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Performance of Hydrodynamic Journal Bearing Operating under Transient Wear

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The paper deals with a theoretical study concerning the effect of transient wear on the performance of hydrodynamic journal bearing. The eccentricity ratios are considered 0.3, 0.6 and for varying wear depth from 0.1 to 0.5 for the analysis of purpose. The Reynolds equations governing the flow of lubricant in the clearance space of a hydrodynamic journal bearing system with varying wear depth has been numerically solved using Galerkin's FEM. The regime is assumed to be isothermal. The positive pressure zone is established using Reynolds boundary condition through iteratively. The various performance parameters which include static, dynamic and stability terms of worn journal bearing are presented with respect to relative wear depth. The journal centre motion trajectories are also obtained by numerically integrated linear and non-linear equation of motion using fourth order Runge-Kutta method. For this a computer program in Mat lab was developed based on analysis to draw the linear and non linear motion trajectories and in order to validate the developed program, the numerically simulated results computed from the present study are compared with the already published results in literature. For analysis purpose the initial values of velocities and displacement are taken as $\bar{X} = \bar{Z} = 0.0$ and $\bar{x} = \bar{z} = 0.005$. The results help in predicting bearing life for smooth operation.

Keywords: FEM, aspect ratio, worn journal bearing.

1. Introduction

Hydrodynamic journal bearings, while operating run over a number of revolutions and are conditional to rubbing during start and stop operation. Under these transient periods, the bearing bush progressively worn out due to abrasive method. Due to this, the geometry of bearing gets altered there by changing its performance. In the past since 1957 many investigators have examined psychological measurement of several destruct bearings [1–4]. A geometric model considers wear was first proposed by Dufrane [5]. Kumar and Mishra [6, 7] found that due to wear the friction, flow rate increases and the load capacity, stability are reduces.

For smooth operation of bearing, several investigators have studied transient response of circular journal bearing systems by discretized time and numerically integrated linearized and nonlinear equations of motion of journal bearing system to study solid and porous bearings having L/D ratio of 1.0 with journal bearing axes parallel as well as skewed [8]. The stability analysis and dynamic transient behavior of shaft and support motions for both the stable and unstable operating speeds has been studied [9]. The study on linear and non linear transient motion analysis of a flexible shell journal bearing having L/D ratio of 1.0 and found that non linear motion equation give higher stability than linear equation of motion [10].

The transient response of plane hydrodynamic journal bearing system during acceleration and deacceleration periods has also investigated [11]. The transient response study of hydrodynamic journal bearing lubricated with non-newtonian lubricants using cubic shear stress law has been investigated [12]. The bearing stiffness characteristics, transient vibration, and frequency response of hydrodynamic journal bearing also obtained for both linear and nonlinear bearing simulations [13]. For a particular frequency of loading, the effects of mass, amplitude of load vibration and frequency of journal speed on stability of journal bearing has been investigated [14]. Non linear transient analysis of journal bearing stability also studied and found that the initial conditions play an important role on the behavior of the orbit [15]. The non-linear time transient analysis of non-Newtonian lubricant hydrodynamic journal bearings having different power law index and aspect ratios was also performed [16]. The dynamic parameters like, stiffness and damping coefficients, critical speed and whirl ratio are calculated for various L/D ratios and nonlinear time transient analysis for stability of hydrodynamic journal bearings under various dynamic loads [17].

A study of bearing dynamics is important for enhancing smooth bearing life. There appears a very little work in the published literature, dealing with the time transient analysis of worn hydrodynamic journal bearing. The current theoretical study is a holistic approach for a depth understanding about the static, dynamic and stability aspect of hydrodynamic journal bearing owing to wear. In order to account the wear, a numerical model for wear depth used by Dufrane [5] has been considered in the analysis. The governing equation for hydrodynamic lubrication is solved using FEM. The details have been given in section 2.

2. Analysis

The conventional Reynolds equation for an incompressible, Newtonian lubricant in the clearance space of a finite journal bearing system is given below in non-

dimensional form as [18]:

$$\frac{\partial}{\partial \alpha} \left(\frac{\bar{h}^3}{12} \frac{\partial \bar{p}}{\partial \alpha} \right) + \frac{\partial}{\partial \beta} \left(\frac{\bar{h}^3}{12} \frac{\partial \bar{p}}{\partial \beta} \right) = \frac{\Omega}{2} \frac{\partial \bar{h}}{\partial \alpha} + \frac{\partial \bar{h}}{\partial \tau} \quad (1)$$

The conventional Reynolds Eq. (1) is solved using FEM, which is described below.

2.1. Fluid-film thickness

The worn bearing zone geometry is shown in Fig. 1. A non-dimensional parameter δ_w has been taken and consider as the measure of wear. Dufrane et al. [5] presume an abrasive wear model by considering due to frequently start and start operation of bearing. Taking the values of computed journal center coordinates (\bar{X}_J and \bar{Z}_J) and defect value δ_w is added to the calculated fluid-film thickness, expressed as:

$$\bar{h} = 1 - \bar{X}_J \cos \alpha - \bar{Z}_J \sin \alpha + \delta_w \quad (2)$$

The eccentricity ratio is given by

$$\varepsilon = \sqrt{|\bar{X}_J|^2 + |\bar{Z}_J|^2} \quad (3)$$

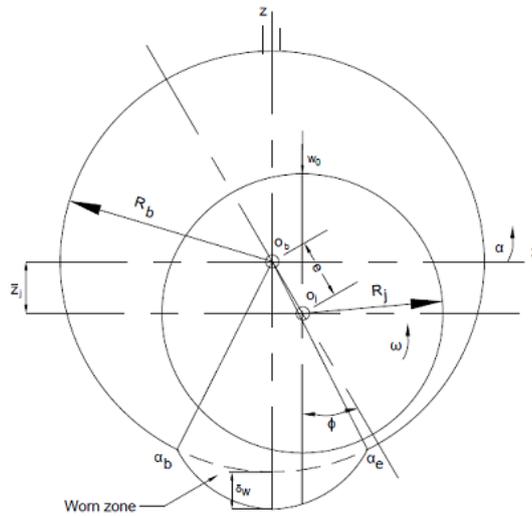


Figure 1 Worn bearing geometry

2.2. FEM formulation

The lubricant domain is discretised using four noded quadrilateral isoparametric elements. The entire domain of pressure field results global linear equations and expressed as [18]:

$$[\bar{F}]_{N \times N} \{\bar{p}\}_{N \times 1} = \{\bar{Q}\}_{N \times 1} + \Omega \{\bar{R}_H\}_{N \times 1} + \dot{x} \{\bar{R}_{X_J}\}_{N \times 1} + \dot{z} \{\bar{R}_{Z_J}\}_{N \times 1} \quad (4)$$

where N – number of nodes in the entire domain, involving the pressure $\{\bar{p}\}$ and the flow $\{\bar{Q}\}$ as nodal variables.

2.3. Boundary conditions

The boundary condition used for the solution are described as:

$$\left. \begin{array}{l} \text{Nodes on external boundary, } \bar{P} = 0.0 \quad \text{at } \beta = \pm 1 \\ \text{At the trailing edge of positive region, i.e, } \bar{p} = \frac{\partial \bar{p}}{\partial \alpha} = 0.0 \end{array} \right\} \quad (5)$$

Eq. (5) can be solved to give both pressure and flow simultaneously because at each node one of the two variables is known.

2.4. Stability parameters

Considering two degrees of freedom system, 2x2 fluid-film stiffness and 2x2 fluid-film damping coefficients can be computed using the expressions given below. The fluid-film stiffness and damping coefficients are defined as [18]:

$$\left[\begin{array}{cc} \bar{S}_{xx} & \bar{S}_{xz} \\ \bar{S}_{zx} & \bar{S}_{zz} \end{array} \right] = - \left[\begin{array}{cc} \frac{\partial \bar{F}_x}{\partial \bar{x}} & \frac{\partial \bar{F}_x}{\partial \bar{z}} \\ \frac{\partial \bar{F}_z}{\partial \bar{x}} & \frac{\partial \bar{F}_z}{\partial \bar{z}} \end{array} \right] \text{ and } \left[\begin{array}{cc} \bar{C}_{xx} & \bar{C}_{xz} \\ \bar{C}_{zx} & \bar{C}_{zz} \end{array} \right] = - \left[\begin{array}{cc} \frac{\partial \bar{F}_x}{\partial \dot{\bar{x}}} & \frac{\partial \bar{F}_x}{\partial \dot{\bar{z}}} \\ \frac{\partial \bar{F}_z}{\partial \dot{\bar{x}}} & \frac{\partial \bar{F}_z}{\partial \dot{\bar{z}}} \end{array} \right] \quad (6)$$

For a small disturbance of journal centre from its static equilibrium position, the hydrodynamic forces generated in the journal can be treated as linear function of the velocity and displacement vectors. The linearized equation of disturbed motion of the journal in the non –dimensional form is given by [4]:

$$\left[\begin{array}{cc} \bar{M}_j & 0 \\ 0 & \bar{M}_j \end{array} \right] \left\{ \begin{array}{c} \ddot{\bar{X}}_j \\ \ddot{\bar{Z}}_j \end{array} \right\}_{|\bar{t}} = \left\{ \begin{array}{c} \delta \bar{F}_x \\ \delta \bar{F}_z \end{array} \right\}_{|\bar{t}} = - \left[\begin{array}{cc} \bar{S}_{xx} & \bar{S}_{xz} \\ \bar{S}_{zx} & \bar{S}_{zz} \end{array} \right] \left\{ \begin{array}{c} \bar{X}_j \\ \bar{Z}_j \end{array} \right\}_{|\bar{t}} - \left[\begin{array}{cc} \bar{C}_{xx} & \bar{C}_{xz} \\ \bar{C}_{zx} & \bar{C}_{zz} \end{array} \right] \left\{ \begin{array}{c} \dot{\bar{X}}_j \\ \dot{\bar{Z}}_j \end{array} \right\}_{|\bar{t}} \quad (7)$$

Where the subscript \bar{t} refers to the instantaneous values and the stiffness and damping coefficients are evaluated at the static equilibrium position of journal centre. For a large disturbance of journal centre, the stiffness and damping coefficients no longer remain constant and their values are changed for every new position of journal. For this, the non-linearized equation of disturbed motion of the journal is used and is given by:

$$\left[\begin{array}{cc} \bar{M}_j & 0 \\ 0 & \bar{M}_j \end{array} \right] \left\{ \begin{array}{c} \ddot{\bar{X}}_j \\ \ddot{\bar{Z}}_j \end{array} \right\}_{|\bar{t}} = \left\{ \begin{array}{c} \delta \bar{F}_x \\ \delta \bar{F}_z \end{array} \right\}_{|\bar{t}} = \left\{ \begin{array}{c} \bar{F}_x - \bar{F}_{xo} \\ \bar{F}_z - \bar{F}_{zo} \end{array} \right\}_{|\bar{t}} \quad (8)$$

where \bar{F}_{xo} and \bar{F}_{zo} are the fluid-film force components at the static equilibrium position. The fluid-film force components (\bar{F}_x and \bar{F}_z) at time \bar{t} , are evaluated at each time step after establishing the pressure field corresponding to the position of the journal at that particular time. Using Rouths' criteria, the stability margin of the journal bearing system, in term of critical mass \bar{M}_c , is obtained.

3. Results and discussion

The study of transient motion through numerical simulation has been performed using flow chart as shown in Fig. 2. The performance characteristics for a worn journal bearing system with various wear depth have been presented and discussed in this section. The influence of wear depth on the performance characteristics of worn journal bearing system have been computed by developed computer code in MAT LAB. The aspect ratios (L/D) 1.0 and eccentricity ratio of 0.2 and 0.6 are considered for the analysis of purpose. For analysis purpose the initial values of velocities and displacement are taken as $\bar{X} = \bar{Z} = 0.0$ and \bar{x} and $\bar{z} = 0.005$.

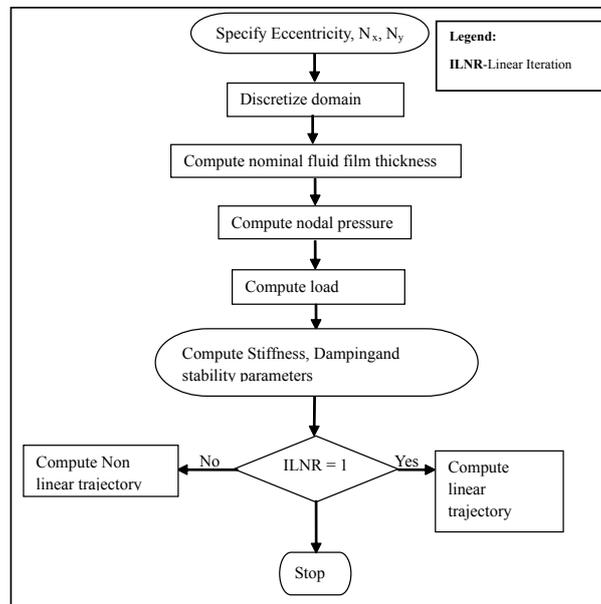
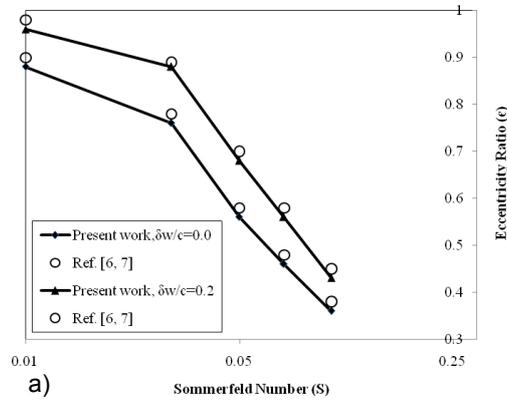


Figure 2 Overall solution process scheme

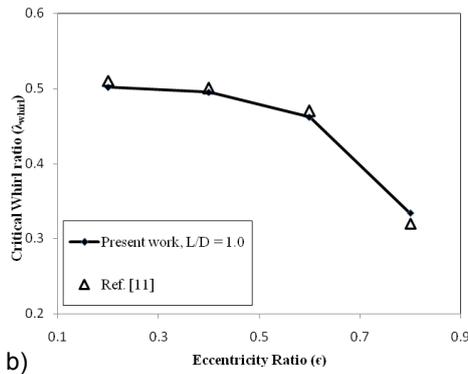
In order to validate the developed program, the numerically simulated results for eccentricity ratio (ε) corresponding to different values of the Sommerfeld number for wear depth $\bar{\delta}_w = 0.0$ and 0.2 are computed from the present study and compared with the already published results in literature [6, 7] as shown in Fig. 3a and results for eccentricity ratio (ε) corresponding to different values of the critical whirl ratio as shown in Fig. 3b for L/D ratio of 1.0 are also computed. The results are observed to be in good agreement with the published work and thus establishes the accuracy of developed code. The bearing geometric and operating parameter values are selected based on the published literature and are shown in Table 1.

Table 1 Operating and geometric parameters of worn hydrodynamic journal bearing

Eccentricity ratio (ε)	0.3 and 0.6
Speed parameter (Ω)	1.0
Wear depth parameter (δ_w)	0.0 – 0.5
Aspect ratio (L/D)	1.0
Clearance ratio (ccR)	0.001



a)



b)

Figure 3 a) Eccentricity ratio versus Sommerfeld number, b) Critical Whirl ratio versus Eccentricity ratio

The simulated results of worn journal bearing with respect to relative wear depth for L/D ratio of 1.0 is presented. In Fig. 4 it is observed that the value of minimum fluid-film thickness for a specified relative wear depth increases with the increase in eccentricity ratio and wear depth. At lower value of fluid film thickness the pressure

is high as it observed in Fig. 5, the value of maximum pressure is high for a wear depth up to 0.2. It is also observed that the value of minimum fluid-film thickness increases morapidly when $\bar{\delta}_w > 0.2$.

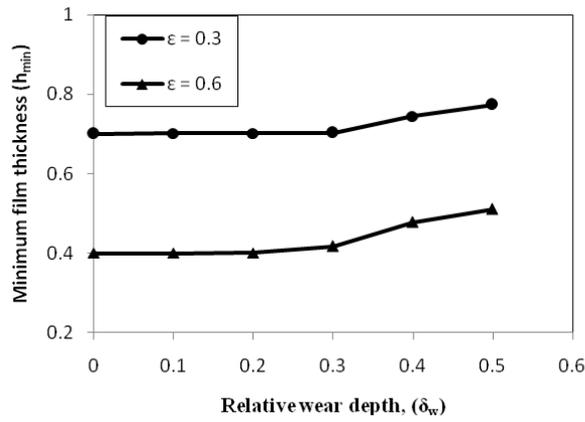


Figure 4 Minimum fluid-film thickness versus relative wear depth

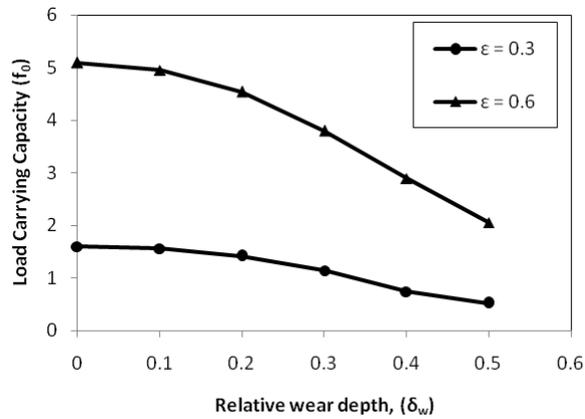


Figure 5 Load carrying capacity versus relative wear depth

In Fig. 5 it is observed that the value of Load carrying capacity at different values of eccentricity ratio decreases with the increase in wear depth. The value of load carrying capacity depends on pressure. When pressure increases the load carrying capacity also increases and vice versa. It is also observed that the value of Load

carrying capacity decreases more rapidly when $\bar{\delta}_w > 0.3$. In Fig. 6 the effect of wear on the value of maximum pressure is significantly higher with eccentricity ratio of 0.6 and decreases with an increase in wear depth. Actually at higher eccentricity ratio the journal tends to go inside the cavity formed due to wear and thus variation in the value of pressure gradients occurs. It is also observed that the value of maximum pressure for a specified relative wear depth first increase upto to wear depth of 0.2 and then decreases with an increase in wear depth.

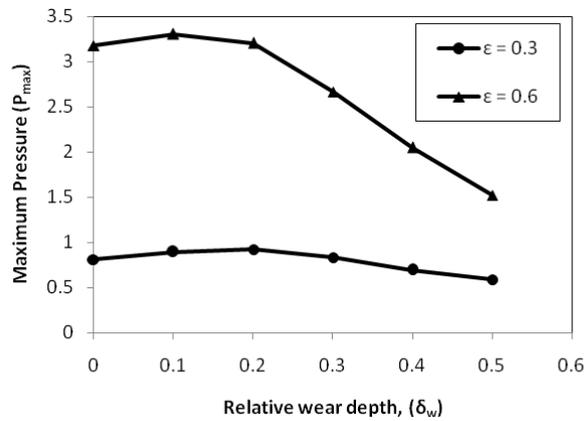


Figure 6 Maximum pressure versus relative wear depth

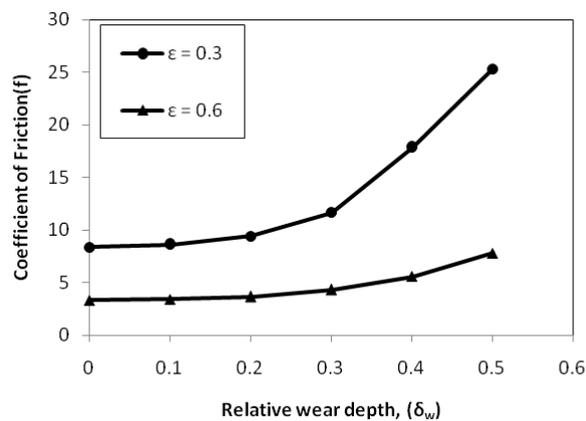


Figure 7 Coefficient of friction versus relative wear depth

In Fig. 7 friction coefficient is increased with increasing in wear depth and decrease with increased in eccentricity ratio. It is also observed that the value of coefficient of friction increases more rapidly when $\bar{\delta}_w > 0.2$. The results shows, that there is judicious need to run the worn bearing up to certain wear depth limits from the view point of frictional power loss and load carrying capacity of fluid film journal bearing. The direct fluid-film stiffness coefficients \bar{S}_{xx} and \bar{S}_{zz} as shown in Fig. 8 and Fig. 9 shows a decreasing trend with an increase in wear depth. The value of stiffness coefficients for direct fluid-film stiffness coefficient \bar{S}_{xx} and \bar{S}_{zz} due to wear found to be more at higher eccentricity ratio of 0.6.

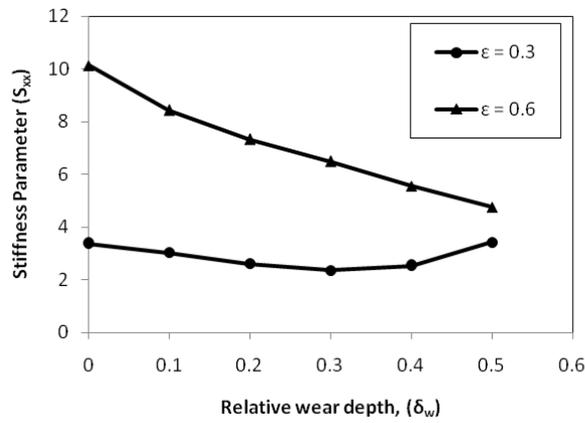


Figure 8 Direct fluid-film stiffness coefficient versus relative wear depth

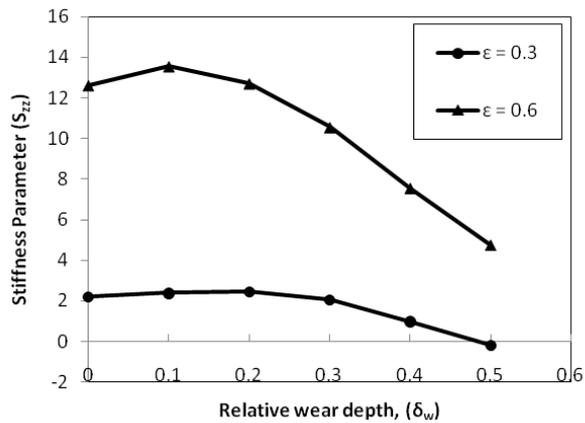


Figure 9 Direct fluid-film stiffness coefficient versus relative wear depth

The direct damping coefficient parameters \bar{C}_{xx} and \bar{C}_{zz} as shown in Fig. 10 and Fig. 11 shows decreasing trend with an increase in wear depth and eccentricity ratio.

The linear and non-linear transient responses of worn journal bearing with respect to varying wear depth and for a fixed eccentricity ratio is shown in Tab. 2.

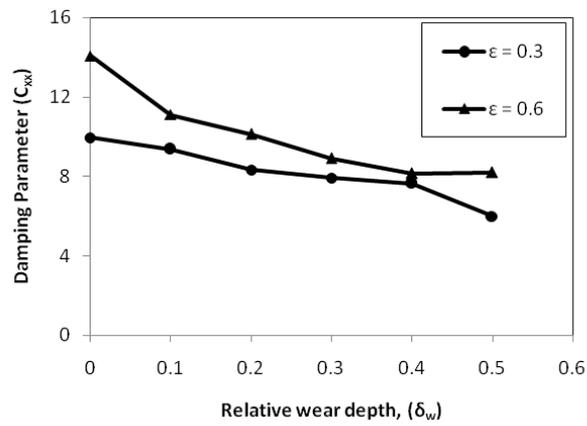


Figure 10 Direct fluid-film damping coefficient versus relative wear depth

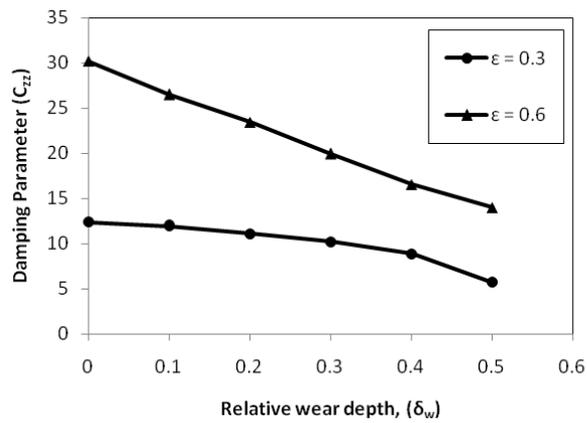


Figure 11 Direct fluid-film damping coefficient versus relative wear depth

Table 2 Transient characteristics of worn hydrodynamic journal bearing at eccentricity of 0.3 and 0.6.

Wear depth	Eccentricity ratio	Critical Mass	Transient linear response	Transient nonlinear response
0.0	0.3	$M_c = 0.9M_j$	Stable	Stable
		$M_c = M_j$	Limit cycle	Limit cycle
		$M_c = 1.1M_j$	Unstable	Unstable
	0.6	$M_c = 0.9M_j$	Stable	Stable
		$M_c = M_j$	Limit cycle	Limit cycle
		$M_c = 1.1M_j$	Unstable	Unstable
0.2	0.3	$M_c = 0.9M_j$	Stable	Stable
		$M_c = M_j$	Limit cycle	Limit cycle
		$M_c = 1.1M_j$	Unstable	Unstable
	0.6	$M_c = 0.9M_j$	Stable	Stable
		$M_c = M_j$	Limit cycle	Limit cycle
		$M_c = 1.1M_j$	Unstable	Unstable
0.4	0.3	$M_c = 0.9M_j$	Stable	Stable
		$M_c = M_j$	Limit cycle	Stable
		$M_c = 1.1M_j$	Unstable	Unstable
	0.6	$M_c = 0.9M_j$	Stable	Stable
		$M_c = M_j$	Limit cycle	Unstable
		$M_c = 1.1M_j$	Unstable	Unstable

From Tab. 2 it is observed that the effect of wear on stability of journal bearing is greater at eccentricity ratio of 0.6. It is also observed that transient response of linearized equation are approximate same in all varying wear depth and eccentricity ratio. But in case of non linear transient response, the stability of journal bearing altered. The non-linear journal centre motion trajectories of worn journal bearing with respect to varying wear depth and for a eccentricity ratio of 0.3 and 0.6 is shown in Figs. 12–17. It is observed that non-linearized equation of gives limit cycle when $M_c = M_j$ and $\bar{\delta}_w < 0.2$ and it become unstable at wear depth of 0.4 and eccentricity ratio is equals to 0.6. as shown in Fig. 17. It is also observed that with varying wear depth, non-linearized equation gives stable position of journal when $M_c = 0.9M_j$ in both eccentricity ratio of 0.3 and 0.6 and gives Unstable position of journal when $M_c = 1.1M_j$.

The results indicated about the lifespan and use of bearing in the rotating machinery. It also predicted life of worn bearing for smooth operation. It is observed that stability margin increases with the increase in wear depth up to $\bar{\delta}_w = 0.3$ and after that it decreases. From results it is advisable for the use of worn bearing in rotating machinery up to wear depth of 0.3.

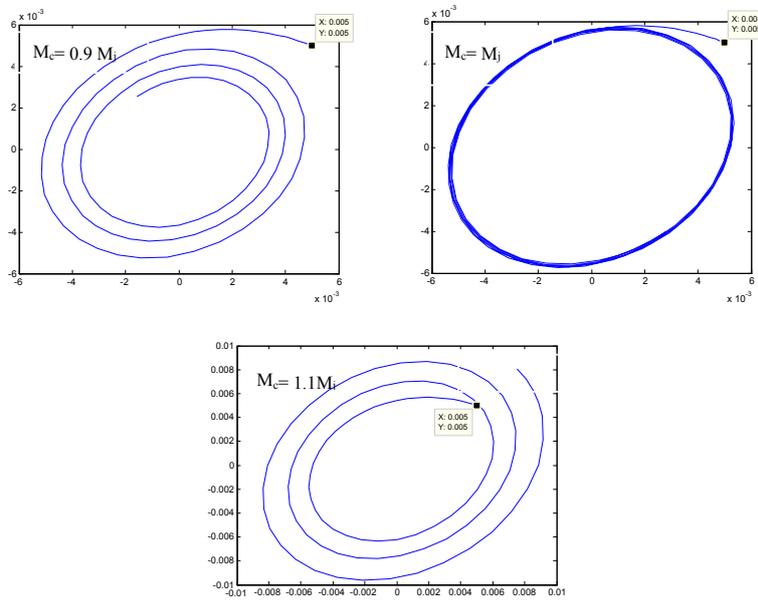


Figure 12 Non linear trajectories for hydrodynamic journal bearings at eccentricity ratio of 0.3 and $\bar{\delta}_w = 0.0$

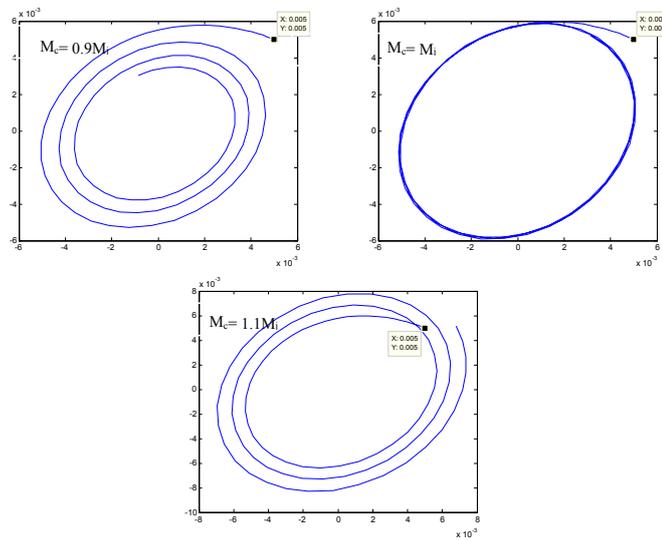


Figure 13 Non linear trajectories for hydrodynamic journal bearings at eccentricity ratio of 0.3 and $\bar{\delta}_w = 0.2$

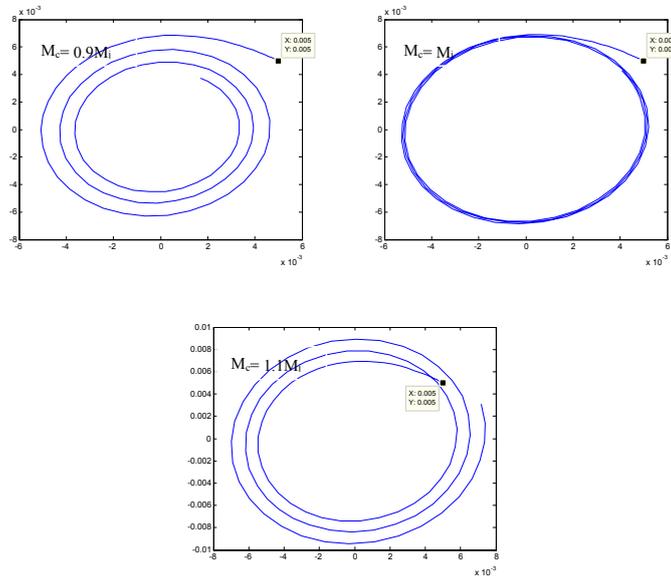


Figure 14 Nonlinear trajectories for hydrodynamic journal bearings at eccentricity ratio of 0.3 and $\bar{\delta}_w = 0.4$

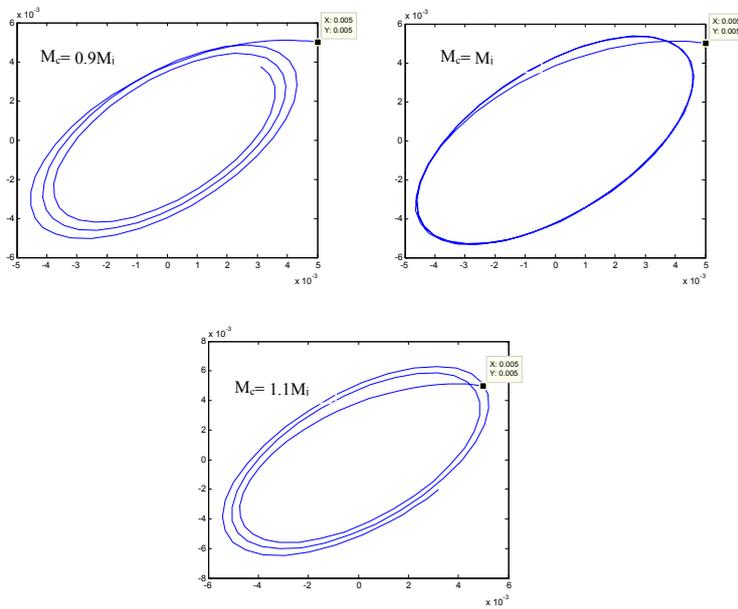


Figure 15 Nonlinear trajectories for hydrodynamic journal bearings at eccentricity ratio of 0.6 and $\bar{\delta}_w = 0.0$

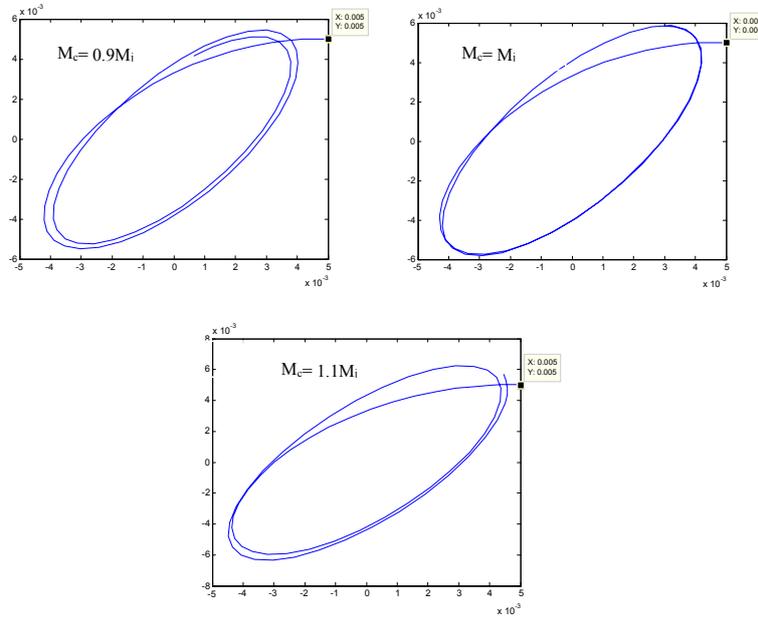


Figure 16 Non linear trajectories for hydrodynamic journal bearings at eccentricity ratio of 0.6 and $\delta_w = 0.2$

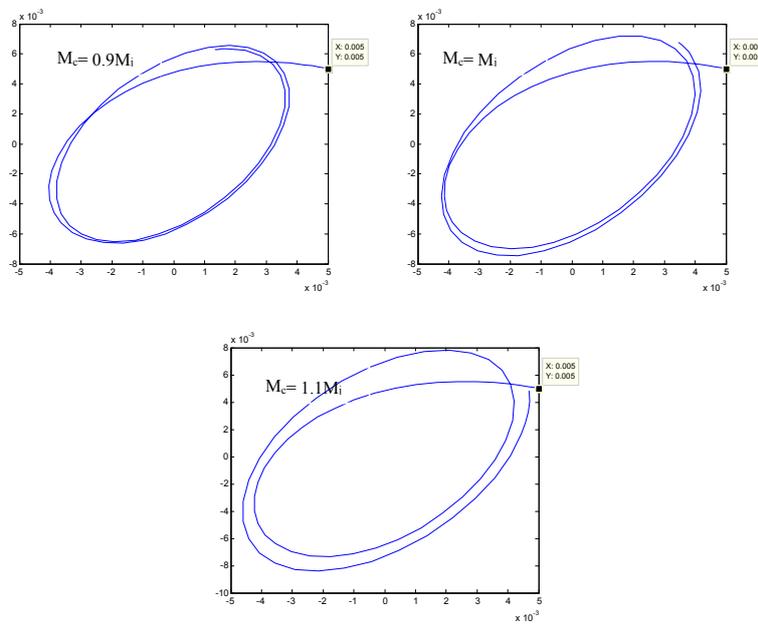


Figure 17 Nonlinear trajectories for hydrodynamic journal bearings at eccentricity ratio of 0.6 and $\delta_w = 0.4$

4. Conclusion

This study investigates the effect of varying wear depth on the performance of worn hydrodynamic journal bearings. Based on results, following conclusions can be drawn:

1. The effect of wear on all static and dynamic parameters is greater at higher eccentricity ratio.
2. The stability of worn hydrodynamic journal bearings becomes progressively high at higher eccentricity ratio. It increases first with an increase in wear depth and decreases when $\bar{\delta}_w > 0.3$.
3. The results indicated about the lifespan and use of bearing in the rotating machinery. It is advisable for the use of worn bearing in rotating machinery up to wear depth of 0.3.

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Nomenclature:

Dimensional parameters:

- c – radial clearance, mm,
 C_{ij} – fluid-film damping coefficients (i,j = x,z), N mm⁻²,
 D – journal diameter, mm,
 e – journal eccentricity, mm,
 F – fluid-film reaction ($\partial h/\partial t \neq 0$), N,
 F_x, F_z – fluid-film reaction components in X and Y direction ($\partial h/\partial t \neq 0$), N,
 h – nominal fluid-film thickness, mm,
 L – bearing length, mm,
 M_c, M_J – critical mass and Mass of journal, Kg,
 N – rotational speed, rpm,
 O_B – center of the bearing,
 O_j – center of the journal,
 p – pressure, N.mm-2,
 p_s – reference pressure, N mm⁻² ($\mu_r \omega_J R_J^2/c^2$),
 R_j, R_b – radius of journal and bearing, mm,
 S – Sommerfeld number,
 S_{ij} – fluid-film stiffness coefficients (i,j=X,Z), N mm⁻¹,
 t – time, s,
 W – load capacity, N,
 W_o – external load, N,
 x – circumferential coordinate,
 y – axial coordinate,
 X_J, Z_J – journal center coordinate,
 X, Y, Z – Cartesian coordinate system,
 z – coordinate along film thickness.

Greek letters:

- μ – lubricant viscosity, Pa s,
 α – angular coordinate, rad,
 ϕ – attitude angle, rad,
 F – eccentricity ratio, e/c ,
 δ_w – wear depth, mm.

Non-dimensional parameters:

- $\bar{C}_{ij} = C_{ij} \left(\frac{c^3}{\mu_r R_J^4} \right) \bar{F}_x$
 $\bar{F}_z = (F_x, F_z/p_s R_J^2) \bar{h}, \bar{h}_{\min} = h/c$
 $h_{\min}/c\bar{p} = (p/p_s) \bar{p}_{\max} = p_{\max}/p_s$
 $\bar{S}_{ij} = S_{ij} \left(\frac{c}{p_s R_J^2} \right)$
 $\bar{\tau} = t (c^2 p_s / \mu_r R_J^2)$
 $\bar{W}_o = \frac{W_o}{p_s R_J^2}$
 $(\bar{X}_J, \bar{Z}_J) = (X_J, Z_J)/c$
 $(\bar{X}, \bar{Z}) = (X, Z)/c$
 $\alpha, \beta = (x, y)/R_J$
 $\varepsilon = e/c$
 L/D – aspect ratio,

$\bar{\delta}_w = \delta_w/c$ – worn depth,
 $\Omega = \omega_J (\mu_r R_J^2 / c^2 p_s)$ – speed parameter.

Matrices:

$[\bar{F}]$ – assembled fluidity matrix,

$\{\bar{p}\}$ – nodal pressure vector,

$\{\bar{Q}\}$ – nodal flow vector,

$\{\bar{R}_H\}$ – column vectors due to hydrodynamic terms,

$\{\bar{R}_{X_J}\}, \{\bar{R}_{Z_J}\}$ – global right hand side vectors due to journal center linear velocities.

Subscripts and superscript:

b – Bearing

J – Journal