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The Effect of Heat Laser Pulse on Generalized Thermoelasticity for Micropolar Medium

Mohamed I. A. Othman
Department of Mathematics, Faculty of Science
Taif University, P.O. 888, Taif, Saudi Arabia
Zagazig University, P.O. Box 44519, Egypt
m_i_a_othman@yahoo.com

M. Marin
Department of Mathematics
Transilvania University of Brasov, Romania
m.marin@unitbv.ro

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We introduced the coupled theory, Lord–Schulman theory with one relaxation time and Green-Lindsay theory with two relaxation times to study the influence of thermal loading due to laser pulse on generalized micropolar thermoelasticity. The bounding plane surface is heated by a non–Gaussian laser beam with the pulse duration of 8 ps. The methodology applied here is the use of normal mode analysis to solve the problem of thermal loading due to laser pulse to obtain the exact expressions for the displacement components, force stresses, temperature, couple stresses and microrotation. The distributions of the considered variables are illustrated graphically. Comparisons are made with the results predicted by three theories in the presence of laser pulse and for different values of time.

Keywords: heat laser pulse, micropolar, thermoelasticity, Lord–Schulman, Green-Lindsay.

1. Introduction

Eringen's micropolar theory of elasticity [1] is now well known and does not need much introduction and in this theory, a load across a surface element is transmitted by a force vector along with a couple stress vector. The motion is characterized by six degrees of freedom three of translation and three of the microrotation. Scarpetta [2], Passarella [3] and Eringen [4] developed the linear theory of micropolar elasticity. Tauchert et al. [5] also derived the basic equations of the linear theory of micropolar thermoelasticity. Dost and Tabarrok [6] presented the micropolar generalized thermoelasticity by using Green-Lindsay theory. Chandrasekhariah [7] formulated a theory of micropolar thermoelasticity which includes heat-flux among the constitutive variables. Othman and Singh [8] studied the effect of rotation on generalized micropolar thermoelasticity in a half-space under five theories. Othman and Song [9] investigated the effect of thermal relaxation and magnetic field on generalized micropolar thermoelastic medium.

A generalized theory of linear micropolar thermoelasticity that admits the possibility of "second sound" effects was established by Boschi and Iesan [10]. Kumar and Singh [11] extended the micropolar thermoelasticity with stretch given by Eringen [12]. Marin et al. [13] studied an extension of the domain of influence theorem for anisotropic thermoelastic material with voids. Marin et al. [14] investigated the localization in time of solutions for thermoelastic micropolar materials with voids. Very rapid thermal processes under the action of an ultra-short laser pulse are interesting from the standpoint of thermoelasticity since they require an analysis of the coupled temperature and deformation fields. This means that the absorption of the laser pulse energy results in a localized temperature increase, which in turn causes thermal expansion and generates rapid movements in the structure elements, thus causing the rise of vibrations. This mechanism has attracted considerable attention due to the extensive application of pulsed laser technologies in material processing and non-destructive detection and characterization. Due to the advancement of pulsed lasers, fast burst nuclear reactors and particle accelerators, etc. which can supply heat pulses with a very fast time rise by Bargmann [15] and Boley [16], generalized thermoelasticity theory is receiving serious attention. At present mainly two different models of generalized thermo-elasticity are being extensively used one proposed by Lord and Shulman [17] and the other proposed by Green and Lindsay [18]. Lord and Shulman (L-S) theory suggests one relaxation time and according to this theory, only Fourier's heat conduction equation is modified; while (G-L) theory suggests two relaxation times and both the energy equation and the equation of motion are modified. The so-called ultra-short lasers are those with the pulse duration ranging from nanoseconds to Femto-seconds in general. In the case of ultra-short-pulsed laser heating, the high-intensity energy flux and ultra-short duration laser beam, have introduced situations where very large thermal gradients or an ultra-high heating speed may exist on the boundaries by Hussain et al. [19]. Othman and Abd-Elaziz [20] studied the effect of thermal loading due to laser pulse in generalized thermoelastic medium with voids in the dual-phase-lag model.

Othman, et al. [21] investigated the effect of magnetic field on a rotating thermoelastic medium with voids under thermal loading due to laser pulse with energy dissipation. Othman and Edeeb [22] studied the 2-D problem of a rotating thermoelastic solid with voids and thermal loading due to laser pulse under three theories. These problems are based on the more realistic elastic model since earth; the moon and other planets have angular velocity.

The present paper is motivated by micropolar theory given by Eringen [4]. Here, the normal mode method is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are represented graphically. Numerical results for the field quantities are given and illustrated graphically in the presence of laser pulse and for different values of time.

2. Formulation of the problem

We consider a homogeneous and micropolar thermoelastic half-space ($z \geq 0$), the rectangular Cartesian coordinate system (x, y, z) having originated on the surface $y = 0$. All quantities considered are functions of the time $t$and the coordinates x and z, $u = (u,\, 0,\, w)$ is the dynamic displacement vector and the micro-rotation vector is $\phi = \ (0, \phi_2, 0)$.

3. Basic equations

The system of governing equations of a linear micropolar thermoelasticity without body forces and body couples consists of:
- Equation of motion:

$$\sigma_{ji,j} = \rho u_{i,\,tt} \tag{1}$$

$$\varepsilon_{ijk}\sigma_{jk} + m_{ji,j} = J\rho\phi_{i,\,tt} \tag{2}$$

– The constitutive laws:

$$\sigma_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + k(u_{j,i} - \varepsilon_{ijk}\phi_k) - \gamma_1(T + \nu T_{,t})\delta_{ij} \tag{3}$$

$$m_{ij} = \alpha\,\phi_{r,r}\delta_{ij} + \beta\,\phi_{i,j} + \gamma\,\phi_{j,i} \tag{4}$$

– The heat conduction equation:

$$KT_{,jj} = \rho\, C_E(1 + \tau_0\frac{\partial}{\partial t})T_{,t} + (1 + n_0\tau_0\frac{\partial}{\partial t})(T_0\gamma_1 u_{i,it} - \rho Q) \tag{5}$$

Where, λ, μ are Lamé constants, $k,\ \alpha, \beta, \gamma$ are micropolar constants, ρ is the density, C_E is the specific heat at constant strain, $\tau_0,\ \nu$ are the relaxation times, σ_{ij} are the components of stress, u_i are the components of displacement vector, K is the thermal conductivity, J is the current density vector, m_{ij} is the couple stress tensor, ε_{ijk} is the alternate tensor, T is the temperature distribution, T_0 is the reference temperature chosen so that $|(T - T_0)/T_0| < \ 1$, ϕ microrotation vector, δ_{ij} is the Kronecker delta, $\gamma_1 = (3\,\lambda + 2\,\mu + k)\,\alpha_t$, while α_t is the linear thermal expansion coefficient.
The laser pulse given by the heat input [19]:

$$Q \ = I_0 f(t) g(x) h(z) \tag{6}$$

where, I_0 is the energy absorbed, the temporal profile $f(t)$ is represented as:

$$f(t) = \frac{t}{t_0^2}\exp(\frac{-\,t}{t_0}) \tag{7}$$

Here t_0 is the pulse rise time.

$$g(x) = \frac{1}{2\pi r^2}\exp\,(\frac{-x^2}{r^2}) \tag{8}$$

Where, r is the beam radius.

$$h(z) \ = \ \gamma^*\exp\,(-\gamma^* z) \tag{9}$$

From Eqs. (7–9) into Eq. (6),

$$Q = \frac{I_0 \gamma^*}{2\pi r^2 t_0^2} t \exp(\frac{-x^2}{r^2} - \frac{t}{t_0}) \exp(-\gamma^* z) \tag{10}$$

The constitutive equations can be written as:

$$\sigma_{xx} = \lambda e + (2\mu + k)\frac{\partial u}{\partial x} - \gamma_1 (1 + \nu \frac{\partial}{\partial t}) T \tag{11}$$

$$\sigma_{yy} = \lambda e - \gamma_1 (1 + \nu \frac{\partial}{\partial t}) T \tag{12}$$

$$\sigma_{zz} = \lambda e + (2\mu + k)\frac{\partial w}{\partial z} - \gamma_1 (1 + \nu \frac{\partial}{\partial t}) T \tag{13}$$

$$\sigma_{xz} = \mu \frac{\partial u}{\partial z} + (k + \mu)\frac{\partial w}{\partial x} + k\phi_2 \tag{14}$$

$$\sigma_{zx} = \mu \frac{\partial w}{\partial x} + (k + \mu)\frac{\partial u}{\partial z} - k\phi_2 \tag{15}$$

$$m_{xy} = \gamma \frac{\partial \phi_2}{\partial x} \tag{16}$$

$$m_{zy} = \gamma \frac{\partial \phi_2}{\partial z} \tag{17}$$

4. Method of solution

From Eqs. (11–17) into Eqs. (1–5):

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + \mu)\frac{\partial e}{\partial x} + (\mu + k)\nabla^2 u - k\frac{\partial \phi_2}{\partial z} - \gamma_1 (1 + \nu \frac{\partial}{\partial t}) \frac{\partial T}{\partial x} \tag{18}$$

$$\rho \frac{\partial^2 w}{\partial t^2} = (\lambda + \mu)\frac{\partial e}{\partial z} + (\mu + k)\nabla^2 w + k\frac{\partial \phi_2}{\partial x} - \gamma_1 (1 + \nu \frac{\partial}{\partial t}) \frac{\partial T}{\partial z} \tag{19}$$

$$J\rho \frac{\partial^2 \phi_2}{\partial t^2} = k(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) + \gamma \nabla^2 \phi_2 - 2k\phi_2 \tag{20}$$

$$K\nabla^2 T = \rho C_E (1 + \tau_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial t} + (1 + n_0 \tau_0 \frac{\partial}{\partial t})(T_0 \gamma_1 \frac{\partial e}{\partial t} - \rho Q) \tag{21}$$

For simplifications we shall use the following non-dimensional variables:

$$(x',\, z') = \frac{\eta_0}{C_0}(x,\, z) \qquad (u',\, w') = \frac{\rho \eta_0 C_0}{\gamma_1 T_0}(u,\, w) \qquad (t',\, \tau_0',\, \nu') = \eta_0\, (t,\, \tau_0, \nu)$$

$$\theta' = \frac{T}{T_0} \qquad \sigma_{ij} = \frac{\sigma_{ij}}{\gamma_1 T_0} \qquad \phi_2' = \frac{\rho\, C_0^2}{\gamma_1 T_0}\phi \tag{22}$$

$$m_{ij}' = \frac{\eta_0}{C_0 \gamma_1 T_0} m_{ij} \qquad Q' = \frac{\gamma_1^2}{\rho C_0^2} Q$$

where: $\eta_0 = \frac{\rho C_E C_0^2}{K}$, $\gamma_1 = (3\lambda + 2\mu + k)\, \alpha_t$ and $C_0^2 = (\lambda + 2\mu + k)\,/\rho$.

Eqs. (18–21) take the following form (dropping the dashes for convenience):

$$\frac{\partial^2 u}{\partial t^2} = (\frac{\lambda+\mu}{\rho C_0^2})\frac{\partial e}{\partial x} + \frac{(k+\mu)}{\rho C_0^2}\nabla^2 u - \frac{k}{\rho C_0^2}\frac{\partial \phi_2}{\partial z} - (1+\nu\frac{\partial}{\partial t})\frac{\partial \theta}{\partial x} \tag{23}$$

$$\frac{\partial^2 w}{\partial t^2} = (\frac{\lambda+\mu}{\rho C_0^2})\frac{\partial e}{\partial z} + (\frac{\mu+k}{\rho C_0^2})\nabla^2 w + \frac{k}{\rho C_0^2}\frac{\partial \phi_2}{\partial x} - (1+\nu\frac{\partial}{\partial t})\frac{\partial \theta}{\partial z} \tag{24}$$

$$\frac{J\rho C_0^2}{\gamma}\frac{\partial^2 \phi_2}{\partial t^2} = \nabla^2\phi_2 - \frac{2kC_0^2}{\gamma\eta_0}\phi_2 + \frac{kC_0^2}{\gamma\eta_0^2}(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) \tag{25}$$

$$\nabla^2\theta = (1+\tau_0\frac{\partial}{\partial t})\frac{\partial \theta}{\partial t} + \varepsilon\,(1+n_0T_0\frac{\partial}{\partial t})\frac{\partial e}{\partial t} \;-\; Q_0 f^*\exp\left(-\,\gamma^* z\right) \tag{26}$$

where: $\varepsilon = \frac{\gamma_1^2 T_0}{\rho K \eta_0}$, $Q_0 = \frac{\rho\eta_0^2\gamma^* I_0}{2\pi K T_0 a^2 t_0^2}$, $f^* = [1+n_0(\tau_0 - t)]\exp(\frac{-t}{t_0} - \frac{x^2}{r^2})$.
We introduce the displacement potentials $q(x,\,z,\,t)$and $\psi(x,\,z,\,t)$ which related to displacement components:

$$u = \frac{\partial q}{\partial x} + \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial q}{\partial z} - \frac{\partial \psi}{\partial x}, e = \nabla^2 q, \quad \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = \nabla^2\psi \tag{27}$$

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form:

$$\begin{aligned}&[u, w, \theta, \phi_2, q, \psi, \sigma_{ij}, m_{ij}](x,z,t)\\ &= [u^*, w^*, \theta^*, \phi_2^*, q^*, \psi^*, \sigma_{ij}^*, m_{ij}^*](z)\exp(bt + i\,ax)\end{aligned} \tag{28}$$

Using Eqs. (27, 28) into Eqs. (23–26):

$$(\mathrm{D}^2 - a_1)q^* - a_2\theta^* = 0 \tag{29}$$

$$(a_3\mathrm{D}^2 - a_4)\psi^* - a_5\phi_2^* = 0 \tag{30}$$

$$(\mathrm{D}^2 - a_6)\phi_2^* + a_7(\mathrm{D}^2 - a^2)\psi^* = 0 \tag{31}$$

$$(\mathrm{D}^2 - a_8)\theta^* - a_9(\mathrm{D}^2 - a^2)q^* = -\,Q_0\;f\exp(-\gamma^* z) \tag{32}$$

where: $\mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}z}$, $a_1 = a^2 + b^2$, $a_2 = 1 + \nu b$, $a_3 = \frac{\mu+k}{\rho C_0^2}$, $a_4 = a^2 a_3 + b^2$, $a_5 = \frac{k}{\rho C_0^2}$, $a_6 = a^2 + \frac{J\rho C_0^2 b^2}{\gamma} + \frac{2kC_0^2}{\gamma\eta_0}$, $a_7 = \frac{kC_0^2}{\gamma\eta_0^2}$, $a_8 = a^2 + b + \tau_0 b^2$, $a_9 = \varepsilon(b + n_0\tau_0 b^2)$, $f = f^*\exp(-\,bt - iax)$.
Eliminating q^* between Eqs. (29) and (32):

$$(\mathrm{D}^4 - A\mathrm{D}^2 + B)q^*(z) = -\,a_2 Q_0 f\exp(-\,\gamma^* z) \tag{33}$$

In similar manner we arrive at:

$$(\mathrm{D}^4 - A\mathrm{D}^2 + B)\theta^*(z) = -\,Q_0(\gamma^* - a_1)f\exp(-\,\gamma^* z) \tag{34}$$

Eliminating ψ^* and ϕ_2^* between Eqs. (30) and (31):

$$(\mathrm{D}^4 - C\,\mathrm{D}^2 + F)\psi^*(z) = 0 \tag{35}$$

$$(\mathrm{D}^4 - \; C\,\mathrm{D}^2 + F\,)\phi_2^*(z) = 0 \tag{36}$$

where: $A = a_1 + a_8 + a_2 a_9$, $B = a_1 a_8 + a_2 a_9 a^2$, $C = \frac{a_4 + a_3 a_6 - a_5 a_7}{a_3}$, $F = \frac{a_4 a_6 - a_5 a_7 a^2}{a_3}$.
Eqs. (33, 34) can be factored as:

$$(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)\; q^*(z) = -\; a_2 Q_0 f \exp(-\;\gamma^* z) \tag{37}$$

$$(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)\; \theta^*(z) = -\; Q_0(\gamma^* - a_1) f \exp(-\;\gamma^* z) \tag{38}$$

where: k_n^2 $(n = 1, 2)$ are the roots of the characteristic equation of Eqs. (33) and (34).
Eqs. (35), (36) can be factored as:

$$(\mathrm{D}^2 - \alpha_1^2)(\mathrm{D}^2 - \alpha_2^2)\; \{\psi^*(z), \phi_2^*(z)\} = 0 \tag{39}$$

where, α_s^2 $(s = 1, 2)$ are the roots of the characteristic equation of Eqs. (33) and (34).
The general solution of Eqs. (33–36), bound as $z \to \infty$, is given by:

$$q(x, z, t) = \sum_{n=1}^{2} M_n \exp(-k_n z + bt + iax) + Q_0 a_2 L_1 f^* \exp(-\gamma^* z) \tag{40}$$

$$\theta(x, z, t) = \sum_{n=1}^{2} H_{1n} M_n \exp(-\; k_n z + bt + iax) + Q_0(\gamma^* - a_1) L_1 f^* \exp(-\gamma^* z) \tag{41}$$

$$\psi(x, z, t) = \sum_{n=1}^{2} R_n \exp(-\;\alpha_n z + bt + iax) \tag{42}$$

$$\phi_2(x, z, t) = \sum_{n=1}^{2} H_{2n} R_n \exp(-\;\alpha_n z + bt + iax) \tag{43}$$

where: $L_1 = \frac{-1}{\gamma^{*4} - A\gamma^{*2} + B}$, $H_{1n} = \frac{k_n^2 - a_1}{a_2}$, $H_{2n} = \frac{a_3 \alpha_n^2 - a_4}{a_5}$, $n = 1, 2$,
Eq. (27) together with Eqs. (40) and (42) give:

$$u(x, z, t) = \sum_{n=1}^{2} [iaM_n \exp(-k_n z) - \alpha_n R_n \exp(-\;\alpha_n z)] \exp(bt + iax)$$
$$-Q_0 L_1 a_2 (\frac{2x}{r^2})\, f^* \exp(-\;\gamma^* z) \tag{44}$$

$$w(x, z, t) = \sum_{n=1}^{2} [-k_n M_n \exp(-k_n z) - iaR_n \exp(-\;\alpha_n z)] \exp(bt + iax)$$
$$-Q_0 L_1 a_2\, \gamma^* f^* \exp(-\;\gamma^* z) \tag{45}$$

Substituting from Eq. (22) in Eqs. (11–17) and with the help of Eqs. (41–45) we obtain the components of stresses and tangential couple stress:

$$\sigma_{xx}(x, z, t) = \sum_{n=1}^{2} [H_{3n} M_n \exp(-k_n z) - H_{4n} R_n \exp(-\;\alpha_n z)] \exp(bt + iax)$$
$$+Q_0 (f_1 - f_2) \exp(-\;\gamma^* z) \tag{46}$$

$$\sigma_{yy}(x,z,t)=\sum_{n=1}^{2}H_{5n}M_n\exp(-\,k_nz+bt+iax)$$
$$+Q_0(f_3-f_2)\exp(-\,\gamma^*z) \tag{47}$$
$$\sigma_{zz}(x,z,t)=\sum_{n=1}^{2}[H_{6n}M_n\exp(-k_nz)-H_{4n}R_n\exp(-\,\alpha_nz)]\exp(bt+iax)$$
$$+Q_0(f_4-f_2)\exp(-\,\gamma^*z) \tag{48}$$
$$\sigma_{xz}(x,z,t)=\sum_{n=1}^{2}[H_{7n}M_n\exp(-k_nz)+H_{8n}R_n\exp(-\alpha_nz)]\exp(bt+iax)$$
$$+Q_0f_5\exp(-\gamma^*z) \tag{49}$$
$$\sigma_{zx}(x,z,t)=\sum_{n=1}^{2}[H_{9n}M_n\exp(-k_nz)+H_{10n}R_n\exp(-\,\alpha_nz)]\exp(bt+iax)$$
$$+Q_0f_5\exp(-\,\gamma^*z) \tag{50}$$
$$m_{xy}(x,z,t)=\sum_{n=1}^{2}\frac{i\,a\,\gamma\,\eta_0^2}{\rho\,C_0^4}H_{2n}R_n\exp\,(-\,\alpha_nz) \tag{51}$$
$$m_{zy}(x,z,t)=\sum_{n=1}^{2}\frac{-\,\gamma\,\eta_0^2}{\rho\,C_0^4}\alpha_nH_{2n}R_n\exp(-\,\alpha_nz) \tag{52}$$

where: M_n and $R_n (n = 1,\,2)$ are some parameters and:

$$H_{3n}=-\,a^2+\frac{\lambda}{\rho C_0^2}k_n^2-H_{1n}+\nu bH_{1n}\qquad H_{4n}=i\,a\,\alpha_n(\frac{\lambda}{\rho\,C_0^2}-1)$$

$$H_{5n}=\frac{\lambda}{\rho C_0^2}(-\,a^2+k_n^2-H_{1n}+\nu bH_{1n})\qquad H_{6n}=k_n^2-\frac{\lambda}{\rho C_0^2}a^2-H_{1n}+\nu bH_{1n}$$

$$H_{7n}=\frac{-ia}{\rho C_0^2}[\mu k_n+(\mu+k)]\qquad H_{8n}=\frac{1}{\rho C_0^2}[\mu\alpha_n^2+a^2(\mu+k)+kH_{2n}]$$

$$H_{10n}=\frac{1}{\rho C_0^2}[\mu a^2+\alpha_n^2(\mu+k)-kH_{2n}]$$

$$f_1=L_1f^*[a_2(\frac{-\,2}{r^2}+\frac{4x^2}{r^4})+\frac{\lambda}{\rho C_0^2}a_2\gamma^*-(\gamma^*-a_1)]$$

$$f_2=\nu L_1(\gamma^*-a_1)[n_0+\frac{1+n_0(\tau_0-t)}{t_0}]\exp(\frac{-x^2}{r^2}-\frac{t}{t_0})$$

$$f_3=L_1f^*[\frac{\lambda a_2}{\rho C_0^2}(\frac{-2}{r^2}+\frac{4x^2}{r^4})+\frac{\lambda}{\rho C_0^2}a_2\gamma^*-(\gamma^*-a_1)]$$

$$f_4=L_1f^*[\frac{\lambda a_2}{\rho C_0^2}(\frac{-2}{r^2}+\frac{4x^2}{r^4})+a_2\gamma^*-(\gamma^*-a_1)]$$

$$f_5=\frac{a_2\gamma^*L_1f^*}{\rho C_0^2}(\frac{-2x}{r^2})(k-2\mu)$$

5. Boundary conditions

In order to determine the parameters $M_n, R_n (n = 1, 2)$ we need to consider the boundary condition at $z = 0$ as follows:
Thermal boundary condition:

$$\frac{\partial \theta}{\partial z} = p_2 \exp(bt + iax) \tag{53}$$

Mechanical boundary condition:

$$\sigma_{xx} = -p_1 \exp(bt + iax),\ \sigma_{xz} = 0, \quad m_{zy} = 0 \tag{54}$$

Substituting the expression of the variables considered into the above boundary conditions, we can obtain the following equations satisfied by the parameters:

$$\sum_{n=1}^{2} - k_n H_{1n} M_n = p_2 \tag{55}$$

$$\sum_{n=1}^{2} (H_{3n} M_n + H_{4n} R_n) = -p_1 \tag{56}$$

$$\sum_{n=1}^{2} (H_{7n} M_n + H_{8n} R_n) = 0 \tag{57}$$

$$\sum_{n=1}^{2} H_{2n} R_n = 0 \tag{58}$$

Solving Eqs. (55–58) for $M_n, R_n\ (n = 1, 2)$ by using the inverse of matrix method as follows:

$$\begin{pmatrix} M_1 \\ M_2 \\ R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} -k_1 H_{11} & -k_2 H_{12} & 0 & 0 \\ H_{31} & H_{32} & H_{41} & H_{42} \\ H_{71} & H_{72} & H_{81} & H_{82} \\ 0 & 0 & -\alpha_1 H_{13} & -\alpha_2 H_{14} \end{pmatrix}^{-1} \begin{pmatrix} p_2 \\ -p_1 \\ 0 \\ 0 \end{pmatrix} \tag{59}$$

6. Numerical results and discussions

The analysis is conducted for a magnesium crystal-like material. Following Othman and Singh [8] the values of physical constants are:

$$T_0 = 298K^\circ, \qquad \lambda = 9.4 \times 10^{10} Nm^{-2}, \qquad \mu = 4.0 \times 10^{10} Nm^{-2},$$

$$k = 1.0 \times 10^{10} Nm^{-2}, \qquad \rho = 1.74 \times 10^3\ kg/m^3, \qquad \gamma = 0.779 \times 10^{-9} N,$$

$$J = 0.2\times10^{-15} m^2, \qquad C_E = 1.04\times10^3\ kg\ m^{-3}, \qquad K = 1.7\times10^2 Jm^{-1}s^{-1}\deg^{-1},$$

$$\alpha_t = 7.4033\times10^{-7} K^{-1}, \quad b = b_0 + i\,\xi, \quad b_0 = 0.6, \qquad \xi = 0.1, \qquad p_1 = 1, \quad p_2 = 1.$$

where: b_0 is the complex time constant.
The laser pulse parameters are $I_0 = 10^7 J$, $r = 100\,\mu m$, $\gamma^* = 8\mathrm{m}^{-1}$, $t_0 = 8$ p. sec.

The comparisons have established for two values of time ($t = 0.1,\ 0.9$) in the context of the coupled theory (CD), (L-S) and (G-L) theories, on the surface plane $x = 1$. The numerical technique outlined above is used for the distribution of the real part of the non-dimensional displacements $u$and w, the non-dimensional temperature θ, the distributions of non-dimensional stresses $\sigma_{xx}$and σ_{xz}, the non-dimensional micro-rotation ϕ_2 and the non-dimensional couple stress $m_{xy}$for the problem, the results are shown in Figs. 1-7. The graphs show the six curves predicted by three different theories of thermoelasticity. In this figure, the solid lines represent the solution in the (CD) theory, the dashed lines represent the solution derived using the (L-S) theory and the dotted lines represent the solution in the generalized (G-L) theory.

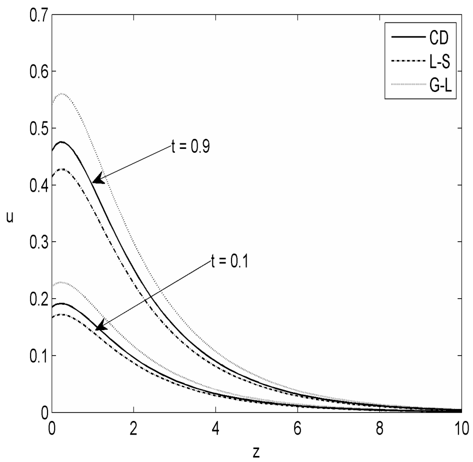

Figure 1 Variation of thermal displacement u with horizontal distance z

In Fig. 1, the displacement component u is plotted against the distance z, it is observed that the displacement u for (G-L) theory is greater than that of the (L-S) theory and the (CD) theory. It is clear that the values of solutions for ($t = 0.9$) are greater than that for ($t = 0.1$). Fig. 2 shows the distribution of displacement components w in the context of the three theories; it noticed that the distribution of w decreases with the increase of the distance z, and the values of solutions for ($t = 0.9$) are greater than that for ($t = 0.1$). Fig. 3 explains the distribution of temperature θ, against the distance z, this figure shows the similar behaviors as those of figure 2. Fig. 4 explains the distribution of normal stress σ_{xx} against the distance z, we see that the values of solutions for ($t = 0.9$) are smaller than that for ($t = 0.1$) and the normal stress σ_{xx} for (G-L) theory is greater than that of the (L-S) theory and the (CD) theory. Fig. 5 investigates the distribution of tangential stress σ_{xz} versus the distance z. The values of σ_{xz} for (CD) theory are small compared to those in the other theories. The values of σ_{xz} start from zero, which agree with the boundary conditions.

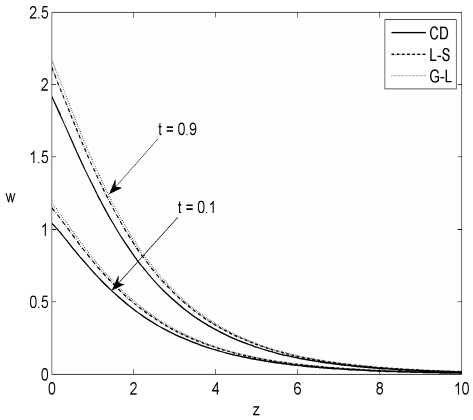

Figure 2 Variation of normal displacement w with horizontal distance z

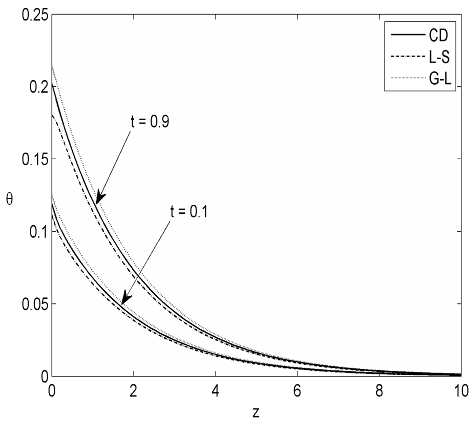

Figure 3 Variation of conductive temperature θ with horizontal distance z

Fig. 6 depicts the variations of the normal micro-rotation ϕ_2 against the distance z, from this figure we see that the curves start from the same value at $z = 0$, the values of solutions for $(t = 0.1)$ are smaller than that for $(t = 0.9)$. Fig. 7 exhibits the values of tangential couple stress m_{xy} against the distance z. It is clear that the values of solutions for (CD) theory are large in comparison with those for (L-S) and (G-L) theories. The values of solutions for $(t = 0.9)$ are smaller than that for $(t = 0.1)$.

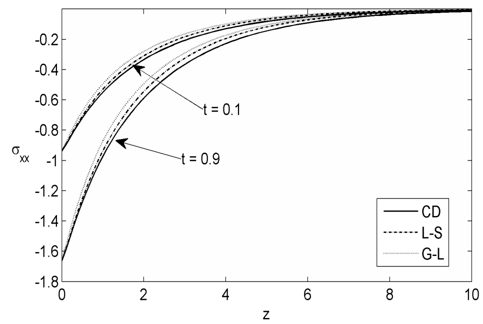

Figure 4 Variation of normal stresses σ_{xx} with horizontal distance z

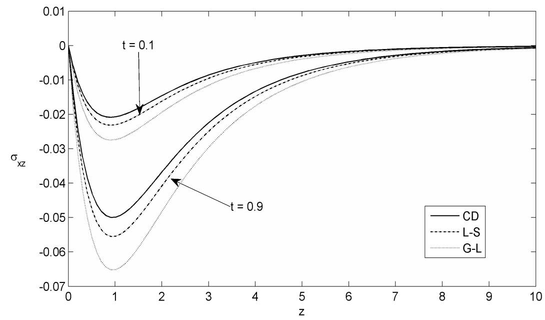

Figure 5 Variation of tangential stresses σ_{xz} with horizontal distance z

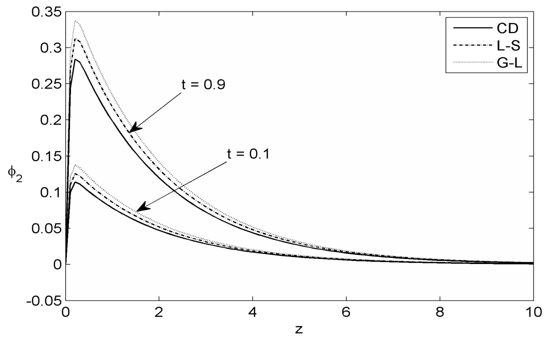

Figure 6 Variation of micro-rotation ϕ_2 with horizontal distance z

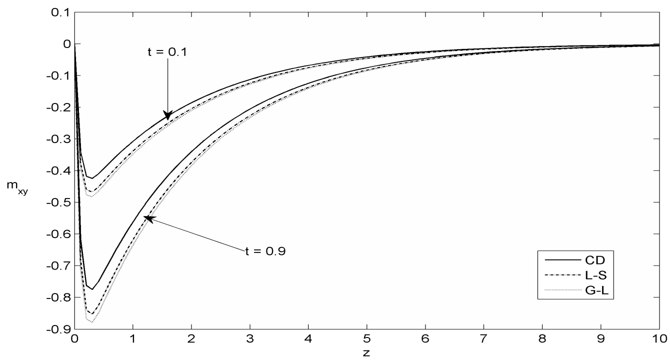

Figure 7 Variation of couple stress $m_{xy}$with horizontal distance z

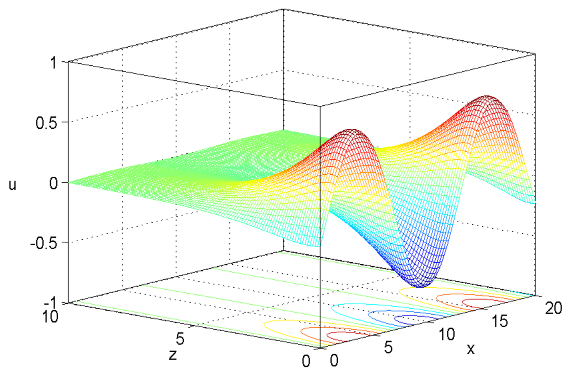

Figure 8 (3D) Horizontal component of displacement $u$against both components of distance based on (CD) theory, in the presence of laser pulse

Figs. 8–10 are giving 3D surface curves for the physical quantities, i.e., the horizontal displacement u and the tangential stress component $\sigma_{xz}$and the micro-rotation component ϕ_2 of the thermal shock problem in the presence of laser pulse at $(t = 0.1)$ in the context of the (CD) theory. These figures are very important to study the dependence of these physical quantities on the vertical component of distance. The curves obtained are highly depending on the vertical distance from origin, all the physical quantities are moving in wave propagation.

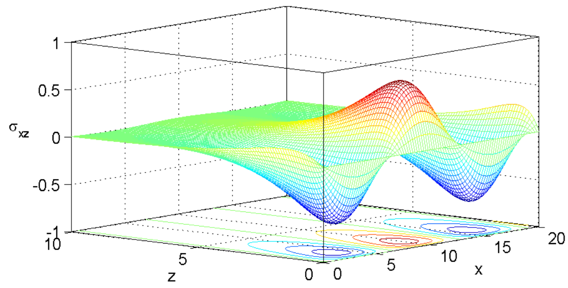

Figure 9 (3D) Distribution of stress component σ_{xz} against both components of distance based on (CD) theory, in the presence of laser pulse

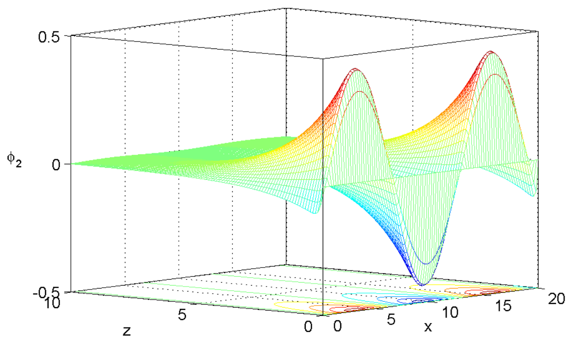

Figure 10 (3D) Distribution of micro-rotation ϕ_2 against both components of distance based on (CD) theory, in the presence of laser pulse

7. Conclusions

According to the above results, we can conclude that:

1. The time parameter t in the current model has significant effects on all the fields.

2. The value of all physical quantities converges to zero with an increase in distance z and all the functions are continuous.

3. The comparison of different theories of thermoelasticity, *i.e.* (CD), (L-S) and (G-L) theories are carried out.

4. Analytical solutions based upon normal mode analysis for thermoelastic problem in solids have been developed and used.

5. The deformation of a body depends on the nature of the applied forces and thermal loading due to laser pulse as well as the type of boundary conditions.

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