

## Effect of Residual Stresses on the Stress Intensity Factor of Cracks in a Metal Matrix Composite: Numerical Analysis

S. RAMDOUM  
F. BOUAFIA  
B. SERIER  
H. FEKIRINI

*Laboratory of Mechanic, Physics of Materials  
University of Sidi Bel Abbes, Algeria  
ramdoun-sara@hotmail.com*

Received (10 September 2017)

Revised (24 September 2017)

Accepted (30 September 2017)

In this work, the finite element method was used to determine the stress intensity factors as a function of crack propagation in metal matrix composite structure, A three-dimensional numerical model was developed to analyze the effect of the residual stresses induced in the fiber and in the matrix during cooling from the elaboration temperature at room temperature on the behavior out of the composite. Added to commissioning constraints, these internal stresses can lead to interfacial decohesion (debonding) or damage the matrix. This study falls within this context and allows cracks behavioral analysis initiated in a metal matrix composite reinforced by unidirectional fibers in ceramic. To do this, a three-dimensional numerical model was analyzed by method of finite element (FEM). This analysis is made according to several parameters such as the size of the cracking defects, its propagation, its interaction with the interface, the volume fraction of the fibers (the fiber-fiber interdistance), orientation of the crack and the temperature.

*Keywords:* Finite Element Method (FEM), crack, matrix, fiber, temperature.

### 1. Introduction

Composite materials generally have excellent rigidity thanks to reinforcing materials. Due to this property, these materials are widely used in industry, particularly in the aviation, aerospace, marine, automotive, civil engineering etc. However, during the elaboration of composites, residual stresses are born during the cooling process of the elaboration temperature to room temperature. These stresses are due to the difference of rigidity of and coefficient of thermal expansion between the fiber and the matrix. This difference weakens the adhesion between these two components and thus enhances initiation of microcracks. The behavior of cracks is studied in terms of variation of the stress intensity factor in modes I and II. Several research

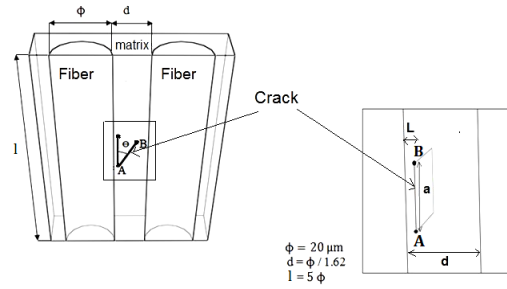
studies have been devoted to the analysis of these constraints on the mechanical behavior of composites. A model provided by Zhang et al. [1] a new tool to gain a thorough knowledge of residual stresses in multiphase materials. As Patricia P. et al. [2] presented in a study, the formation of thermal residual stress in thermoplastic composites and experimental techniques to detect these constraints by treating the effect of these properties. Liu H. T. et al. [3] studied analytically the effect of thermal residual stresses in composite materials. A study is made by Goffredo de Portu et al. [4] on the residual thermal stresses which arise due to the difference in coefficients of thermal expansion between the matrix and the reinforcing material. An elastic-plastic behavior of the composite Al/SiC and an analysis of residual stresses introduced by the cooling process were studied by Jae-Heoung Chun et al. [5] the large difference in thermal expansion coefficients (CTE) between the silicon carbide fiber (SCS-2) and the Aluminum 6061 tends to produce high residual stresses matrix. This analysis is based on the successive approximation scheme with the plastic flow model Prandtl-Reuss and criterion of Von Mises. M.M Aghdam et al. [6], Matteo Galli et al. [7] and MJ Mahmoodi MJ et al. [8] presented another model to study the damage effects on the interface SiC/Ti in the elastic-plastic behavior of unidirectional composites metal matrix (MMC). According Zeng Yi et al. [9] and M. Safarabadi [10] addresses the factors responsible for the formation of residual stresses in composites and their effects on the fiber and matrix properties. This author presents the analytical, numerical and experimental methods for prediction of thermal residual stress. S. Gasparyan [11] and Gilles Lubineau [12], they proposed a relatively simple method, based on observation of the displacement field associated with the creation of a transverse crack in a laminate, and advanced models in damage taking into account the residual stresses and microcracks. Mukherjee S. et al. [13] used a mechanical approach to fracture to examine the process of interfacial delamination in the composite metal matrix (MMC) during a push-out test on fiber, Benedikt B. et al. [14], H. Li et al. [15] and G. Maier et al. [16] shown that the residual stress state can be adjusted by treatment at low temperature and subsequent heating to room temperature. On the other hand, it has been demonstrated by Gentz M. et al. [17] that residual stresses in the composite unidirectional graphite fiber/polyamide are significantly affected by aging at elevated temperature. Behavior out of composite metal matrix differs from that of fiber-matrix interfacial decohesion. Indeed, the sub-interfacial fatigue microcracks can be initiated in one of these two components and their propagation leads to the destruction of the material. Interfacial cracks may exist from mechanisms of internal shear stress, the level of these stresses and the adhesion energy fiber-matrix determine the mechanical behavior of composite added to commissioning constraints, these constraints can be fatal for the composites.

The originality of this work lies in the analysis of the behavior of cracks initiated in a metal matrix composite reinforced by unidirectional fiber ceramic. To do this, this analysis is made according to the effect of the thermal loading, its location, its orientation, its propagation, its interaction with the interface, the fiber volume fraction and the defect size. The volume fraction of the fibers is the interdistance between fiber-fiber. The behavior of cracks is studied in terms of variation of the stress intensity factor in modes I and II. The finite element method is used for the determination of these factors.

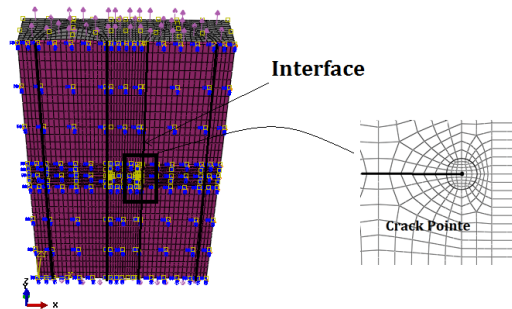
## 2. Finite Element modeling

### 2.1. Model validation

Using the finite element method is most adapted to the mechanical problems, this study was performed using the ABAQUS software finite element version 6.13 (Hibbit, Karlsson Sorensen [18]). The elementary structure is three-dimensional, this structure is composed of a parallelepiped-shaped having two capillary along its main axis, where two cylindrical fiber will be inserted. This structure schematically shows the boundary condition imposed with a crack in the matrix contains a central longitudinal crack of matrix/fiber interface size "a" initiated in the matrix in Fig. 1. The fiber volume fraction is expressed in terms its diameter  $\varphi$  and the distance "d" between two fibers. Due to the geometric symmetry of the structure, its half was modeled.



**Figure 1** Model cracked analyzed



**Figure 2** Boundary conditions and mesh of geometrical model

Whose boundary conditions and symmetry applied to the analyzed structure are:

$$U_Y = U_{RX} = U_{RZ} = 0$$

where (o, z, x) (symmetry condition with respect to y).

These boundary conditions are dependent on the symmetry of the geometry fixed in the computer code Abaqus (Fig. 2a). The structure has been meshed by brick elements, type C3D20RH (20-node element (Fig. 2b)) with a total of 15273 elements. Reinforcements Al<sub>2</sub>O<sub>3</sub> are linear elastic isotropic with Young's modulus  $E_f = 345$  GPa and Poisson's ratio  $\nu = 0.27$ , a thermal expansion coefficient  $\alpha = 8.8e-006$ . The matrix on Aluminum is considered isotropic elastic material with a Young's modulus  $E_m = 67.5$  GPa and a Poisson's ratio  $\nu = 0.33$ , and a coefficient of thermal expansion  $\alpha = 23,5e-006$ . (B. Serier et al. [19], Ramdoun S. et al. [20]).

The analyzed model has a central crack initiated in the matrix.

### 3. Results and discussion

The elaboration of composite materials is generally made in relatively high temperatures according to the nature of matrix. As a result, residual stress appears at the fiber-matrix interface due to the difference of thermal expansion coefficients of these two constituents. Several studies as Sellam S. et al. [21] have shown that the level and distribution of these stresses depended on the nature of the two components bonded together, the difference of physical properties (thermal expansion coefficient) and the temperature of junction (temperature of elaboration).

The residual stresses generated during the implementation of the composite material during the cooling process of the elaboration temperature to room temperature, are introduced into the fiber, the matrix and the very close vicinity of their interface. We have made a numerical analysis by Abaqus software to evaluate the residual stresses (Fig. 3) which appear during cooling due to thermal expansion coefficient differences of the two constituents. Indeed, the embodiment temperature of the composite, the metal matrix contract much more than the ceramic fiber, the result of the shear stress at the fiber-matrix interface due to the equalization of elastic deformations of the matrix and the fiber:

$$\varepsilon_m = \alpha_m(T_0 - T) \quad \text{and} \quad \varepsilon_f = \alpha_f(T_0 - T) \quad (1)$$

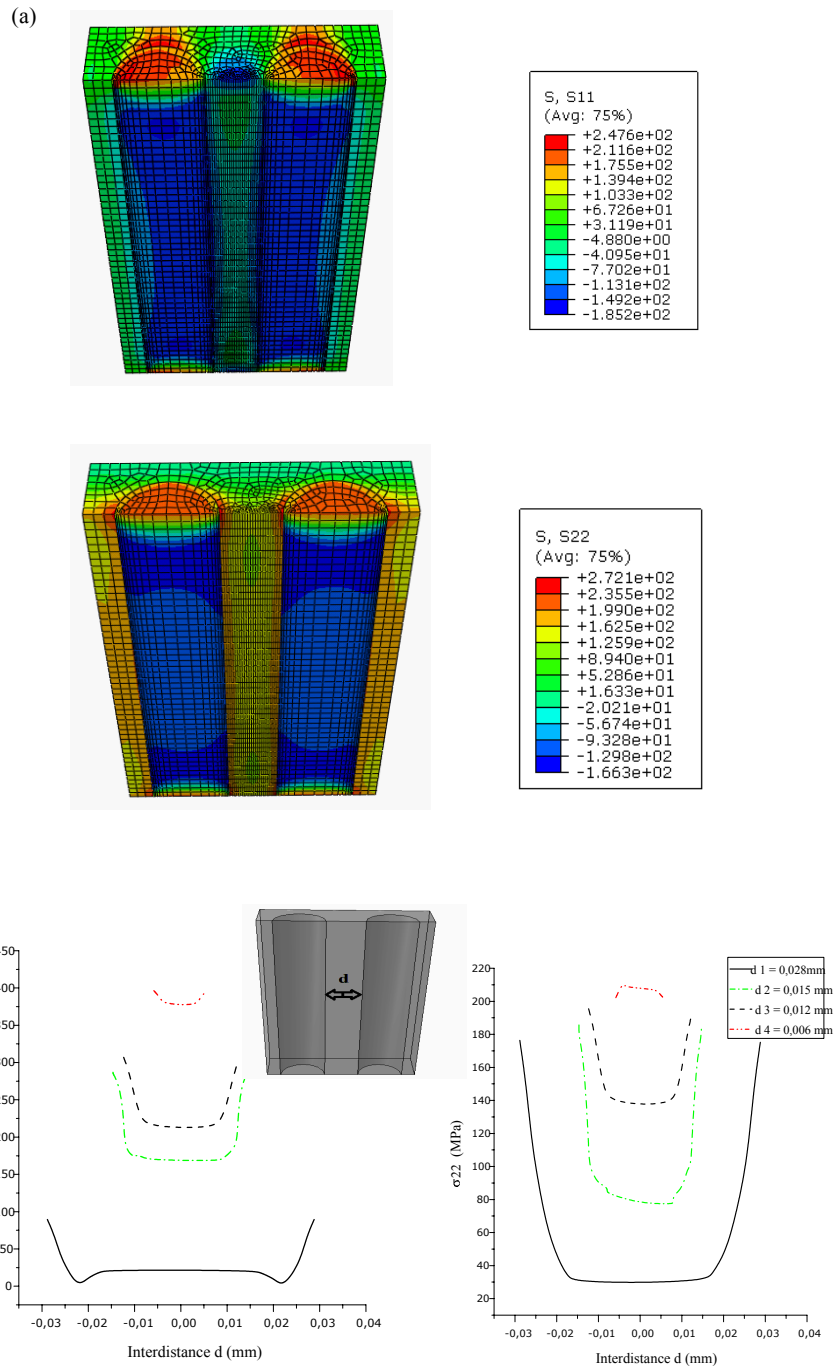
These residual stresses ( $\sigma_R$ ), depend not only on the difference between the thermal expansion coefficients of the matrix and fiber, the difference between the temperature at which the thermoelastic deformation disappears and the elaboration temperature, but also the modulus of elasticity " $E$ " and Poisson's ratio " $\nu$ " of the two constituents (fiber and matrix):

$$\sigma_R = \frac{(\alpha_m - \alpha_f)(T - T_0)}{\frac{1+\nu_m}{2E_m} + \frac{1-2\nu_f}{E_f}} \quad (2)$$

$\alpha_m$  et  $\alpha_f$  in the both of Eq. (1-2) are the thermal expansion coefficients of the matrix and the fiber respectively.

$(T_0 - T)$  is the temperature deviation from the reference temperature.

In the following the effect of the internal stresses on crack behavior is analyzed in terms of variation of the stress intensity factor in the open mode (mode I) and shear mode (modes II).



**Figure 3** Variation of residual stresses according to the Fiber-Fiber interdistance for  $\Delta T = 600^\circ\text{C}$

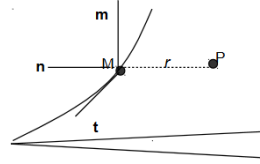
### 3.1. Calculation of the stress intensity factor

GENIAUT Samuel [22] described a method of calculating  $K_I$ ,  $K_{II}$  in 2D (plane and axisymmetric). It is usable with the order, the accuracy of the results of the extrapolation method of movement breaks is significantly improved if the mesh is quadratic. To crack a meshed, it is recommended to use so-called elements of "Barsoum" in crack tip (elements whose backgrounds nodes are situated at a quarter of the edges)  $K_I$ ,  $K_{II}$ , both of a mesh crack (conventional finite elements) for not a crack mesh (finite element enriched: method X-FEM) with:

- $t, n$  – in the crack plane  $M$ ,
- $t$  – vector tangent to the crack in  $M$ ,
- $n$  – normal vector to the crack tip in  $M$ ,
- $m$  – vector normal to the crack plane in  $M$ ,
- $U_m$  – jump of displacement between the crack lips:  
 $[U_m] = (U_{upperlip} - U_{lowerlip})_m$ ,
- $R = \|MP\|$  where  $P$  is a point in the plane normal to the crack tip  $M$ , situated on the lips.

$$K_1(M) = \lim_{r \rightarrow 0} \left( \frac{E}{8(1-\nu^2)} [U_m] \sqrt{\frac{2\pi}{r}} \right) \quad (3)$$

$$K_2(M) = \lim_{r \rightarrow 0} \left( \frac{E}{8(1-\nu^2)} [U_n] \sqrt{\frac{2\pi}{r}} \right) \quad (4)$$

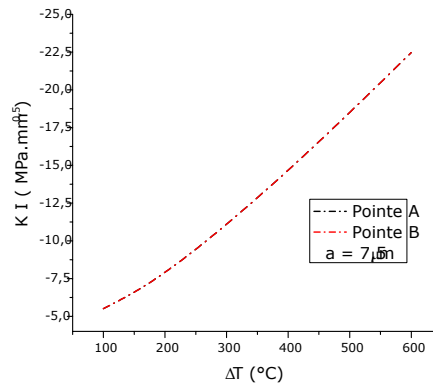


### 3.2. Effect of temperature

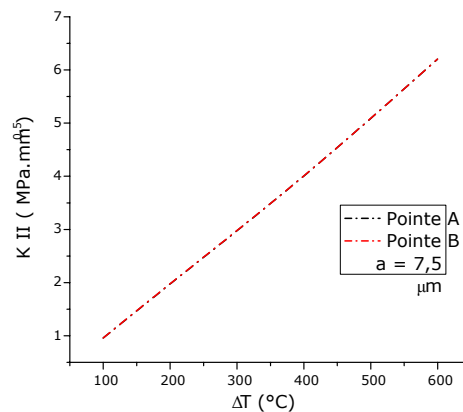
In this study, one will analysis the elaboration temperature effect on a crack behavior initiated in the matrix composite, the very close vicinity of the fiber parallel to the longitudinal axis of the reinforcement. The results thus obtained are shown in (Fig. 4(a)). This last, illustrates the variation of the stress intensity factor in mode I as a function of this temperature, it's clearly shows that the internal stresses thermally induced in the matrix near the interface affect on the two fronts of cracking, denoted here pointes A and B, such as crack closure stress. This behavior is defined by negative values of this failure criterion.

In mode II, the stress intensity factor is more important that the temperature is high (Fig. 4(b)). This rupture setting is equally distributed on the two fronts cracking. Indeed, the values of the resulting factors of these two points are identical whatever the temperature. The results illustrated in this figure show that the internal stresses promote instability of the crack in mode II.

(a) Mode I



(b) Mode II

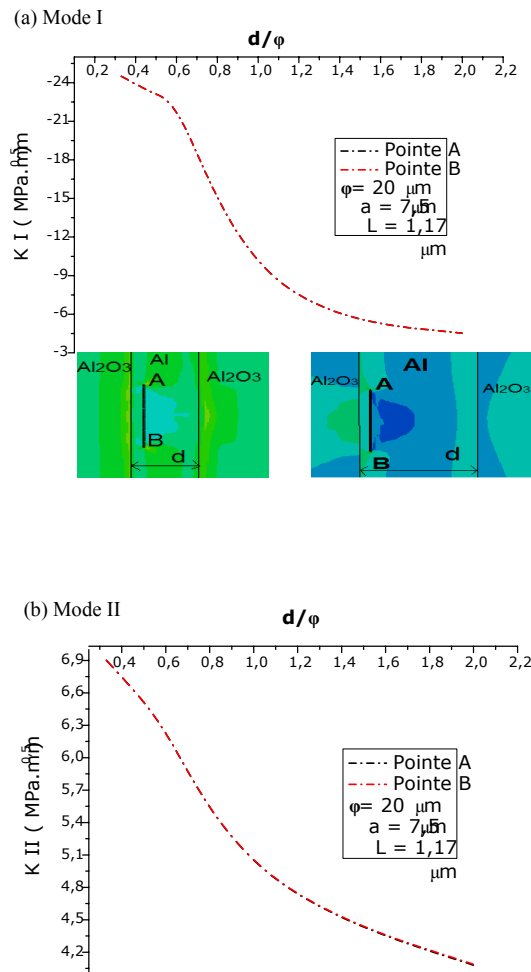


**Figure 4** Variation of the stress intensity factor according to the elaboration temperature added to the commissioning constraints.  $\Delta T$  (°C) +  $\sigma = 50$  MPa

### 3.3. Effect of fiber-fiber interdistance

The interdistance "d" between the fibers indirectly determines the volume fraction of reinforcement (Fig. 1). This last plays an important role on the level and the distribution of internal stresses. In what follows, we analyze its effect on the behavior of a crack defined above. In Fig. 5 is given the variation of the stress intensity factor in mode I and II, based on the distance between the fibers as defined according to the size of reinforcement " $\Phi$ ". The analysis of this figure shows that a close arrangement of fibers promotes instability in mixed mode II of such a crack. This instability is defined by an intensification of this rupture parameter (SIF).

A behavior is explained by the interaction effect of the residual stresses field strongly localized to the matrix in close proximity of the fiber. This field is particularly important that the fibers are located very close to one another. such a distribution of the reinforcement, induced in the matrix, the internal stresses are much more intense than when the fiber is isolated.



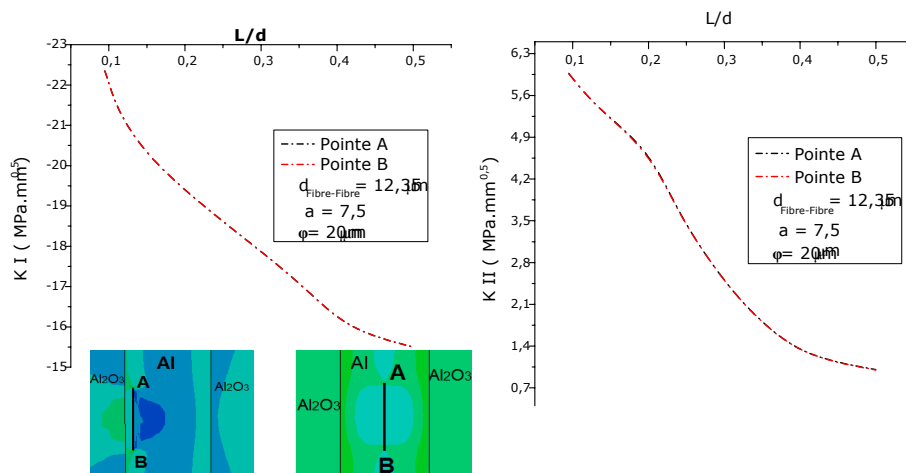
**Figure 5** Variation of the stress intensity factor a function of the fiber-fiber interdistance:  $\Delta T = 600^\circ\text{C} + \sigma = 50 \text{ MPa}$

#### 4. Effect of crack-interface distance

The crack position relative to the interface on the stress intensity factor, is analyzed in the following, this position defined by "L" (Fig. 1) is characterized according to the distance between the crack and the fiber. The effect of crack-interface interdistance on the stress intensity factor is given by Fig. 6. This figure shows that such



a crack initiated in the heart of the matrix is almost stable. The rupture factors in mode II is practically negligible. A tendency of this crack to the interface leads to an augment in these parameters. This indicates clearly that the residual stresses are intensively localized in the matrix in very close vicinity of the interface. This explains the strong values of this criteria propagation. The risk of propagation is greatest when the crack is initiated near the reinforcement.



**Figure 6** Variation of the stress intensity factor a function of the interface-fiber interdistance:  $\Delta T = 600^\circ\text{C} + \sigma = 50\text{MPa}$

## 5. Effect of the crack orientation

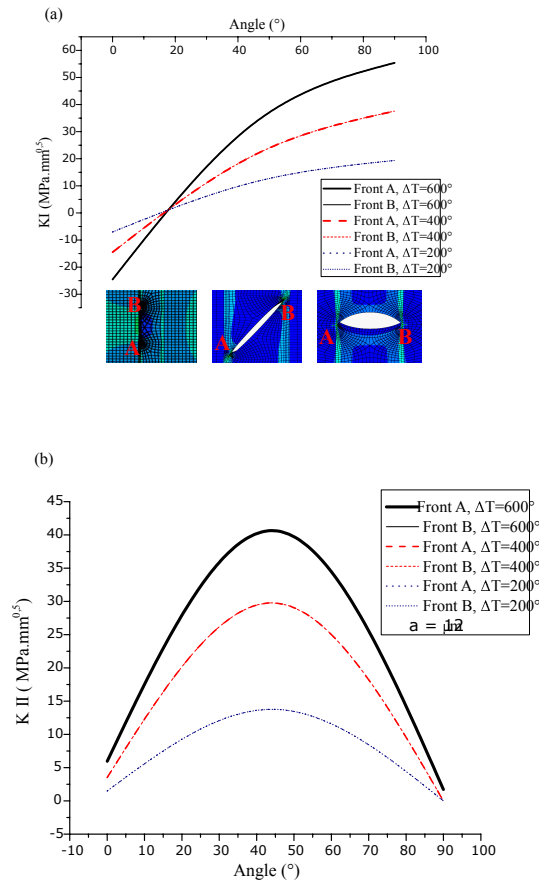
The composites materials elaborated at high temperatures induce more intense residual stresses. The shears stresses are very highly localized at the interface makes the fiber in compression and matrix in tension. The most unstable crack fronts are located in the high stress concentration zones; a tendency of the crack to the transverse axis of the fiber promotes the most dangerous propagation mode. In fact such a defect develops pure opening mode. The growth in mode I of the two fronts of this crack, is highly unstable in the vicinity of the fiber, and can be deviated simply or doubling towards the interface. This deviation may be caused by the compressive internal stresses induced in the fiber. These stresses generate the closure of the crack. Our results show that under the effect of the residual stresses induced during the elaboration of the metal matrix composite, a crack initiated in the metal, is propagated to the ceramic reinforcement in modes I and II. The predominant mode of growth of such cracks depends on its orientation compared with the interface. Its kinetics of propagation is closely related to its size and intensity of the stress field through which the crack front, and in the both of orientations; the K I and K II values are identical in the two fronts of cracks.

Under the effect of residual stresses:

A crack initiated in the matrix parallel to the fiber-matrix interface propagates essentially in mode II (Fig. 7(b)).

A crack oriented at  $45^\circ$  propagates in mode I and II.

A crack oriented perpendicular to the fiber-matrix interface propagates in pure mode I.



**Figure 7** Variation of stress intensity factors (KI, KII) according to the crack orientation added to the commissioning constraints

The results obtained show that the presence of residual stresses favors the risk of rupture of the matrix, these constraints can be intensified by the presence of defects, such as for example the interface, pores etc., constitute a major risk for the ruin of composites. The latter is all the more likely as these internal stresses of thermal origin are added to the constraints of commissioning. This behavior is in good agreement with that resulting from other works among which we can cite, Hobbiebrunken Thomas et al. [23], Lei Yang et al. [24] studied by a method of finite

elements (FEM), the effect of residual thermal stresses in the event of damage to cross-matrix composite reinforced polymer fibers. Particularly, the influence of mechanical and thermal properties of the two fiber-matrix materials on the formation of thermal residual stresses depends on the temperature.

## 6. Conclusion

The results obtained in this work allow drawing the following conclusions:

1. The variation of the stress intensity factor in mode I as a function of temperature clearly shows that the internal stresses acting on the two fronts of cracking, and not only as closure stress in mode II, this behavior is defined by negative values of this failure criterion; the mechanical energy is distributed evenly within two points of the crack defect.
2. A very close arrangement of the fibers promotes instability in mode II of such a crack, this instability is defined by an intensification of this rupture parameter, such behavior is explained by the interaction effect of the residual stress field strongly localized to the matrix in close proximity of the fiber.
3. The residual stresses are intensively localized in the matrix in very close vicinity of the interface, this explains the strong values of these two criteria propagation, and the risk of spread is greatest when the crack is initiated near the reinforcement.
4. A Crack initiated in the metal, is propagated to the ceramic reinforcement in modes I and II. The predominant method of such a crack depends on its orientation relative to the interface; its kinetics of propagation is closely related to the size and intensity of the stress field through which crack fronts.

## References

- [1] Zhang, X. X., Xiao, B. L., Andrä, H., Ma, Z. Y.: Multi-scale modeling of the macroscopic, elastic mismatch and thermal misfit stresses in metal matrix composites, *Composite Structures*, 125, 176–187, **2015**.
- [2] Parlevliet, P. P., Bersee, H. E. N., Beukers, A.: Residual stresses in thermoplastic composites – a study of the literature. Part III: Effects of thermal residual stresses, *Composites Part A: Applied Science and Manufacturing*, 6, A2038, 1581–1596, **2007**.
- [3] Liu, H. T., Sun, L. Z.: Effects of thermal residual stresses on effective elastoplastic behavior of metal matrix composites, *International Journal of Solids and Structures*, 8, 41, 2189–2203, **2004**.
- [4] de Portu, G., Micele, L., Guicciardi, S., Fujimura, S., Pezzotti, G., Sekiguchi, Y.: Effect of residual stresses on the fracture behaviour of notched laminated composites loaded in flexural geometry, *Composites Science and Technology*, 65, 1501–1506, **2005**.
- [5] Chun, H.-J., Daniel, I. M., Wooh, S.-C.: Residual thermal stresses in a filamentary SiC/Al composite, *Composites Engineering*, 5, 425–436, **1995**.
- [6] Aghdam, M. M., Smith, D., Javier, M. J.: Finite element micromechanical modelling of yield and collapse behaviour of metal matrix composites, *Journal of the Mechanics and Physics of Solids*, 3, 48, 499–528, **2000**.

- [7] **Galli, M., Cugnoni, J., Botsis, J.:** Numerical and statistical estimates of the representative volume element of elastoplastic random composites, *European Journal of Mechanics - A/Solids*, 33, 31–38, **2012**.
- [8] **Mahmoodi, M. J., Aghdam, M. M., Shakeri, M.:** Micromechanical modeling of interface damage of metal matrix composites subjected to off-axis loading, *Journal Materials and Design*, 31, 829–836, **2010**.
- [9] **Yi Zeng, Xiang Xiong, Dini Wang, Liang Wu:** Residual thermal stresses in carbon/carbon–Zr–Ti–C composites and their effects on the fracture behavior of composites with different performs, *Journal Carbon*, 81, 597–606, **2015**.
- [10] **Safarabadi, M.:** Understanding residual stresses in polymer matrix composites, *Journal Residual Stresses in Composite Materials*, 197–232, **2014**.
- [11] **Gasparyan, S.:** Determination of residual stresses in metallic composites, *Journal of Materials Processing Technology*, 1–3, 178, 14–18, **2006**.
- [12] **Lubineau, G.:** Estimation of residual stresses in laminated composites using field measurements on a cracked sample, *Composites Science and Technology*, 68, 13, 2761–2769, **2008**.
- [13] **Mukherjee, S., Ananth, C. R., Chandra, N.:** Effect of residual stresses on the interfacial fracture behavior of metal-matrix composites, *Composites Science and Technology*, 57, 11, 1501–1512, **1997**.
- [14] **Benedikt, B., Kumosa, M., Predecki, P. K., Kumosa, L., Castelli, M. G., Sutter, J. K.:** An analysis of residual thermal stresses in a unidirectional graphite/PMR-15 composite based on X-ray diffraction measurements, *Composites Science and Technology*, 61, 14, 1977–1994, **2001**.
- [15] **Li, H., Li, J. B., Sun, L. Z., Wang, Z. G.:** Modification of the residual stress state in a SiC<sub>p</sub>/6061Al composite by low-temperature treatment, *Journal Composites Science and Technology*, 57, 2, 165–172, **1997**.
- [16] **Maier, G., Hofmann, F.:** Performance enhancements of polymermatrix composites by changing of residual stresses, *Composites Science and Technology*, 68, 9, 2056–2065, **2008**.
- [17] **Gentz, M., Benedikt, B., Sutter, J. K., Kumosa, M.:** Residual stresses in unidirectional graphite fiber/polyimide composites as a function of aging, *Composites Science and Technology*, 64, 10–11, 1671–1677, **2004**.
- [18] **Hibbit, Karlsson & Sorensen Inc.:** ABAQUS, User’s Manual, 6.13, **2013**.
- [19] **Serier, B., Bachir Boudjra, B., Belhouari, M.:** Finite element analysis of bimaterial interface notch crack behaviour, *Computational Materials Science*, 27, 517–522, **2003**.
- [20] **Ramdoun, S., Serier, B., Fekirini, H., Bouafia, F.:** ‘Numerical Analysis of the Effect of Residual Stresses on the Behavior of Cracks in a Metal Matrix Composite, *9th International Conference on Advanced Computational Engineering and Experimenting*, **2015**.
- [21] **Souad, S., Serier, B., Bouafia, F., Bachir Boudjra, B. A., Hayat, S. S.:** Analysis of the stresses intensity factor in alumina–Pyrex composites, *Journal Computational Materials Science*, 72, 68–80, **2013**.
- [22] **Geniaut, S.:** Calcul des facteurs d’intensité des contraintes, [http://www.codeaster.org/doc/v12/fr/man\\_r/r7/r7.02.08.pdf](http://www.codeaster.org/doc/v12/fr/man_r/r7/r7.02.08.pdf) **2012**.
- [23] **Hobbiebrunken, T., Fiedler, B., Hojo, M., Ochiai, S., Schulte, K.:** Microscopic yielding of CF/epoxy composites and the effect on the formation of thermal residual stresses, *Composites Science and Technology*, 10, 65, 1626–1635, **2005**.

- [24] **Yang, L., Yan, Y., Ma, J., Liu, B.:** Effects of inter-fiber spacing and thermal residual stress on transverse failure of fiber-reinforced polymer-matrix composites, *Computational Materials Science*, 68, 255–262, **2013**.

