΔK by ΔK_{eff} and thus obtain the dimensions of the cyclic plasticized zone:

$$r_{pc} = r_{pm} U^{1+n'} \tag{12}$$

Using the hypothesis suggested by Tomkins [23], the fatigue crack growth model is described in function of $\Delta\varepsilon_p$and r_{pc} as follow:

$$\frac{da}{dN} = \Delta\varepsilon_p r_{pc} \tag{13}$$

In order to calculate$\Delta\varepsilon_p$, they use the equation $\Delta\sigma_N/2 = k^{'} \left(\Delta\varepsilon_p/2\right)^{n'}$which describes the hysteresis loops, where $\Delta\sigma_N$ is the variation of stress in the ligament. They expressed the plastic deformation as an amplitude function of the stress intensity factor:

$$\Delta\varepsilon_p = \left[\frac{\Delta K}{(\sqrt{\pi a})k'(1-a/w)}\right]^{\frac{1}{n'}} \tag{14}$$

The propagation rate is then equal to:

$$\frac{da}{dN} = \frac{(0,5U)^{1+n'}}{(k'\sqrt{\pi})^{1/n'}\,\sigma_e^{1+n'}} \left(\frac{\left(\sqrt{a/w}\right)^{n'(1-n')}}{\sqrt{a/w}\,(1-a/w)}\right)^{1/n'(\sqrt{w})^{1-n'-1/n'}(\Delta K)^{1+n'+1/n'}} \tag{15}$$

This equation has been verified for many materials (aluminum, mild steel, stainless steel and copper) and the calculated and measured values of parameters C and m of the Paris' law agree with it.

Two of the parameters of equation (15), (a) and (w) are geometric and easily measurable. The other three U, σ_e, k' and n' depends on the material. The rate decreases when the elasticity limit increases, and it increases if R increases, which matches the experimental results. Parameters σ_e, k' and n' might be easily calculated, but U (except for some aluminum alloys) is less well known. However, there is not enough experimental evidence to determine the variation of the rate as a function of crack length.

Pugno [24] developed a crack propagation model based on the Wöhler curve (S-N). They expressed da/dN in the parameters of the relationship Basquin that assimilates the limited endurance zone N_f to straight line:

$$\sigma_D \Delta\sigma_m^b = N_\infty \Delta\sigma_D^b = N_f \Delta\sigma^b = \overline{C} N_D < N_f < N_\infty \tag{16}$$

They proposed the following expression:

$$\frac{da}{dN} = C(\Delta\sigma\sqrt{\pi(a + (\frac{\Delta\sigma^{b-m}}{C\overline{C}\pi^{m/2}(m/2-1)})^{1/(m/2-1)})})^m \tag{17}$$

Generally speaking, All these models cannot be applied; each describes a situation and becomes unsuitable as soon as a parameter of the experience varies. So, to release the results of a model, the influence of the intrinsic parameters (Young's modulus, grain size, yield strength, toughness) and extrinsic (specimen dimensions, and environmental effects) on the propagation speed of a fatigue crack should be considered.

In numerical investigation, Patel [25] have proposed a crack growth model for multiple surface cracks. This model was used in the study of multiple interacting and coalescing semi-elliptical cracks and plastic zones at crack tip.

In recent investigations, [26, 27] have developed exponential models for 2024 T351, 2024 T3 and 7020 T7 aluminum alloys respectively. This model is described by the following equations:

$$a_j = a_i e^{m_{ij}} (N_j - N_i) \tag{18}$$

$$m_{ij} = \frac{\ln\left(\frac{a_j}{a_i}\right)}{(N_j - N_i)} \tag{19}$$

Fatigue crack growth behavior dependents strongly on initial crack length and previous load history. The specific crack growth rate (m) is correlated by another parameter (l) which takes into account the two crack driving forces ΔK and $K_{\max}$ as well as materials parameters K_C, E, σ_e and is defined by the following equations (Eq. 20 and Eqs. 21, 22):

$$l = [(\frac{\Delta K}{K_C})(\frac{K_{\max}}{K_C})(\frac{\sigma_e}{E})]^{1/4} \tag{20}$$

$$m = A'.l^3 + B'l^2 + C'l + D' \qquad \text{for 2024 T3 and 7020 T7} \tag{21}$$

$$m = A'.l^4 + B'l^3 + C'l^2 + D'l + E' \qquad \text{for 2024 T351} \tag{22}$$

In a recent study, [28] have developed analytical fatigue crack growth rate model based on crack closure expression, effective cyclic plastic zone and the low cycle fatigue properties. Comparative results demonstrate that the fatigue crack growth

rate estimated by the theoretical model closely approximates the experimental results. The predicted model is limited in isotropic material.
The objective of this work is to study the evolution of analytical model for fatigue crack grown rate (FCGR) and the influence of the elastic properties with cyclic hardening (E, σ_e, n') based on the properties obtained experimentally from [1].

2. Analytical model

The relation (1) is valid between $\Delta K_{th} \leq \Delta K \leq \Delta K_{IC}$. By integrating equation (1), the number of cycles is given by:

$$\mathrm{N}_f^{\mathrm{P}} = \int_a^{a_c} \frac{1}{C(\Delta K)^m} da \tag{23}$$

It is the same for the relationship (24) [29]:

$$\frac{\Delta \varepsilon_{\mathrm{P}}}{2} = \varepsilon'_f (2N_f^B)^{c'} \tag{24}$$

We obtain:

$$N_f^B = \frac{1}{2} (\frac{1}{2\varepsilon'_f})^{\frac{1}{c'}} (\Delta \varepsilon_P)^{\frac{1}{c'}} \tag{25}$$

Based on the assumptions made by Pugno [24], who considered that there is equality between the lifetime obtained by the relation of Paris and the one determined by the law of Basquin from the Wöhler curve, so we have:

$$\mathrm{N}_f^{\mathrm{P}} = N_f^B \tag{26}$$

where N_f^P and N_f^B are the final lifetimes obtained by Paris and Manson-Coffin laws respectively.
For crack lengths between a_0 and a_c and an exponent of Paris $m > 2$, the number of cycles is given by:

$$\mathrm{N}_f^{\mathrm{P}} = \frac{2(a)^{\frac{2-m}{2}}}{C((\Delta\sigma)^m)(\pi^{m/2})(m-2)} \tag{27}$$

The equality (26) will be written as follow:

$$\frac{1}{2} (\frac{1}{2\varepsilon'_f})^{\frac{1}{c'}} (\Delta \varepsilon_P)^{\frac{1}{c'}} = \frac{2(a)^{\frac{2-m}{2}}}{C((\Delta\sigma)^m)(\pi^{m/2})(m-2)} \tag{28}$$

From the relationship (28), the expression of the plastic deformation is drawn and written the following way:

$$(\Delta \varepsilon_P) = (2\varepsilon'_f) \left(\frac{4a^{\frac{2-m}{2}}}{C((\Delta\sigma)^m)(\pi^{m/2})(m-2)} \right)^{c'} \tag{29}$$

with $c' = \frac{-2}{m.(n'+1)}$ is obtained by Glinka [30] and $\Delta\sigma^2 = \frac{\Delta K^2}{\pi.a}$ equation (29) can be written as follows:

$$(\Delta \varepsilon_P) = (2\varepsilon'_f) \left(\frac{4a}{C(m-2)})^{(\frac{2}{m})} (\Delta K^{-2} \right)^{(\frac{-1}{\mathrm{n}'+1})} \tag{30}$$

The ray of the cyclic plastic zone is given by the relationship:

$$r_c = \frac{1}{2\pi\sigma_e^2}(\Delta K)^2 \tag{31}$$

Replacing ΔK obtained from equation (9) and introducing it into (31), the expression of the radius of the plastic zone is:

$$r_c = \frac{1}{2\pi\sigma_e^2}[(\frac{\Delta\varepsilon_P E}{2\sigma_e})^{\frac{1+n'}{2}}\frac{((1+n')\pi a)^{\frac{1}{2}}}{(2\sigma_e)^{-1}}]^2 \tag{32}$$

$$r_c = [(\frac{E}{2\sigma_e})^{1+n'}2a(1+n')](\Delta\varepsilon_P)^{1+n'} \tag{33}$$

$$r_c = (\frac{2\varepsilon'_f E}{2\sigma_e})^{1+n'}2a(1+n')[\frac{C(m-2)}{4a}]^{\frac{2}{m}}(\Delta K)^2 \tag{34}$$

Substituting Equation (33) into Equation (13), the crack propagation is given by:

$$\frac{da}{dN} = (\Delta\varepsilon_p)^{2+n'}2a(1+n')(\frac{\mathrm{E}}{2\sigma_e})^{1+n'} \tag{35}$$

Substituting Equation (30) into Equation (35), the crack propagation can be written in a form similar to that of the Paris law:

$$\frac{da}{dN} = A\,(\Delta K)^B \tag{36}$$

where the constants A and B are defined as:

$$A = (\frac{E}{\sigma_e})^{1+n'}2(1+n')(\varepsilon'_f)^{n'+2}\left(\frac{(C(m-2))^{(2n'+4)}}{4^{(2n'+4)}(a)^{(2n'+4-\mathrm{mn'}-\mathrm{m})}}\right)^{(\frac{1}{m(n'+1)})}$$

$$B = \frac{2n'+4}{n'+1}$$

3. Application and discussion

The above model presents the fatigue crack growth based on elastic properties due to the plasticity by cyclic hardening at the crack tip and the constants of the Paris law. All properties parameters for the steel 12NC6 in three states, used for the validation and comparison of the results obtained by the developed model to the experimental results Ould Chikh and al [1], are reported in table 1. In addition, the variability of the elastic properties by hardening parameters is evaluated to show their influence on fatigue crack growth.

In order to test the validity of the proposed model, three applications will be given when the (CT) specimen is used according to ASTM code (ASTM E647-00) with the same applied loading defined in experimental tests Ould Chikh and al [1]. The geometrical parameters and applied loads for specimens are defined in Table 2.

Table 1 Mechanical properties of 12NC6 steel

Material	E (GPa)	σ_e (MPa)	n'	k' (MPa)	C	m	ε_f '	Reference
Mat 1	222	1070	0.062	1604	4.02e-8	2.80	0.20	[1]
Mat 2	187	880	0.034	1052	5.43e-9	2.98	0.22	[1]
Mat 3	177	270	0.233	885	2.87e-9	3.20	0.23	[1]

The stress intensity factor range for specimen is given by the equation:

$$\Delta K = \Delta\sigma f(\frac{a}{w}) \tag{37}$$

where $\Delta\sigma = \frac{\Delta P}{B\sqrt{w}}$; $\Delta P = P_{\max} - P_{\min}$ define the amplitude loading, $B$and w are the thickness and width of the tested specimens respectively.

Table 2 Experimental conditions obtained for tested all material 12NC6 steel

B (mm)	W (mm)	Frequency (Hz)	P_{max} (KN)	P_{min} (KN)	Reference
15	80	30	10	1	[1]

Firstly, the theoretical evaluation of the plastic zone size in the vicinity of the crack tip is handicapped by the distorted picture of the stress field in front of a fatigue crack. So far practically no information on the Young's modulus, cyclic hardening exponent and the Paris law parameter within the plastic zone attending a fatigue crack is available. In this study we propose the cyclic plastic zone size versus by the properties of the different parameters (Eq. 34).

Fig. 1 shows that the evolution in cyclic plastic zone size versus the stress intensity factor range have the same trend comparatively for three materials to those obtained experimentally Ould Chikh and al [1], it can be seen that the results gave a good correlation between experimental data and predicted the cyclic plastic zone size from various materials, thus establishing confidence in the results of the model for materials fracture analysis.

In the testes of the model for fatigue crack growth range (Eq. 36) developed by this work, it is of interest to take into account the crack growth effect which are the different material properties such as Young's modulus, tensile strength, cyclic hardening exponent and the celebrated Paris law.

The values of the materials properties listed in Table 2 are plotted against for experimental curves results [1]. Fig. 2 shows that the experimental and computed values lie near a line drawn through the origin experimental data [1] suggesting a good agreement between them. However, our approach, in this study, is very direct and perhaps explains the variability of different material properties in a more straightforward way.

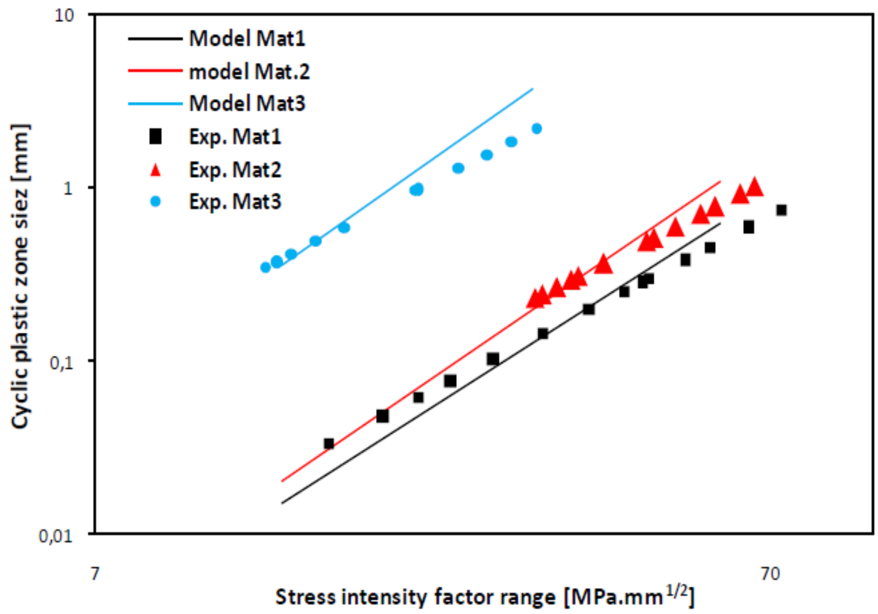

Figure 1 Cyclic plastic zone size versus stress intensity factor range

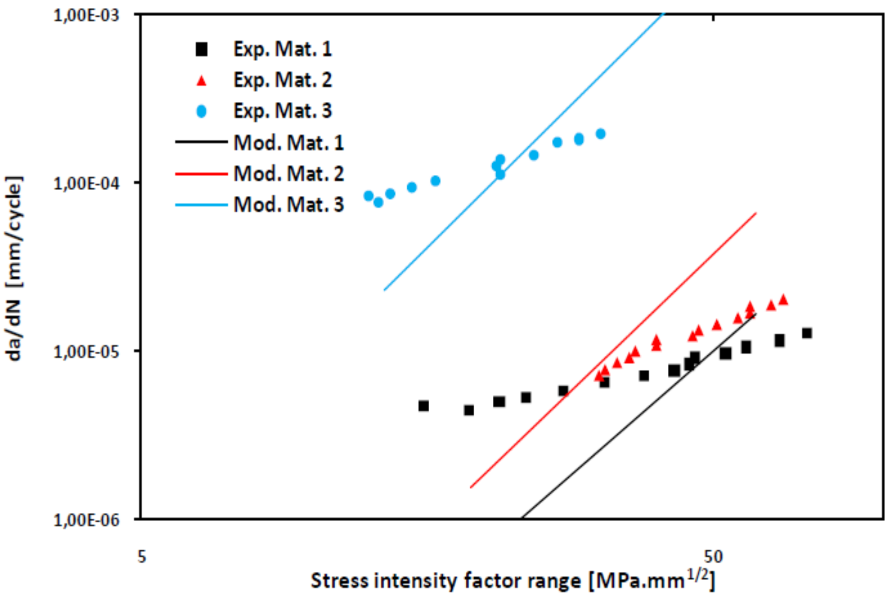

Figure 2 Fatigue crack growth rate versus stress intensity factor range

4. The influence of variability on elastic properties and cyclic hardening exponent

In these respects, the proposed approach has the advantage of being an "interpolation procedure to variability" between the elastic properties (tensile strength (σ_e), Young's modulus E) and cyclic hardening (n') within the cyclic plastic zone size and fatigue crack growth range, hence avoids the risk associated with the inevitable "extrapolation" nature of the many other phenomenological but essentially empirical models. These works correctly modeled suggest a qualitative experimental verification, but the model, in the present form, remains essentially speculative.

4.1. *In cyclic plastic zone size*

Fig. 3 show the variability of the tensile strength (σ_e) on the evolution of cyclic plastic zone size versus the stress intensity factor range (ΔK). The influence of the tensile strength is well marked on the evolution of the cyclic plastic zone size. It is noticed that the plastic zone size increases when the tensile strength (σ_e) decreases. Several studies [31, 1, 22, 32, 33] can be remarked the phenomena.

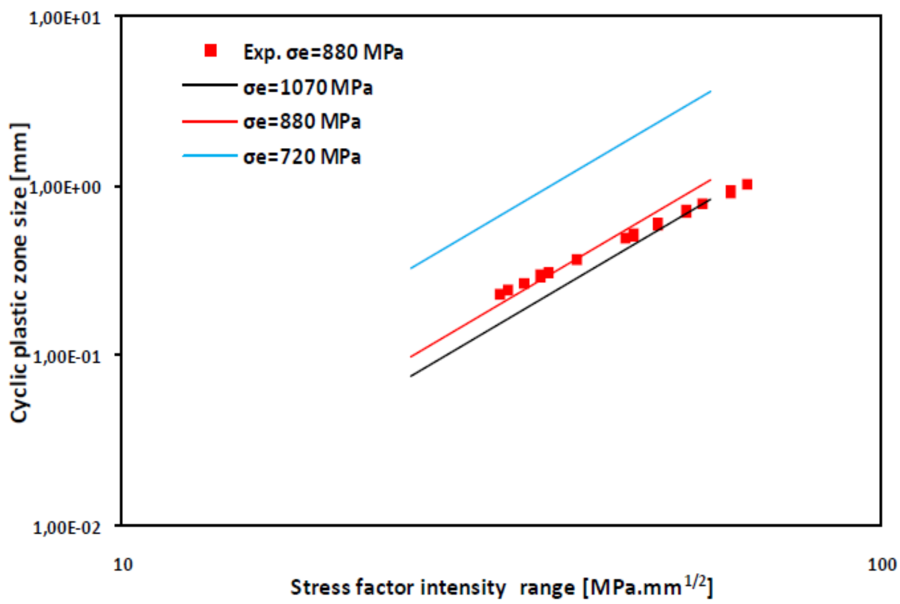

Figure 3 Cyclic plastic zone size versus stress intensity factor range with variability to stress field

Also, the cyclic plastic zone size as a function of stress intensity factor range at different value of Young's modulus (E = 222–187–177 GPa), are plotted shows Fig. 4. Effectually, this variability was influenced slightly proportional the cyclic plastic zone size, signified the Young's modulus is depended of rigidity within hardens materials, can be increase (or decrease) slowly the cyclic plastic zone size for all depends direction of the evaluation [34, 35], as the value E = 187 GPa increase to 222 GPa the cyclic plastic zone size increase to approximate mean value 0,06 mm. Therefore, this indicates that the relationships (Eq. 34) can successfully attentive the influences by Young's modulus.

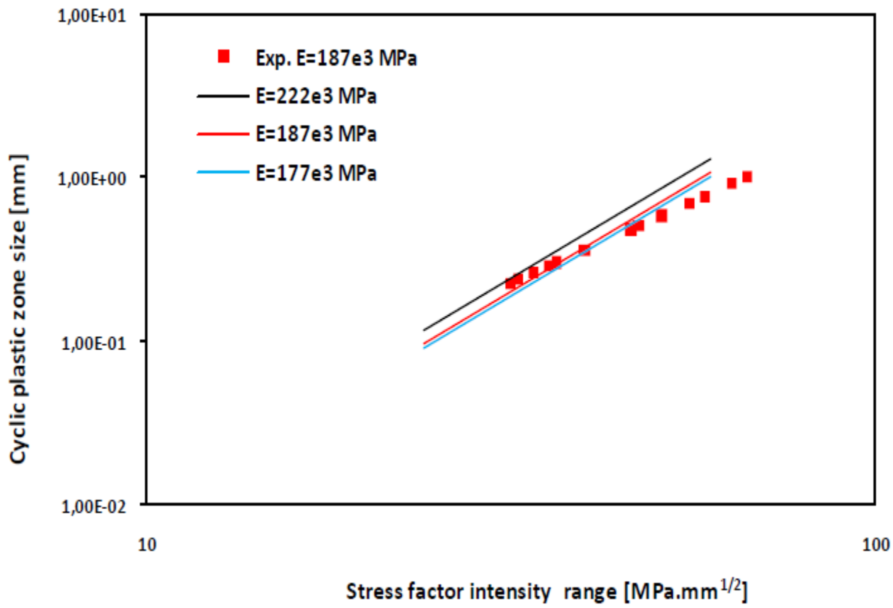

Figure 4 Cyclic plastic zone size versus stress intensity factor range with variability to Young's modulus

Further, the evaluate of cyclic hardening exponent within the relationship to cyclic plastic zone size versus the stress intensity factor range is clearly identified, shown Fig. 5, the influence to variability of cyclic hardening exponent increase when the cyclic plastic zone size increase, to explain by the cyclic plastic zone size near at crack tip becomes higher which induce a brittle (higher hardening exponent n') or ductile (lower hardening exponent n') fractures [33, 36]. The fracture type is related to the level of cyclic hardening exponent.

4.2. *In fatigue crack growth*

Fig. 6 shows the velocity to fatigue crack growth rate versus the stress intensity factor range with three different value to tensile strength (σ_e) into relationship (Eq. 36). The influence to variability it seems clearly suitable thus the velocity to fatigue crack growth rate increase when the tensile strength decreases. However,

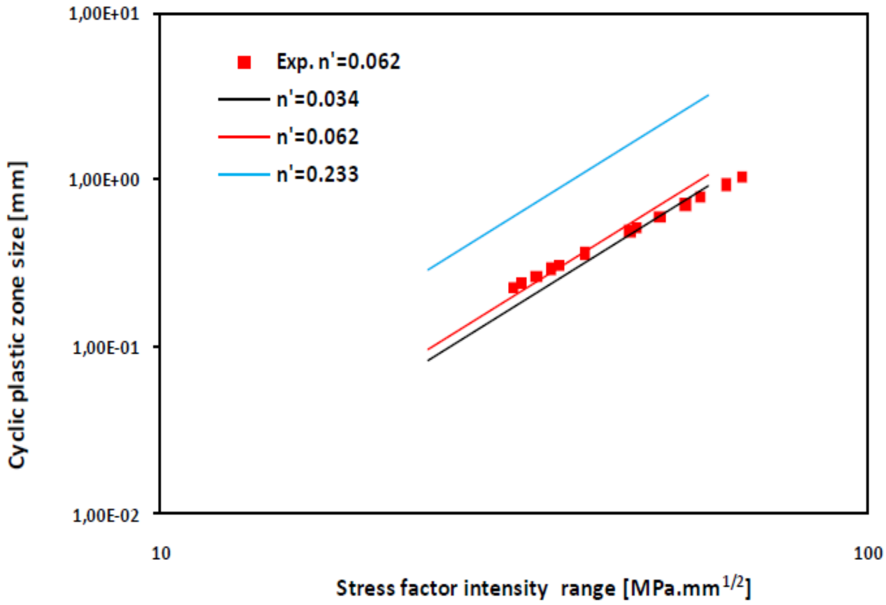

Figure 5 Cyclic plastic zone size versus stress intensity factor range with variability to cyclic hardening exponent

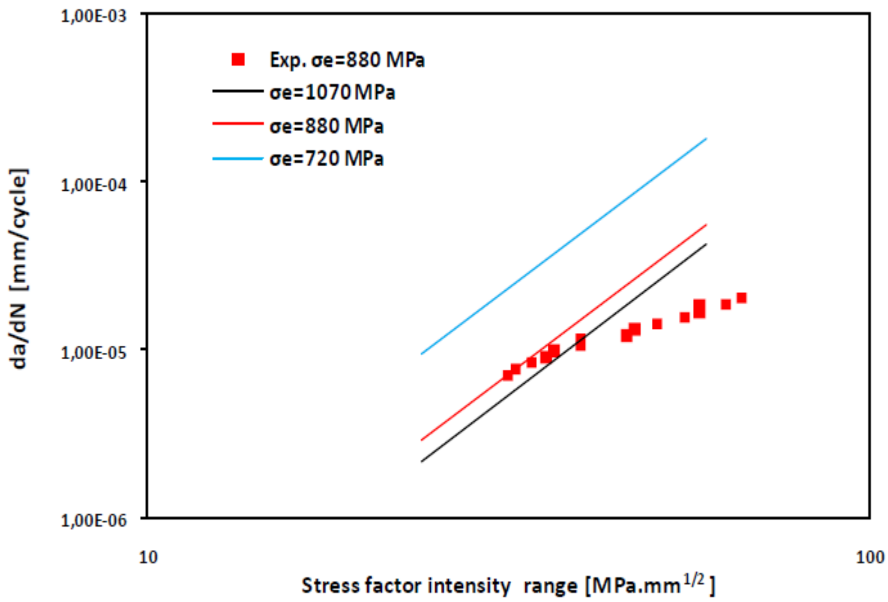

Figure 6 Fatigue crack growth rate versus stress intensity factor range with variability to stress field

these remarks completed to obtain such relationship (Eq. 34) Fig. 3, we can say that the variability of tensile strength is an important parameter into cyclic plastic zone size within this model proposed for fatigue crack growth rate (Eq. 36), which characterizes the stress field and strain at the crack tip.
The fatigue crack growth rates for Aluminum alloys are much more rapid than steel for a given ΔK [1, 36].However, when the influence to variability by Young's Modulus for each metal exhibit about different behavior from fatigue crack growth. Figure 7 shows the fatigue crack growth rate as a function the stress intensity range at different value for Young's modulus for 12NC6 steel study, it is shown that there exist weakly influence by variability to this parameter such as the fatigue crack growth rate increase slowly when increase the Young's modulus.

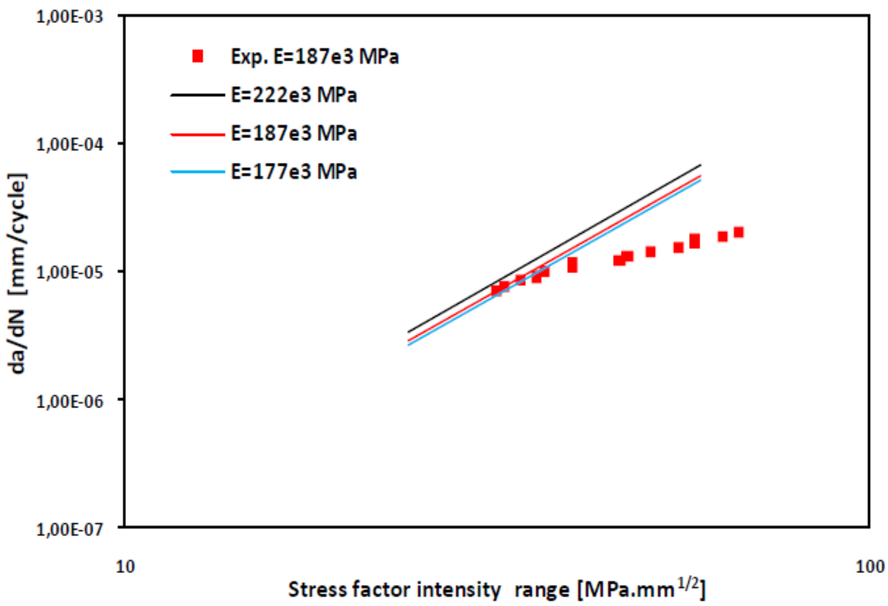

Figure 7 Fatigue crack growth rate versus stress intensity factor range with variability to Young's modulus

Other, at the crack tip on course fatigue cycle, the cyclic plastic zone size effect themo-mechanical modified material behavior, presents an important hardening. There exist evident influences of cyclic hardening exponent (n') on the evolution of fatigue crack growth versus stress intensity factor range ΔK shown in figure 8, the influences to variability clearly of the cyclic hardening exponent (n') increase when the fatigue crack growth range increase, the high level cyclic hardening exponent (n') has been obtained the fracture materials [31, 32, 34, 18, 22].

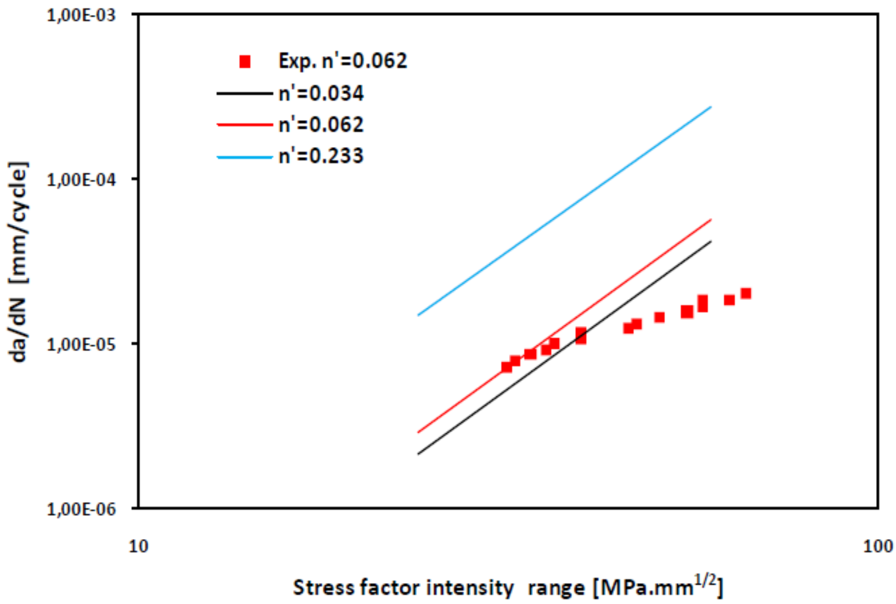

Figure 8 Fatigue crack growth rate versus stress intensity factor range with variability to cyclic hardening exponent

5. Conclusion

This paper focuses on propose model analytical for fatigue crack growth at the crack tip with different materials properties parameters (Tensile strength, Young's modulus and hardening exponent). However, the variability influenced the cyclic plastic zone size behavior. This works suggested notice remarks, for considering the different materials parameters into model proposed form the fatigue crack growth rate can be seen an excellent agreement with experimental data [1], the cyclic plastic zone size depends also on many variables parameters, the variability of these different parameters studies have been influence the cyclic plastic zone size within effects such as fatigue crack growth rate.

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