

## Vibration of Skew Plate with Circular Variation in Thickness and Poisson's Ratio

Amit SHARMA  
*Amity University Haryana  
Gurgaon, India  
dba.amitsharma@gmail.com*

Received (1 August 2017)

Revised (19 August 2017)

Accepted (12 September 2017)

The present study analyzes the natural vibration of non homogeneous visco elastic skew plate (parallelogram plate) with non uniform thickness under temperature field. Here non homogeneity in the plate's material arises due to circular variation in Poisson's ratio. Also the circular variation in thickness causes non uniformity in the shape of the plate. Bi linear temperature variation on the plate along both the axes is being viewed. The equation of motion related to frequency modes are solved by Rayleigh Ritz method. The findings of the present analysis are presented with the help of tables.

*Keywords:* circular variation, Poisson's ratio, skew plate, temperature field.

### 1. Introduction

Vibration is the periodicity of oscillation through an equilibrium point. The natural frequency is the frequency at which system tends to oscillate when there is no external force. The vibration phenomena is undesirable because its waste energy and efficiency of the system. Therefore, scientists and researchers are keen to study the vibration and how to optimize (minimize) the vibration. A good and quite significant work has been provided.

Leissa [1] studied plate vibration using classical plate theory. Timoshenko and Goodier [2] provided theory of elasticity (engineering problems) in his excellent book. Yamada and Irie [3] provided a review of large number of papers (approximately 200) on the vibration of plate. Mackerle [4] studied vibration analysis of beams, plates and shells using finite element method. Parametric study on thick plate vibration using FSDT has been studied by Kalita and Haldar [5]. Gupta and Khanna [6] discussed the natural vibration of visco-elastic rectangular plate with linearly varying thickness variation in both directions. Sharma et. al [7, 8, 9] provided analysis of natural vibration of non homogeneous rectangular plate (isotropic

and orthotropic) and square plate under thermal gradient. The theoretical study of vibration of non homogeneous skew plate (parallelogram plate) with thickness variation (exponential and sinusoidal variation) under temperature field has been given by Khanna and Kaur [10, 11]. Ansari [12] studied forced response of polar orthotropic tapered circular plates resting on elastic foundation. Tariverdilo et. al [13] discussed asymmetric free vibration of circular plate in contact with incompressible fluid. Zhou and Zheng [14] applied moving least square ritz method (numerical method) to study vibration of skew plates. The effect of structural parameter on vibration of non homogeneous visco elastic rectangular plate has been studied by Khanna and Kaur [15]. Gupta and Sharma [16] studied free vibration of non-homogeneous trapezoidal plates with variable thickness under thermal gradient.

The main aim of the present analysis is to optimize the vibration frequency by taking appropriate variation in plate's parameter. Here we take circular variation (new aspect) in thickness and in Poisson's ratio. Here we also calculate the effect of other plate's parameters to vibrational frequency. The results of the present paper are also compared with published results of [10] and [11].

## 2. Differential equation of motion

The differential equation of motion and time function for visco elastic plate with thickness variation is given by [6]:

$$\left[ \begin{array}{l} D_1 (\Phi_{,xxxx} + 2\Phi_{,xxxy} + \Phi_{,yyyy}) + 2D_{1,x} (\Phi_{,xxx} + \Phi_{,xyy}) \\ + 2D_{1,y} (\Phi_{,yyy} + \Phi_{,yxx}) + D_{1,xx} (\Phi_{,xx} + \nu \Phi_{,yy}) \\ + D_{1,yy} (\Phi_{,yy} + \nu \Phi_{,xx}) + 2(1-\nu) D_{1,xy} \Phi_{,xy} \end{array} \right] - \rho k^2 g \Phi = 0 \quad (1)$$

$$\ddot{T} + k^2 \tilde{D}T = 0 \quad (2)$$

Here comma followed by suffix is known as partial derivative of  $\Phi$  with respect to independent variable and double dot represent the second derivative with respect to  $t$ . Also  $D_1 = \frac{Yg^3}{12(1-\nu^2)}$  is called flexural rigidity of the plate.

Now the expression for kinetic energy  $T_s$  and strain energy  $V_s$  is given by [7]:

$$T_s = \frac{1}{2} k^2 \rho \int \int g \Phi^2 dy dx \quad (3)$$

$$V_s = \frac{1}{2} \int \int D_1 \left\{ (\Phi_{,xx})^2 + (\Phi_{,yy})^2 + 2\nu \Phi_{,xx} \Phi_{,yy} + 2(1-\nu) (\Phi_{,xy})^2 \right\} dy dx \quad (4)$$

A non uniform and non homogeneous visco elastic parallelogram plate having skew angle  $\theta$  is shown in Fig. 1.

The skew coordinates of the plates are:

$$x = \zeta + \psi \sin \theta, \quad y = \psi \cos \theta \quad (5)$$

The boundary conditions of the plate in skew coordinates are:

$$\zeta = 0 \quad \zeta = a \quad \text{and} \quad \psi = 0 \quad \psi = b \quad (6)$$

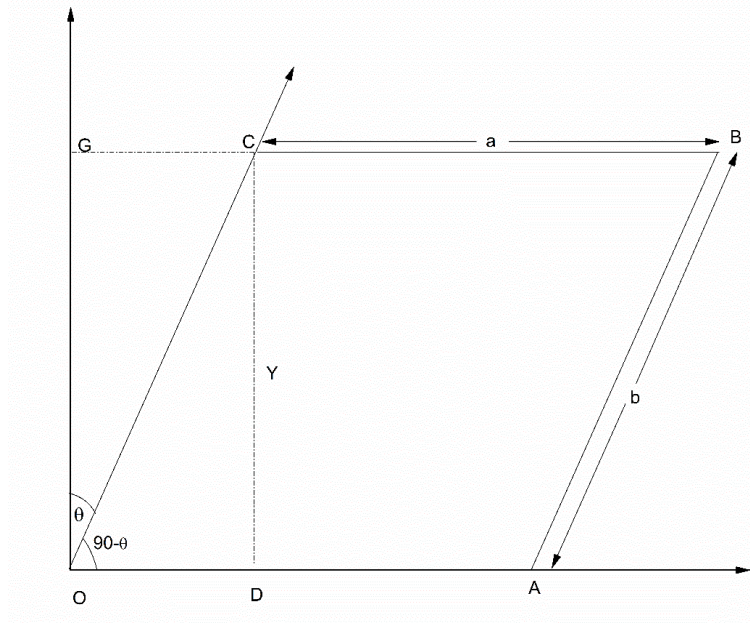


Figure 1 Parallelogram plate with skew angle  $\theta$

Using (5), (3) and (4) becomes:

$$T_s = \frac{1}{2} k^2 \rho \cos \theta \int_0^b \int_0^a g \Phi^2 d\zeta d\psi \quad (7)$$

$$V_s = \frac{1}{2 \cos^3 \theta} \int_0^b \int_0^a D_1 \left[ \begin{array}{l} (\Phi_{,\zeta\zeta})^2 - 4 \sin \theta (\Phi_{,\zeta\zeta}) (\Phi_{,\zeta\psi}) \\ + 2 (\sin^2 \theta + \nu \cos^2 \theta) (\Phi_{,\zeta\zeta}) (\Phi_{,\psi\psi}) \\ + 2 (1 + \sin^2 \theta - \nu \cos^2 \theta) (\Phi_{,\zeta\psi})^2 \\ - 4 \sin \theta (\Phi_{,\zeta\psi}) (\Phi_{,\psi\psi}) + (\Phi_{,\psi\psi})^2 \end{array} \right] d\zeta d\psi \quad (8)$$

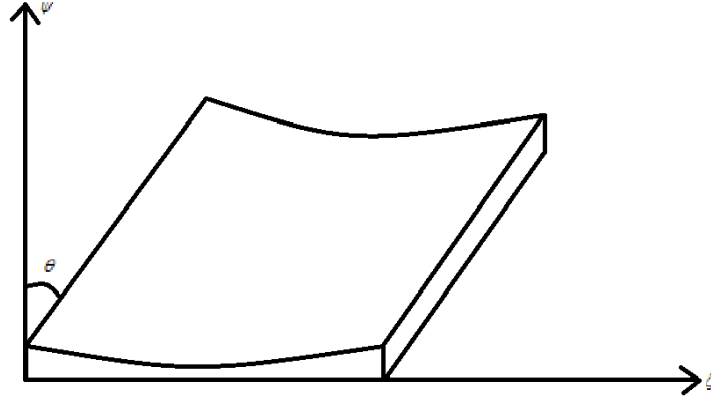
### 2.1. Assumptions

Due to the wide range and general scope of vibrations, we require little limitations in the form of assumptions in this present study.

1. The thickness of the plate is assumed to be circular in one dimension as shown in Fig. 2.

$$g = g_0 \left[ 1 + \beta \left( 1 - \sqrt{1 - \frac{\zeta^2}{a^2}} \right) \right] \quad (9)$$

where  $\beta$ , ( $0 \leq \beta \leq 1$ ) is known as tapering parameter. Thickness of the plate become constant i.e.,  $g = g_0$  at  $\zeta = 0$



**Figure 2** Parallelogram plate with circular variation in  $\zeta$  direction

2. We consider plate's material to be non homogeneous. Therefore, either density or Poisson's ratio varies. Here we keep density of the plate is constant and Poisson's ratio varies circularly in one dimension as

$$\nu = \nu_0 \left[ 1 - m \left( 1 - \sqrt{1 - \frac{\zeta^2}{a^2}} \right) \right] \quad (10)$$

where  $m$ , ( $0 \leq m \leq 1$ ) is known non homogeneity constant. Poisson's ratio become constant i.e.,  $\nu = \nu_0$  at  $\zeta = 0$ .

3. The temperature variation on the plate is considered to be bi linear i.e., linear in  $\zeta$  direction and linear in  $\psi$  direction as:

$$\tau = \tau_0 \left( 1 - \frac{\zeta}{a} \right) \left( 1 - \frac{\psi}{b} \right) \quad (11)$$

where  $\tau$  and  $\tau_0$  denotes the temperature excess above the reference temperature on the plate at any point and at the origin respectively. The temperature dependence modulus of elasticity for engineering structures is given by:

$$Y = Y_0 (1 - \gamma\tau) \quad (12)$$

where  $Y_0$  is the Young's modulus at mentioned temperature (i.e.,  $\tau = 0$ ) and  $\gamma$  is called slope of variation.

Using (11), (12) becomes:

$$Y = Y_0 \left[ 1 - \alpha \left( 1 - \frac{\zeta}{a} \right) \left( 1 - \frac{\psi}{b} \right) \right] \quad (13)$$

where  $\alpha$ , ( $0 \leq \alpha < 1$ ) is called temperature gradient, which is the product of temperature at origin and slope of variation i.e.,  $\alpha = \gamma\tau_0$ .

Using (9), (10) and (13), flexural rigidity of the plate become:

$$D_1 = \frac{Y_0 g_0^3 \left[ 1 - \alpha \left( 1 - \frac{\zeta}{a} \right) \left( 1 - \frac{\psi}{b} \right) \right] \left[ 1 + \beta \left( 1 - \sqrt{1 - \frac{\zeta^2}{a^2}} \right) \right]^3}{12 \left( 1 - \nu_0^2 \left[ 1 - m \left( 1 - \sqrt{1 - \frac{\zeta^2}{a^2}} \right) \right]^2 \right)} \quad (14)$$

Using (9), (10) and (14), (7) and (8) becomes:

$$T_s = \frac{1}{2} k^2 \rho g_0 \int_0^b \int_0^a (1 + \beta \Lambda) \Phi^2 d\zeta d\psi \quad (15)$$

$$V_s = \frac{Y_0 g_0^3}{24 \cos^4 \theta} \int_0^b \int_0^a \left\{ \begin{array}{l} \frac{[1 - \alpha(1 - \frac{\zeta}{a})(1 - \frac{\psi}{b})](1 + \beta \Lambda)^3}{(1 - \nu_0^2(1 - m \Lambda)^2)} \\ \left[ \begin{array}{l} (\Phi_{,\zeta\zeta})^2 - 4 \left( \frac{a}{b} \right) \sin \theta (\Phi_{,\zeta\zeta})(\Phi_{,\zeta\psi}) \\ + 2 \left( \frac{a}{b} \right) (\sin^2 \theta) \\ + \nu_0 t (1 - m \Lambda) \cos^2 \theta t (\Phi_{,\zeta\zeta})(\Phi_{,\psi\psi}) \\ + 2 \left( \frac{a}{b} \right)^2 (1 + \sin^2 \theta) \\ - \nu_0 (1 - m \Lambda) \cos^2 \theta (\Phi_{,\zeta\psi})^2 \\ - 4 \left( \frac{a}{b} \right)^3 \sin \theta (\Phi_{,\zeta\psi})(\Phi_{,\psi\psi}) \\ + \left( \frac{a}{b} \right)^4 (\Phi_{,\psi\psi})^2 \end{array} \right] \end{array} \right\} d\zeta d\psi \quad (16)$$

where:

$$\Lambda = \left( 1 - \sqrt{1 - \frac{\zeta^2}{a^2}} \right)$$

4. In the present scenario, we are computing frequency modes on clamped (along all the four edges) boundary condition, therefore we have:

$$\begin{aligned} \Phi = \Phi_{,\zeta} = 0 \text{ at } \zeta = 0, a \\ \Phi = \Phi_{,\psi} = 0 \text{ at } \psi = 0, b \end{aligned} \quad (17)$$

Therefore, two term deflection (i.e., maximum displacement) which satisfy (17) could be:

$$\Phi(\zeta, \psi) = \left[ \begin{array}{l} \Omega_1 \left( \frac{\zeta}{a} \right)^2 \left( \frac{\psi}{b} \right)^2 \left( 1 - \frac{\zeta}{a} \right)^2 \left( 1 - \frac{\psi}{b} \right)^2 \\ + \Omega_2 \left( \frac{\zeta}{a} \right)^3 \left( \frac{\psi}{b} \right)^3 \left( 1 - \frac{\zeta}{a} \right)^3 \left( 1 - \frac{\psi}{b} \right)^3 \end{array} \right] \quad (18)$$

where  $\Omega_1$  and  $\Omega_2$  are arbitrary constants.

### 3. Solution for frequency equation and frequency modes

We are using Rayleigh Ritz method (i.e., maximum kinetic energy  $T_s$  must equal to maximum strain energy  $V_s$ ) to solve frequency equation and frequency modes. Therefore, we have:

$$\delta(V_s - T_s) = 0 \quad (19)$$

Using (15) and (16) in (19), we get:

$$\delta (V_s^* - \lambda^2 T_s^*) = 0 \quad (20)$$

where:

$$T_s^* = \int_0^b \int_0^a (1 + \beta \Lambda) \Phi^2 d\zeta d\psi \quad (21)$$

$$V_s^* = \frac{1}{\cos^4 \theta} \int_0^b \int_0^a \left\{ \begin{array}{l} \frac{[1 - \alpha(1 - \frac{\zeta}{a})(1 - \frac{\psi}{b})](1 + \beta \Lambda)^3}{(1 - \nu_0^2(1 - m \Lambda)^2)} \\ \left[ \begin{array}{l} (\Phi_{,\zeta\zeta})^2 - 4(\frac{a}{b}) \sin \theta (\Phi_{,\zeta\zeta})(\Phi_{,\zeta\psi}) \\ + 2(\frac{a}{b})(\sin^2 \theta \\ + \nu_0 t(1 - m \Lambda) \cos^2 \theta) t(\Phi_{,\zeta\zeta})(\Phi_{,\psi\psi}) \\ + 2(\frac{a}{b})^2 (1 + \sin^2 \theta \\ - \nu_0(1 - m \Lambda) \cos^2 \theta)(\Phi_{,\zeta\psi})^2 \\ - 4(\frac{a}{b})^3 \sin \theta (\Phi_{,\zeta\psi})(\Phi_{,\psi\psi}) \\ + (\frac{a}{b})^4 (\Phi_{,\psi\psi})^2 \end{array} \right] \end{array} \right\} d\zeta d\psi \quad (22)$$

and  $\lambda^2 = 12\rho k^2 a^4 / Y_0 g_0^2$  is known as frequency parameter.

(20) consists of two unknowns constants because of substitution of (18). These constants can be determined as follows:

$$\frac{\partial}{\partial \Omega_n} (V_s^* - \lambda^2 T_s^*) \quad n = 1, 2 \quad (23)$$

After simplifying (23), we get homogeneous system of equation:

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (24)$$

where  $d_{11}, d_{12} = d_{21}$  and  $d_{22}$  involve parametric constant and frequency parameter. To get non trivial solution the determinant of the coefficient matrix of (24) must be zero. Therefore, we have

$$\begin{vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{vmatrix} = 0, \quad (25)$$

From (25), we get a quadratic equation (i.e., frequency equation) from which we get two roots (i.e., frequency modes)  $\lambda_1$  (first mode) and  $\lambda_2$  (second mode).

#### 4. Results and discussion

To analyze the behavior of frequency modes, the first two modes of vibration of non homogeneous parallelogram plate corresponding to different values of plate's parameters (i.e., taper constant  $\beta$ , non homogeneity constant  $m$ , thermal gradient  $\alpha$  and skew angle  $\theta$ ) have been computed. The value of  $\nu_0$  is taken 0.345. All the results are displayed with the help of tables.

Table 1 provides frequency modes corresponding to thickness variation in the plate for fixed value of skew angle  $\theta = 30^\circ$  and aspect ratio  $a/b = 1.5$  and for three different values of non homogeneity constant  $m = 0, 0.4, 0.8$  and thermal gradient  $\alpha = 0, 0.4, 0.8$ . From table 1, one can easily get that frequency mode increases for all the three values of non homogeneity constant and thermal gradient, when tapering parameter increases from 0 to 1. The rate of increment is less due to circular

variation in thickness. When the combined value of non homogeneity constant  $m$  and thermal gradient  $\alpha$  on the plate varies from 0 to 0.8, frequency mode decreases.

Table 2 displays vibrational frequency corresponding to non homogeneity in the plate's material for fixed value of skew angle  $\theta = 30^0$  and aspect ratio  $a/b = 1.5$  and for three different values of tapering parameter  $\beta = 0, 0.4, 0.8$  and thermal gradient  $\alpha = 0, 0.4, 0.8$ . From Tab. 2, we conclude that when non homogeneity in the plate's material increases from 0 to 1, vibrational frequency decreases for all the above said three values. The rate of decrement is very less due to circular variation in density. When combined values of tapering parameter  $\beta$  and thermal gradient  $\alpha$  on the plate increases from 0 to 0.8, frequency mode increases.

**Table 1** Thickness (tapering parameter  $\beta$ ) variation in plate vs vibrational frequency ( $\lambda$ ) for  $\theta = 30^0$  and  $a/b = 1.5$

$\beta$	$m = \alpha = 0$		$m = \alpha = 0.4$		$m = \alpha = 0.8$	
	$\lambda_1$	$\lambda_2$	$\lambda_1$	$\lambda_2$	$\lambda_1$	$\lambda_2$
0.0	78.77	313.60	74.25	295.65	69.62	277.26
0.2	81.38	322.95	76.78	304.64	72.08	285.92
0.4	84.11	322.70	79.42	314.00	74.65	294.96
0.6	86.95	342.82	82.16	323.73	77.31	304.33
0.8	89.88	353.28	85.00	333.78	80.07	314.03
1.0	92.90	364.07	87.93	344.15	82.91	324.02

**Table 2** Non homogeneity ( $m$ ) variation in plate's material vs vibrational frequency ( $\lambda$ ) for  $\theta = 30^0$  and  $a/b = 1.5$

$m$	$\beta = \alpha = 0.0$		$\beta = \alpha = 0.4$		$\beta = \alpha = 0.8$	
	$\lambda_1$	$\lambda_2$	$\lambda_1$	$\lambda_2$	$\lambda_1$	$\lambda_2$
0.0	78.77	313.60	80.06	316.59	81.33	319.27
0.2	78.47	312.39	79.73	315.25	80.97	317.80
0.4	78.19	311.25	79.42	314.00	80.64	316.44
0.6	77.92	310.18	79.14	312.84	80.34	315.19
0.8	77.67	309.17	78.47	311.76	80.07	314.03
1.0	77.44	308.23	78.63	310.75	79.82	312.96

Table 3 contains the frequency modes corresponding to temperature variation on the plate for fixed value of skew angle  $\theta = 30^0$  and aspect ratio  $a/b = 1.5$  and for three different values of non homogeneity constant  $m = 0, 0.4, 0.8$  and thickness parameter  $\beta = 0, 0.4, 0.8$ . From Tab. 3, we enlighten the fact that frequency mode decreases when temperature gradient on the plate increases from 0 to 0.8 for all the three mentioned values. The frequency mode increases when the combined value of non homogeneity constant  $m$  and tapering parameter  $\beta$  on the plate varies from 0 to 0.8.

**Table 3** Temperature ( $\alpha$ ) variation on plate *vs* vibrational frequency ( $\lambda$ ) for  $\theta = 30^0$  and  $a/b = 1.5$ 

$\alpha$	$m = \beta = 0$		$m = \beta = 0.4$		$m = \beta = 0.8$	
	$\lambda_1$	$\lambda_2$	$\lambda_1$	$\lambda_2$	$\lambda_1$	$\lambda_2$
0.0	78.77	313.60	83.45	330.03	88.52	347.67
0.2	76.82	305.86	81.46	322.12	86.48	339.57
0.4	74.82	297.92	79.42	314.00	84.40	331.28
0.6	72.76	289.76	77.32	305.68	82.26	322.77
0.8	70.64	281.36	75.17	297.11	80.07	314.03

**Table 4** Skew angle ( $\theta$ ) variation *vs* vibrational frequency ( $\lambda$ ) for  $\alpha = \beta = 0.4$  and  $a/b = 1.5$ 

$\theta$	$m = 0$		$m = 0.4$		$m = 0.8$	
	$\lambda_1$	$\lambda_2$	$\lambda_1$	$\lambda_2$	$\lambda_1$	$\lambda_2$
0	58.48	233.45	58.04	231.56	57.66	229.92
15	63.15	251.42	62.66	249.39	62.25	247.62
30	80.06	316.59	79.42	314.00	78.87	311.76
45	123.03	482.47	122.02	478.49	121.15	475.03
60	251.79	979.83	249.63	971.65	247.77	964.55
75	954.86	3696.81	946.47	3665.73	939.28	3638.74

**Table 5** Comparison of frequency modes of present paper with [10] and [11] corresponding to taper constant for  $\theta = 30^0$  and  $a/b = 1.5$ 

$\beta$	$m = \alpha = 0$		$m = 0, \alpha = 0.2$		$m = 0.2, \alpha = 0$	
	$\lambda_1$	$\lambda_2$	$\lambda_1$	$\lambda_2$	$\lambda_1$	$\lambda_2$
0.0	78.77	313.60	76.82	305.86	78.47	312.39
from [10]	{84.61}	{329.83}	{82.53}	{321.78}	{89.18}	{347.67}
from [11]	{84.61}	{329.83}	{82.53}	{321.78}	{89.18}	{347.67}
0.2	81.38	322.95	79.41	315.11	81.06	321.66
from [10]	{93.86}	{365.64}	{91.66}	{357.06}	{98.99}	{385.82}
from [11]	{96.01}	{375.28}	{93.75}	{366.43}	{101.25}	{395.95}
0.4	84.11	322.70	82.11	324.74	83.77	331.32
from [10]	{104.90}	{407.83}	{102.56}	{398.62}	{110.68}	{430.79}
from [11]	{107.77}	{421.95}	{105.31}	{412.26}	{113.69}	{445.54}
0.6	86.95	342.82	84.92	334.75	86.58	341.35
from [10]	{118.07}	{457.66}	{115.58}	{447.81}	{124.65}	{483.92}
from [11]	{119.76}	{469.45}	{117.10}	{458.89}	{126.39}	{496.00}
0.8	89.88	353.28	87.82	345.09	89.50	351.72
from [10]	{133.79}	{516.63}	{131.11}	{506.04}	{141.31}	{546.84}
from [11]	{131.91}	{517.55}	{129.06}	{506.10}	{139.26}	{547.10}
1.0	92.90	364.07	90.81	355.76	92.50	362.41
from [10]	{152.53}	{586.57}	{149.62}	{575.16}	{161.18}	{621.52}
from [11]	{144.22}	{566.09}	{141.13}	{553.72}	{152.27}	{598.66}



When we move to Tab. 4, it gives frequency modes corresponding to skew angle for fixed value of aspect ratio  $a/b = 1.5$ , thermal gradient and tapering parameter i.e.,  $\alpha = \beta = 0.4$  and variable value of non homogeneity constant i.e.,  $m = 0, 0.4, 0.8$ . From Tab. 4, we can get that frequency mode increases rapidly (there is sharp increment after  $\theta = 45^\circ$ ) for all the three variable values of non homogeneity. When non homogeneity in the plate's material increases from 0 to 0.8, frequency decreases with less rate of decrement (because of circular variation).

## 5. Comparison of result

The results of the present paper is compared with [10] and [11] with the help of table. Table 5 gives the comparison of frequency modes with [10] and [11] corresponding to tapering parameter for three different values of non homogeneity constant and thermal gradient i.e.,  $m = \alpha = 0$ ,  $m = 0, \alpha = 0.2$  and  $m = 0.2, \alpha = 0$ . From Tab. 5, author conclude that frequency for both modes is less when compared to [10] and [11] for all the three above mentioned values of non homogeneity constant and thermal gradient. The frequency variation is also less when compared to [10] and [11].

## 6. Conclusion

From the above result discussion and comparison, author concludes that the frequency modes behave according to plate's parameter variation. The frequency modes and variation in frequency modes in case of circular variation (present paper) is less when compared to exponential variation (as in [10]) and sinusoidal variation (as in [11]) as shown in table 5. The variation in frequency modes is very less due to circular variation in Poisson's ratio as non homogeneity effect when compared to linear, parabolic and exponential variation in density or Poisson's ratio. Therefore, frequency can be optimize by taking appropriate variation in plate's parameters.

## References

- [1] **Leissa A. W.:** Plate vibration research, 1976 – 1980: classical theory, *Shock and Vibration Inform., Center the Shock and Vibration Digest*, 13, 9, 11–22, **1980**.
- [2] **Timoshenko, S. P. and Goodier, J. N.:** Theory of elasticity, *International Journal of Bulk Solids Storage in Silos*, 1.4, **2014**.
- [3] **Yamada, G. and Irie, T.:** Plate vibration research in Japan, *Applied Mechanics Reviews*, 40, 7, 879–892, **1987**.
- [4] **Mackerle, J.:** Finite element vibration analysis of beams, plates and shells, *Shock and Vibration*, 6(2), 97–109, **1999**.
- [5] **Kalita, K. and Haldar, S.:** Parametric study on thick plate vibration using FSDT, *Mechanics and Mechanical Engineering*, 19, 2, 81–90, **2015**.
- [6] **Gupta, A. K and Khanna, A.:** Vibration of visco-elastic rectangular plate with linearly varying thickness variation in both directions, *Journal of Sound and Vibration*, 301, 3–5, 450–457, **2007**.
- [7] **Sharma, A., Raghav, A. K., Sharma, A. K. and Kumar, V.:** A modeling on frequency of rectangular plate, *International Journal of Control Theory of Applications*, 9, 277–282, **2016**.

- [8] **Sharma, A., Sharma, A. K., Raghav, A. K. and Kumar, V.:** Effect of vibration on orthotropic visco-elastic rectangular plate with two dimensional temperature and thickness variation, *Indian Journal of Science and Technology*, 9, 2, pp. 7, **2016**.
- [9] **Sharma, A., Sharma, A. K., Raghav, A. K. and Kumar, V.:** Vibrational study of square plate with thermal effect and circular variation in density, *Romanian Journal of Acoustics and Vibration*, XIII, 146–152, **2016**.
- [10] **Khanna, A. and Arora, P.:** Effect of sinusoidal thickness variation on vibrations of non homogeneous parallelogram plate with bi-linearly temperature variations, *Indian Journal of Science and Technology*, 6(9), 5228–5234, **2013**.
- [11] **Khanna, A. and Arora, P.:** Theoretical study on skew plate under thermal gradient, *Proceedings of the Third International Conference on Soft Computing for Problem Solving. Advances in Intelligent Systems and Computing, Springer*, 259, 173–182, **2014**.
- [12] **Ansari, A. H.:** Forced response of polar orthotropic tapered circular plates resting on elastic foundation, *Advances in Acoustics and Vibration*, ID 1492051, 10 pages, **2016**.
- [13] **Tariverdilo, S., Shahmardani, M., Mirzapour, J. and Shabani, R.:** Asymmetric free vibration of circular plate in contact with incompressible fluid, *Applied Mathematical Modeling*, 37, 228–239, **2013**.
- [14] **Zhou, L. and Zheng, W. X.:** Vibration of skew plates by the MLS-Ritz method, *International. Journal of Mechanical Sciences*, 50, 1133–1141, **2008**.
- [15] **Khanna, A. and Kaur, N.:** Effect of structural parameter on vibration of non homogeneous visco elastic rectangular plate, *Journal of Vibration Engineering and Technology*, 4, 459–466, **2016**.
- [16] **Gupta, A. K. and Shama, P.:** Vibration study of non-homogeneous trapezoidal plates of variable thickness under thermal gradient, *Journal of Vibration and Control*, 22, 5, 1369–1379, **2016**.

### Nomenclature

$a$  – Length of the plate

$b$  – Breadth of the plate

$x, y$  – Coordinates in the plane of plate

$\zeta, \psi$  – Skew coordinates of the plate

$Y$  – Young's modulus

$\nu$  – Poisson's ratio

$\tilde{D}$  – Visco elastic operator

$D_1$  – Flexural rigidity

$\rho$  – Mass density per unit volume of the plate material

$t$  – Time

$\phi(x, y, t)$  – Deflection of plate

$\Phi(x, y)$  – Deflection function

$T(t)$  – Time function

$g$  – Thickness of plate

$\beta$  – Tapering parameter

$m$  – Non homogeneity of the material

$\alpha$  – Temperature gradient

$k^2$  – Constant