

## Rotation, Initial Stress, Gravity and Electromagnetic Field Effect on P Wave Reflection from Stress-Free Surface Elastic Half-Space with Voids under Three Thermoelastic Models

S. M. ABO-DAHAB

*Math. Dept, Faculty of Science  
Taif University 888, Saudi Arabia  
sdahb@yahoo.com*

S. Z. RIDA

R. A. MOHAMED

A. A. KILANY

*Math. Dept., Faculty of Science  
South Valley University, Qena 83523, Egypt*

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The present paper is devoted to investigate the influence of the rotation, thermal field, initial stress, gravity field, electromagnetic and voids on the reflection of P wave under three models of generalized thermoelasticity: Classical and Dynamical coupled model (CD), Lord-Shulman model (LS), Green-Lindsay model (GL). The boundary conditions at stress-free thermally insulated surface are satisfied to obtain Algebraic system of four equations in the reflection coefficients of various reflected waves. It is shown that there exist four plane waves;  $P_1$ ,  $P_2$ ,  $P_3$  and  $P_4$ . In addition, the reflection coefficients from insulated and isothermal stress-free surface for the incident  $P$  wave are obtained. Finally, numerical values of the complex modulus of the reflection coefficients are visualized graphically to display the effects of the rotation, initial stress, gravity field magnetic field, thermal relaxation times and voids parameters.

*Keywords:* electromagnetic, rotation, initial stress, gravity field, thermoelasticity, reflection, voids, relaxation times.

### 1. Introduction

In recent years, more attentions have been given for the rotation effect on waves with thermal field, initial stress magnetic field, electric field and voids under relaxation times because of its utilitarian aspects on Seismic waves, Earthquakes, Volcanoes and Acoustics. In the classical theory of thermoelasticity, when an elastic solid is subjected to a thermal disturbance, the effect is felt in a location far from the

source, instantaneously. This implies that the thermal wave propagates with infinite speed, a physically impossible result. In contrast to conventional thermoelasticity, non-classical theories came into existence during the last part of 20<sup>th</sup> century. For example, Lord and Shulman [1] by incorporating a flux-rate term into Fourier's law of heat conduction, formulated a generalized theory which involves a hyperbolic heat transport equation admitting finite speed for thermal signals. Green and Lindsay [2] by including temperature rate among the constitutive variables, developed a temperature-rate dependent thermoelasticity that does not violate the classical Fourier's law of heat conduction, when body under consideration has center of symmetry and this theory also predicts a finite speed for heat propagation. Chandrasekharaiah [3] referred to this wavelike thermal disturbance as second sound. Singh [4] studied the wave propagation in a generalized thermoelastic material with voids. Dhaliwal and Wang [5] pointed out a heat-flux dependent theory of thermoelasticity with voids. Marin [6] discussed an uniqueness result for body with voids in linear thermoelasticity. Marin [7] discussed on the domain of influence in thermoelasticity of bodies with voids. Ciarletta and Scarpetta [8] investigated Some results on thermoelasticity for dielectric materials with voids. Ciarletta, and Scalia [9] illustrated on the nonlinear theory of non simple thermoelastic materials with voids. Puri and Cowin [10] studied the Plane waves in linear elastic materials with voids. Cowin and Nunziato [11] developed linear elastic materials with voids. Marin [12] illustrated contributions on uniqueness in thermoelasto-dynamics on bodies with voids. Marin and Salca [13] discussed a relation of knopoff-de hoop type in thermoelasticity of dipolar bodies with voids. Birsan [14] discussed existence and uniqueness of weak solution in the linear theory of elastic shells with voids. Kumar and Rani [15] illustrated deformation due to moving loads in thermoelastic body with voids. Tomar [16] studied the wave propagation in a micropolar elastic plate with voids. Rusu [17] discussed on the existence and uniqueness in thermoelasticity of materials with voids. Saccomandi [18] illustrated some remarks about the thermoelastic theory of materials with voids. Ieşan [20] Nonlinear plane strain of elastic materials with voids, De Cicco and Diaco [21] investigated a theory of thermoelastic materials with voids without energy dissipation. Othman and Song [22] studied the effect of rotation on the reflection of magneto- thermoelastic waves under thermoelasticity without energy dissipation. Othman and Song [23] studied reflection of magneto -thermo -elastic waves from a rotating elastic half-space. Abo-Dahab et al. [24] discussed rotation and magnetic field effects on P wave reflection from Stress-Free surface elastic half-space with voids under one thermal relaxation Time. Abd-alla et al. [25] investigated on the reflection of the generalized magneto-thermo-viscoelastic plane waves. Abd-alla and Al-dawy [26] illustrated the reflection phenomena of SV waves in a generalized thermoelastic medium. Biot [27] investigated thermoelasticity and irreversible thermodynamics. Knopoff [28] studied The interaction between elastic wave motions and a magnetic field in electrical. El-Naggar [29] discussed on the initial stress, magnetic field, voids and rotation effects on plane waves in generalized thermoelasticity. El-Sirafy et al. [30] studied effects of voids and rotation on P wave in a thermoelastic half-space under Green-Naghdi theory. Recently, Abo-Dahab et al. [32] investigated wave propagation in fibre-reinforced anisotropic thermoelastic medium subjected to gravity field.

In this paper, we study the  $P$  - wave on an isotropic homogeneous solid half-space under the influence of initial stress, magnetic field, electric field, thermal relaxation times, gravity field and rotation with voids.

**2. Basic equations**

The governing equations for an isotropic, homogeneous elastic solid with generalized thermoelasticity with voids and Lorentz's body forces in the absence of incremental heat flux at reference temperature  $T_0$  are given as follows:

$$P_1, P_2 P_3 \tag{1}$$

$$\rho\eta = \beta e_{kk} + \alpha\Theta + m\Phi \tag{2}$$

$$g^* = -be_{kk} - \xi\Phi + m\Theta \tag{3}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{4}$$

$$\omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j}) \tag{5}$$

$$S_i = \alpha\Phi_{,i} \tag{6}$$

The Maxwell's equation:

$$\tau_{ij} = \mu_e [H_i h_j + H_j h_i - (H_k \cdot h_k) \delta_{ij}] \tag{7}$$

The equation of motion is:

$$\sigma_{ji,j} + F_i = \rho \left[ \ddot{u} + \left( \vec{\Omega} \times \vec{\Omega} \times \vec{u} \right) + \left( 2\vec{\Omega} \times \dot{\vec{u}} \right) \right]_i \tag{8}$$

which tends to:

$$\begin{aligned} & \left( \mu - \frac{p}{2} \right) u_{i,jj} + \left( \lambda + \mu + \frac{p}{2} \right) u_{j,ij} - \beta\Theta_{,i} + b\Phi_{,i} + F_i + G_i \\ & = \rho \left[ \ddot{u} + \left( \vec{\Omega} \times \vec{\Omega} \times \vec{u} \right) + \left( 2\vec{\Omega} \times \dot{\vec{u}} \right) \right]_i \end{aligned} \tag{9}$$

where:

$$F_i = \left( \vec{J} \times \vec{B} \right)_i, \quad G = \rho g \left( -\frac{\partial w}{\partial x}, 0, \frac{\partial u}{\partial x} \right) \tag{10}$$

where,  $\vec{\Omega} \times \vec{\Omega} \times \vec{u}$  is the centripetal acceleration due to the time varying motion only and  $2\vec{\Omega} \times \dot{\vec{u}}$  is the Coriolis acceleration.

For slowly moving medium, the variation of magnetic field and electric field are given by Maxwell's equation as the following form:

$$\begin{aligned} \text{curl } \vec{h} &= \vec{J} + \epsilon_0 \dot{\vec{E}} & \text{curl } \vec{E} &= -\mu_e \dot{\vec{h}} & \text{div } \vec{h} &= 0 \\ \text{div } \vec{E} &= 0 & \vec{E} &= -\mu_e \left( \dot{\vec{u}} \times \vec{H} \right) & \vec{h} &= \text{curl} \left( \vec{u} \times \vec{H}_0 \right) \end{aligned} \tag{11}$$

where:

$$\vec{H} = -\vec{H}_0 + \vec{h}(x, z, t), \quad \vec{H}_0 = (0, H, 0) \quad (12)$$

The equation of heat conduction:

$$K\theta_{,kk} = \left(1 + \tau_2 \frac{\partial}{\partial t}\right) \left[\rho C_e \dot{\Theta} + mT_0 \dot{\Phi}\right] + \gamma T_0 \left(1 + \delta\tau_2 \frac{\partial}{\partial t}\right) \dot{e}_{kk} \quad (13)$$

$$\alpha\Phi_{,kk} - \xi\Phi - bu_{k,k} + m\Theta = \rho\chi\ddot{\Phi} \quad (14)$$

For two-dimensional motion in  $xz$ -plane, equations (9), (13) and (14) written as:

$$\begin{aligned} &\left(\frac{\lambda + 2\mu + \mu_e H^2}{\rho}\right) u_{1,11} + \left(\frac{\lambda + \mu + \mu_e H^2 + P/2}{\rho}\right) u_{3,13} + \left(\frac{\mu - P/2}{\rho}\right) u_{1,33} - gu_{3,1} \\ &- \frac{\gamma}{\rho} \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \Theta_{,1} + \frac{b}{\rho} \Phi_{,1} = \left(\left[1 + \frac{\mu_e^2 \varepsilon_0 H^2}{\rho}\right] \ddot{u}_1 + 2\Omega \dot{u}_3 - \Omega^2 u_1\right) \end{aligned} \quad (15)$$

$$\begin{aligned} &\left(\frac{\lambda + 2\mu + \mu_e H^2}{\rho}\right) u_{3,33} + \left(\frac{\lambda + \mu + \mu_e H^2 + P/2}{\rho}\right) u_{1,31} + \left(\frac{\mu - P/2}{\rho}\right) u_{3,11} + gu_{1,1} \\ &- \frac{\gamma}{\rho} \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \Theta_{,3} + \frac{b}{\rho} \Phi_{,3} = \left(\left[1 + \frac{\mu_e^2 \varepsilon_0 H^2}{\rho}\right] \ddot{u}_3 - 2\Omega \dot{u}_1 - \Omega^2 u_3\right) \end{aligned} \quad (16)$$

$$\begin{aligned} K \left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial z^2}\right) &= \left(1 + \tau_2 \frac{\partial}{\partial t}\right) \frac{\partial}{\partial t} [\rho C_e \Theta + mT_0 \Phi] \\ &+ \gamma T_0 \left(1 + \delta\tau_2 \frac{\partial}{\partial t}\right) \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) \end{aligned} \quad (17)$$

$$\alpha \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2}\right) - \xi\Phi - b \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + m\Theta = \rho\chi \frac{\partial^2 \Phi}{\partial t^2} \quad (18)$$

We study the above basic equations for the following three different theories:

**Table 1** Kenaf fibers and E-glass fibers properties [14]

$\delta$	$\tau_2$	$\tau_1$	Theories
0	0	0	(i) Classical and Dynamical coupled theory (1956) (CD)
1	>0	0	(ii) Lord and Shulman's theory (1967) (LS)
0	>0	>0	(iii) Green and Lindsay's theory (1972) (GL)

The displacement components  $u_1$  and  $u_3$  may be written in terms of the scalar and the vector potential functions,  $\varphi$  and  $\psi$ , respectively, as:

$$u_i = \varphi_{,i} + e_{ijk} \psi_{k,j} \quad (19)$$

which take the form:

$$u = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x} \quad (20)$$

Substituting from Eq. (20) into Eqs. (15)-(18), we get:

$$C_S^2 \nabla^2 \psi + g \frac{\partial \varphi}{\partial x} = (R_H'^2 \ddot{\psi} - \Omega^2 \psi - 2\Omega \dot{\varphi}) \quad (21)$$

$$(C_T^2 + R_H^2) \nabla^2 \varphi - \bar{\beta} \tau_1^{**} \Theta + \bar{b} \Phi - g \frac{\partial \psi}{\partial x} = (R_H'^2 \ddot{\varphi} - \Omega^2 \varphi + 2\Omega \dot{\psi}), \quad (22)$$

$$\omega^* \nabla^2 \Theta = \tau_2^{**} (\dot{\Theta} + \varepsilon_1 \dot{\Phi}) + \tau_{\delta 2}^* \varepsilon \nabla^2 \dot{\varphi} \quad (23)$$

$$\alpha \nabla^2 \Phi - \xi \Phi + m\Theta - b \nabla^2 \varphi = \rho \chi \ddot{\Phi} \quad (24)$$

where:

$$\left. \begin{aligned} C_T^2 &= \frac{\lambda+2\mu}{\rho} & C_S^2 &= \frac{\mu-P/2}{\rho}, & R_H^2 &= \frac{\mu_e H^2}{\rho} & R_H'^2 &= 1 + \frac{\varepsilon_0 \mu_e^2 H^2}{\rho} \\ \omega^* &= \frac{K\rho}{\rho C_e} & \tau_1^{**} &= 1 + \tau_1 \frac{\partial}{\partial t} & \varepsilon &= \frac{\beta T_0}{\rho C_e} & \varepsilon_1 &= \frac{m T_0}{\rho C_e} & \beta &= \frac{\gamma \rho}{\rho} \\ \tau_2^{**} &= 1 + \tau_2 \frac{\partial}{\partial t} & \tau_{\delta 2}^* &= 1 + \delta \tau_2 \frac{\partial}{\partial t} \end{aligned} \right\} \quad (25)$$

Eqs. (21)-(24) are coupled in  $\varphi$ ,  $\Theta$ ,  $\Phi$  and  $\psi$  we see that the SV and P waves affect by the thermal field, voids, rotation and the electromagnetic.

For the analytical solution of Eqs. (21)-(24) in the form of the harmonic travelling wave, we suppose that the solution takes the form:

$$[\varphi, \Theta, \Phi, \psi](x, z, t) = [\varphi_0, \Theta_0, \Phi_0, \psi_0] e^{ik(x \sin \theta + z \cos \theta - vt)} \quad (26)$$

where,  $\varphi_0$ ,  $\Theta_0$ ,  $\Phi_0$  and  $\psi_0$  are arbitrary constants and the pair  $(\sin \theta, \cos \theta)$  denotes the projection of the wave normal onto the xz-plane.

Substituting from Eq. (26) into Eqs. (21)-(24), we obtain:

$$k \left( C_S^2 - R_H'^2 v^2 - \frac{\Omega^2}{k^2} \right) \psi_0 = (ig \sin \theta - 2i\Omega v) \varphi_0 \quad (27)$$

$$k^2 \left( C_T^2 + R_H^2 - R_H'^2 v^2 - \frac{\Omega^2}{k^2} \right) \varphi_0 + \bar{\beta} \tau_1^* \Theta_0 - \bar{b} \Phi_0 - ik(g \sin \theta - 2\Omega v) \psi_0 = 0 \quad (28)$$

$$i\tau_{\delta 2} \varepsilon v k^3 \varphi_0 - (i\tau_2^* v - \omega^* k) k \Theta_0 + i\tau_2^* \varepsilon_1 k \Phi_0 = 0 \quad (29)$$

$$-bk^2 \varphi_0 + [k^2(\alpha - \rho \chi v^2) + \xi] \Phi_0 - m\Theta_0 = 0 \quad (30)$$

Eliminating the constants  $\varphi_0$ ,  $\Theta_0$ ,  $\Phi_0$  and  $\psi_0$  from Eqs. (27)-(30), we obtain:

$$L\Gamma^4 + M\Gamma^3 + N\Gamma^2 + Q\Gamma + R = 0 \quad (31)$$

where:

$$\begin{aligned}
\Gamma &= v^2 \\
L &= 4\Omega g \sin \theta \tau_2^* \left( \frac{\xi}{\omega^3} - \frac{k}{\omega^3} \rho \chi \right) + 4 \frac{\Omega^2}{\omega^2} \tau_2^* (\rho \chi - \xi) \\
M &= g^2 \sin^2 \theta \tau_2^* \left( \frac{\rho \chi}{\omega^2} - \frac{\xi}{\omega^4} \right) + 4\Omega g \sin \theta \left( \frac{k\alpha(\tau_2^* + 1)}{\omega^3} - \frac{k\xi\omega^*}{\omega^5} + \frac{m\varepsilon_1\tau_2^*}{\omega^3} \right) \\
&+ \Lambda_1 \beta \tau_1^* \tau_\delta^* \rho \chi \varepsilon k \frac{4\Omega^2}{\omega^4} (\xi\omega^* - \omega^2 \alpha \tau_2^* - \rho \chi \omega^* \omega^2 - m\varepsilon \omega_1^2 \tau_2^*) \\
&+ \Lambda_1^2 \tau_2^* (\xi + m\varepsilon_1 - \rho \chi \omega^2) \\
N &= \frac{g^2 \sin^2 \theta}{\omega^4} \left( \omega^* \xi - \alpha \tau_2^* - \frac{m\varepsilon_1 \tau_2^*}{\omega^2} \right) + \Lambda_1^2 (\rho \chi \omega^* \omega^2 + \alpha \tau_2^* \omega^2 - \omega^* \xi) \\
&+ \Lambda_2 \tau_2^* \Lambda_1 (\rho \chi \omega^2 - \xi - m) - \frac{4}{\omega^3} \Omega \alpha k \omega^* g \sin \theta + \frac{4}{\omega^2} \Omega^2 \omega^* \alpha \\
&- \Lambda_1 \left( \beta \tau_\delta^* \tau_1^* \varepsilon (\alpha k + \xi) - \beta \tau_1^* \tau_2^* b \varepsilon_1 - \bar{b} m \varepsilon \tau_\delta^* - b \bar{b} \tau_2^* \right) \\
Q &= g^2 \sin^2 \theta \alpha \omega^* + \Lambda_1 \left( -b \bar{b} \omega^* - \Lambda_2 \omega^* \xi - \Lambda_1 \alpha \omega^* \omega^2 \right) - C_s^2 m \varepsilon \bar{b} \tau_\delta^* \\
&- \beta \tau_1^* \tau_\delta^* C_s^2 \rho \chi \varepsilon k \omega^2 - \beta \tau_1^* \tau_2^* b C_s^2 \varepsilon_1 - C_s^2 \rho \chi \omega^2 \tau_2^* \Lambda_3 - b \bar{b} C_s^2 \tau_2^* \\
R &= \Lambda_1 \Lambda_2 (\omega^* \alpha k^2 - \rho \chi \omega^* \omega^2 - \alpha \tau_2^* \omega^2) + -\beta \alpha \tau_1^* \tau_\delta^* C_s^2 \varepsilon \omega^2 + \beta \xi \tau_1^* \tau_\delta^* C_s^2 \varepsilon \\
&+ b \bar{b} C_s^2 \omega^* + C_s^2 \Lambda_3 (\rho \chi \omega^* \omega^2 - \omega^* \alpha - \omega^* \xi + \alpha \tau_2^* \omega^2 + \xi \omega^2 \tau_2^* + m \varepsilon_1 \tau_2^* \omega^2)
\end{aligned}$$

where:

$$\begin{aligned}
\bar{b} &= \frac{b}{\rho} & \bar{\beta} &= \frac{\beta}{\rho} & \tau_1^* &= \tau_1 - i\omega & \tau_2^* &= \tau_2 + \frac{i}{\omega} & \tau_\delta^* &= \delta \tau_2 + \frac{i}{\omega} \\
\Lambda_1 &= \left( R_H'^2 + \frac{\Omega^2}{\omega^2} \right) & \Lambda_2 &= (C_T^2 + C_s^2 + R_H^2) & \Lambda_3 &= (C_T^2 + R_H^2)
\end{aligned}$$

### 3. Reflection coefficients at free surface

Due to the existing of four waves reflect from incident  $P$  wave as shown in Fig. 1, we take into consideration the reflected waves  $P_1, P_2, P_3$  and  $P_4$ ; if the wave normal of the incident wave makes angle  $\theta_o$  with the positive direction of z-axis, and those of reflected  $P_1, P_2, P_3$  and  $P_4$  waves make  $\theta_1, \theta_2, \theta_3$  and  $\theta_4$  also with the z-axis, the displacement potentials  $\varphi$  and  $\psi$  the temperature  $\Theta$  and volume of friction  $\Phi$  take the following forms:

$$\begin{aligned}
\varphi &= A_0 \exp[ik_0(x \sin \theta_0 + z \cos \theta_0) - i\omega t] \\
&+ \sum_{\nu=1}^4 A_\nu \exp[ik_\nu(x \sin \theta_\nu - z \cos \theta_\nu) - i\omega t] \\
\psi &= \zeta_0 A_0 \exp[ik_0(x \sin \theta_0 + z \cos \theta_0) - i\omega t]
\end{aligned} \tag{32}$$

$$+ \sum_{\nu=1}^4 \zeta_{\nu} A_{\nu} \exp[ik_{\nu}(x \sin \theta_{\nu} - z \cos \theta_{\nu}) - i\omega t] \quad (33)$$

$$\Theta = \xi_0 A_0 \exp[ik_0(x \sin \theta_0 + z \cos \theta_0) - i\omega t]$$

$$+ \sum_{\nu=1}^4 \xi_{\nu} A_{\nu} \exp[ik_{\nu}(x \sin \theta_{\nu} - z \cos \theta_{\nu}) - i\omega t] \quad (34)$$

$$\Phi = \eta_0 A_0 \exp[ik_0(x \sin \theta_0 + z \cos \theta_0) - i\omega t]$$

$$+ \sum_{\nu=1}^4 \eta_{\nu} A_{\nu} \exp[ik_{\nu}(x \sin \theta_{\nu} - z \cos \theta_{\nu}) - i\omega t] \quad (35)$$

where:

$$\left. \begin{aligned} \zeta_j &= \frac{ig \sin \theta - 2i\Omega v_j}{k_j \left( C_s^2 - R_H'^2 v_j^2 - \frac{\Omega^2}{k_j^2} \right)} \\ \xi_j &= -\frac{v_j^2 k_j^4 \{ \tau_2^* \varepsilon [k_j^2 (\alpha - \rho \chi v_j^2) + \xi] + b \tau_2^* \varepsilon_1 \}}{\tau_2^* \varepsilon_1 m v_j^2 k_j^2 + (\omega^* k_j - \tau_2^* v_j^2 k_j^2) (k_j^2 (\alpha - \rho \chi v_j^2) + \xi)} \\ \eta_j &= \frac{b k_j^2 + m \xi_j}{k_j^2 (\alpha - \rho \chi v_j^2) + \xi} \end{aligned} \right\} \quad (j = 0, 1, 2, 3, 4) \quad (36)$$

$A_0$  is the amplitudes of the incident  $P$  wave, respectively, and  $A_1, A_2, A_3$  and  $A_4$  are the amplitudes of the reflected  $P_1, P_2, P_3$  and  $P_4$  waves, respectively.

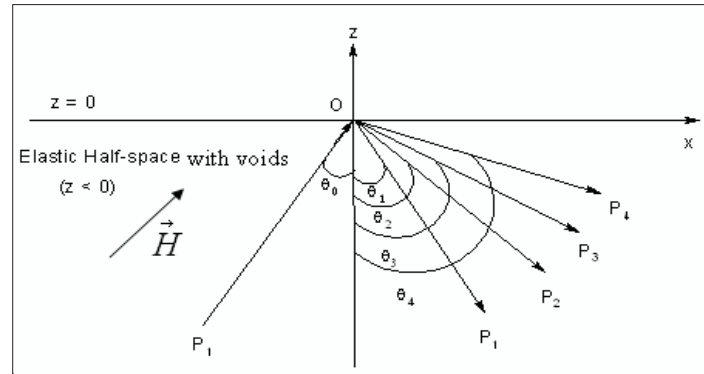


Figure 1 Geometry of the problem

The boundary conditions at the surface interface take the following form:

$$\sigma_{zz} + \tau_{zz} = -P \quad \sigma_{zx} + \tau_{zx} = 0 \quad \frac{\partial \Theta}{\partial z} = 0 \quad \frac{\partial \Phi}{\partial z} = 0 \quad \text{at } z = 0 \quad (37)$$

For the reflected waves, the wave numbers and the reflected angles may be written as:

$$\frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2} = \frac{\sin \theta_3}{v_3} = \frac{\sin \theta_4}{v_4}. \quad (38)$$

Substituting from Eqs. (32)-(35) into the boundary conditions in Eq. (37), we obtain a system of four algebraic equations takes the form:

$$\sum A_{ij} Z_j = C_i, \quad (i, j = 1, 2, 3, 4) \quad (39)$$

where:

$$\begin{aligned} A_{1j} &= \left[ \lambda + \mu(2 \cos^2 \theta_j + \zeta_j \sin 2\theta_j) + \mu_e H_o^2 + \beta \tau_1^* \frac{\xi_j}{k_j^2} - b \frac{\eta_j}{k_j^2} \right] \left( \frac{k_j}{k_1} \right)^2 \\ A_{2j} &= \left( \frac{P}{2} \zeta_j + \mu [-\sin(2\theta_j) + \zeta_j \cos(2\theta_j)] \right) \left( \frac{k_j}{k_1} \right)^2 \\ A_{3j} &= \cos \theta_j \frac{\xi_j}{k_j^2} \left( \frac{k_j}{k_1} \right)^3 \\ A_{4j} &= \cos \theta_j \frac{\eta_j}{k_j^2} \left( \frac{k_j}{k_1} \right)^3 \end{aligned}$$

$C_i$  and  $Z_j$  take the form:

$$\begin{aligned} C_1 &= - \left[ \lambda + \mu(2 \cos^2 \theta_1 + \zeta_1 \sin 2\theta_1) + \mu_e H_o^2 + \beta \tau_1^* \frac{\xi_1}{k_1^2} - b \frac{\eta_1}{k_1^2} \right] \\ C_2 &= -\frac{P}{2} \zeta_1 + \mu [\sin(2\theta_1) - \zeta_1 \cos(2\theta_1)] \quad C_3 = A_{31} \quad C_4 = A_{41} \end{aligned}$$

and:

$$\begin{aligned} Z_1 &= \frac{A_1}{A_0} & Z_2 &= \frac{A_2}{A_0} & Z_3 &= \frac{A_3}{A_0} & Z_4 &= \frac{A_4}{A_0} \\ Z_1 &= \frac{A_1}{A_0} & Z_2 &= \frac{A_2}{A_0} & Z_3 &= \frac{A_3}{A_0} & Z_4 &= \frac{A_4}{A_0} \end{aligned}$$

#### 4. Numerical results, discussion and concluding remarks

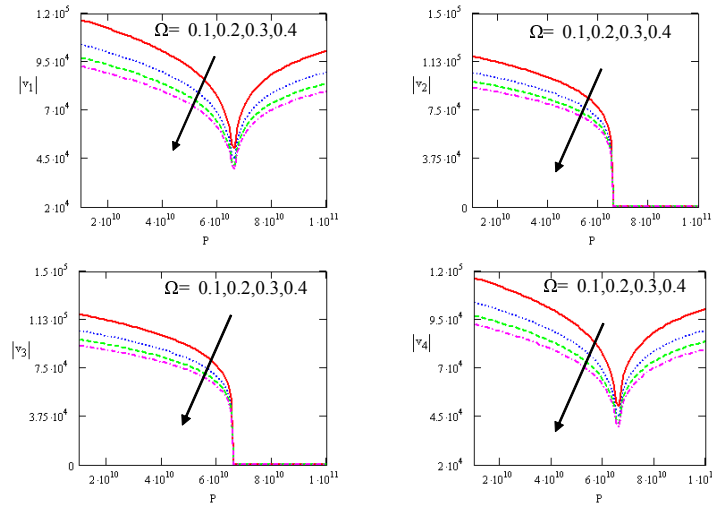
To study the dependence of speeds and reflection coefficients given in Eqs. (31) and (39) on magnetic field, frequency, rotation, relaxation times, and etc., we consider an example where Magnesium Crystal like material is modeled as an isotropic generalized rotating magneto-thermoelastic material with voids for numerical computations. The following elastic and thermal constants at  $T_0 = 298$  K:

$$\begin{aligned} \lambda &= .217 \times 10^{10} \text{N m}^{-2}, & \mu &= 3.278 \times 10^{10} \text{N m}^{-2}, & \sigma &= 0.33, & \omega &= 0.1, \\ C_e &= .104 \times 10^3 \text{Jkg}^{-1} \text{degree}^{-1}, & K &= 1.7 \times 10^2 \text{Wm}^{-1} \text{degree}^{-1}, & k &= 0.04, \\ \rho &= .174 \times 10^3 \text{Kg.m}^{-3}, & \beta &= 2.68 \times 10^6 \text{Nm}^{-2} \text{egree}^{-1}, & \alpha_t &= 0.01, \end{aligned}$$

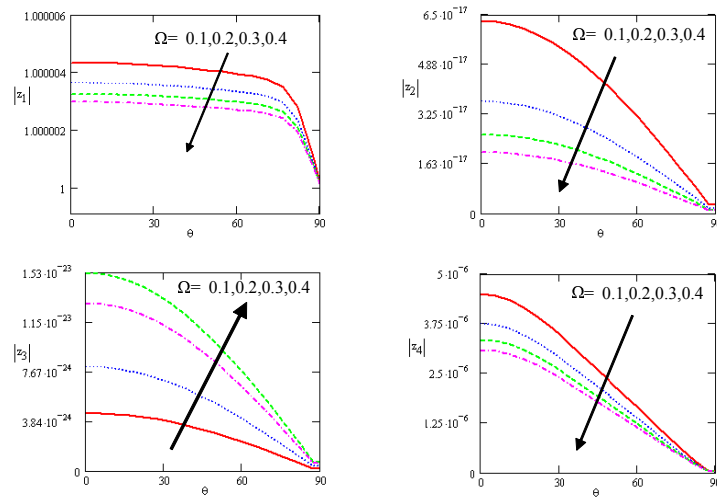
and the following void parameters are given in Abo-Dahab [31] as:

$$\begin{aligned} \chi &= .1753 \times 10^{-15} \text{m}^2, & \alpha &= 3.688 \times 10^{-5} \text{N}, & \xi &= 1.475 \times 10^{10} \text{Nm}^{-2}, \\ b &= .113849 \times 10^{10} \text{Nm}^{-2}, & m &= 2 \times 10^6 \text{Nm}^{-2} \text{degree}^{-2} \end{aligned}$$



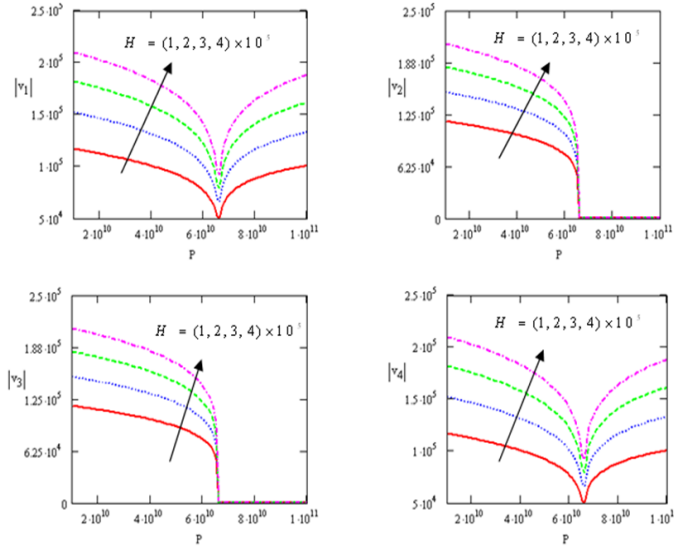


**Figure 2** Variations of the waves velocities under rotation with varies values of initial stress

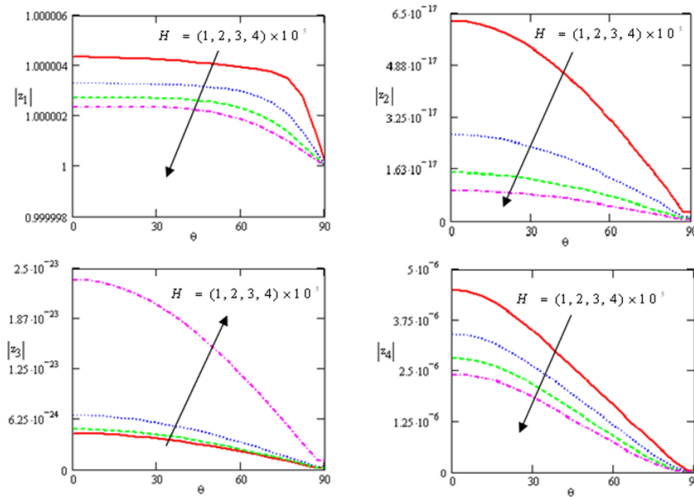


**Figure 3** Variations of the waves amplitudes under rotation with varies values of angle of incidence

Figure 2 displays the variations of the waves velocities under rotation with varies values of initial stress, waves velocities decrease with increasing rotation. Figure 3 displays the variations of the waves amplitudes under rotation with varies values of angle of incidence, amplitudes  $z_1, z_2, z_4$  decrease with increasing rotation but amplitudes  $z_3$  increase with increasing rotation. Figure 4 displays the variations of the waves velocities under magnetic field with varies values of initial stress, waves velocities increase with increasing magnetic field.



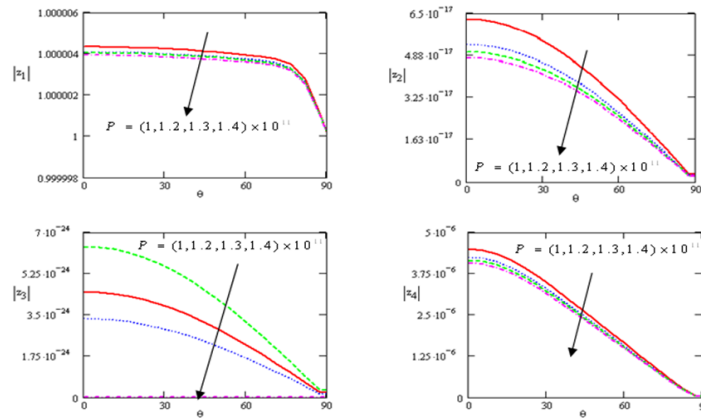
**Figure 4** Variations of the waves velocities under magnetic field with varies values of initial stress



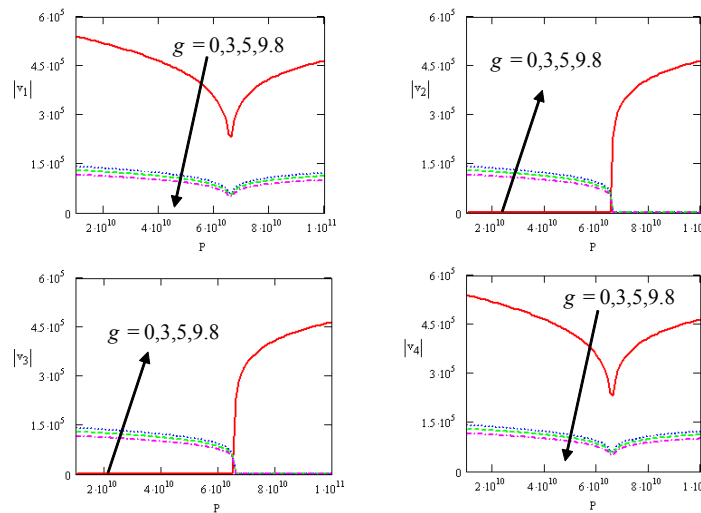
**Figure 5** Variations of the waves amplitudes under magnetic field with varies values of angle of incidence

Figure 5 displays the variations of the waves amplitudes under magnetic field with varies values of angle of incidence, amplitudes  $z_1, z_2, z_4$  decrease with increasing magnetic field but amplitudes  $z_3$  increase with increasing magnetic field. Figure 6 displays the variations of the waves amplitudes under initial stress with varies

values of angle of incidence, the waves amplitudes increase with increasing initial stress.

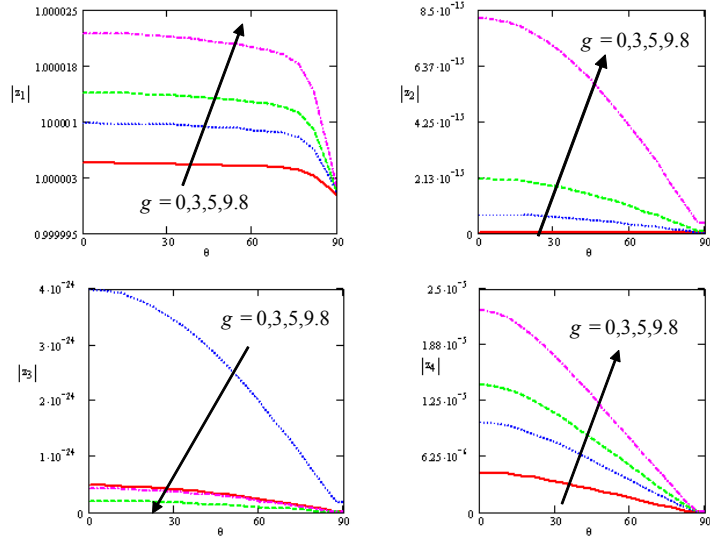


**Figure 6** Variations of the waves amplitudes under initial stress with varies values of angle of incidence

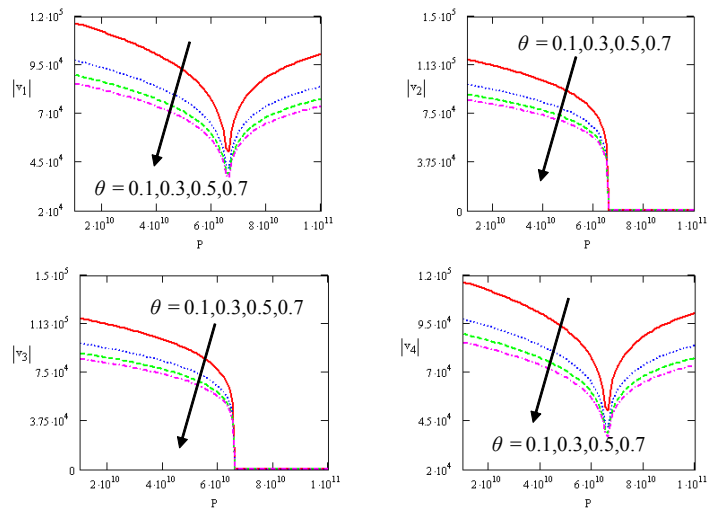


**Figure 7** Variations of the waves velocities under gravity with varies values of initial stress

Figure 7 displays the variations of the waves velocities under gravity with varies values of initial stress, the waves velocities  $v_1$ ,  $v_4$  decrease with increasing gravity but the waves velocities  $v_2$ ,  $v_3$  increase with increasing gravity.



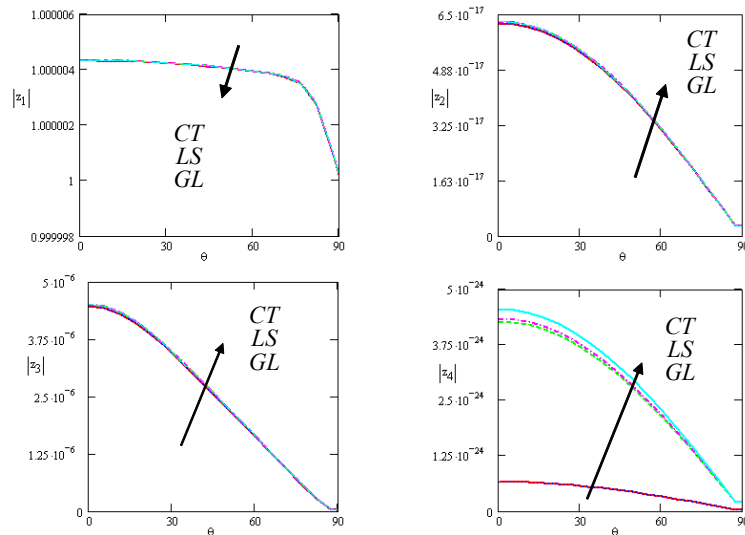
**Figure 8** Variations of the waves amplitudes under gravity with varies values of angle of incidence



**Figure 9** Variations of the waves velocities under angle of incidence with varies values of initial stress

Figure 8 displays the variations of the waves amplitudes under gravity with varies values of angle of incidence, the waves amplitudes  $z_1, z_4$  increase with increasing gravity but  $z_2, z_3$  decrease with increasing gravity.

Figure 9 displays the variations of the waves velocities angle of incidence with varies values of initial stress, the waves velocities decrease with increasing angle of incidence. Figure 10 displays the variations of the waves amplitudes under (CT, LS, GL) models with varies values of angle of incidence, the waves amplitudes  $z_1$  are take the same values (slight change) at different values of three models but the waves amplitudes  $z_1$  and  $z_2$  there are change to three models, the three models take this arrangement  $CT > LS > GL$ , the waves amplitudes  $z_4$  displays the effect is more pronounced.



**Figure 10** Variations of the waves amplitudes under (CT, LS, GL) models with varies values of angle of incidence

### 5. Conclusion

The reflected waves velocity with initial stress and amplitude of reflected wave with the angle of incidence are obtained in the framework of dynamical coupling theory, LS theory, GL theory, the effects of applied magnetic field, initial stress, gravity, rotation and thermal relaxation times are discussed numerically and illustrated graphically.

The following conclusions can be made:

1. The initial stress, thermal relaxation times, rotation, and magnetic field play a significant role of reflected wave.

2. The gravity field has a strong effect on the reflected waves velocity and amplitude of reflected wave.
3. The reflected waves velocity and amplitude of reflected wave depend on the angle of incidence and initial stress.
4. thermal relaxation times are affected strongly on the wave amplitudes  $z_4$  for the incident  $P$  wave.

It is observed that the reflected waves velocity and amplitude of reflected wave change their values in the presence of initial stress, thermal relaxation times, rotation, magnetic field and gravity field.

Finally, we recommend applying the research results in diverse fields because of their utilitarian aspects in Engineering, Geophysics, Geology, Earthquakes, Volcanoes, Structures and other fields.

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**Nomenclature**

$\vec{B}$  – is the magnetic induction vector,  
 $C_e$  – is the specific heat per unit mass,  
 $\vec{E}$  – is the electric intensity vector,  
 $e_{ij}$  – are the components of strain tensor,  
 $\vec{F}$  – is the Loren's body forces vector,  
 $g$  – is the gravity field,  
 $g^*$  – is the intrinsic equilibrated body forces,  
 $\vec{h}$  – is the perturbed magnetic field vector,  
 $\vec{H}$  – is the magnetic field vector,  
 $\vec{J}$  – is the electric current density vector,  
 $k$  – is the wave number,  
 $K$  – is the thermal conductivity,  
 $m$  – is the thermo-void coefficient,  
 $P$  is the initial stress,  
 $S_i$  – are the components of the equilibrated stress vector,  
 $t$  – is the time,  
 $T_0$  – is the natural temperature of the medium,  
 $u_i$  – are the components of the displacement vector,  
 $\alpha, b, \xi$  – are the void material parameters,  
 $\alpha_t$  – is the coefficient of linear thermal expansion,

$$\beta = \alpha_t(3\lambda + 2\mu),$$

$\delta_{ij}$  is the Kronecker delta,  
 $\varepsilon_0$  – is the magnetic permeability,  
 $\eta$  – is the entropy per unit mass,  
 $\mu_e$  – is the magnetic permeability,  
 $\rho$  – is the density,  
 $\sigma_{ij}$  – are the components of the stress vector,  
 $\tau_1, \tau_2$  – the thermal relaxation times ,  
 $\tau_{ij}$  – is the Maxwell's stress tensor,  
 $v$  – is the phase speed,  
 $\chi$  – is the equilibrated inertia,  
 $\omega$  – is the frequency,

$$\Theta = T - T_0, \quad \left| \frac{T - T_0}{T_0} \right| \ll 1,$$

$\Phi$  – is the change in volume fraction field,  
 $\Omega$  – is the angular velocity.