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Ivestigation of the Dynamic Stress State of Foam Media in Cosserat Elasticity

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The paper presents studies on the application of the boundary integral equation method for investigation of dynamic stress state of foam media with tunnel cavities in Cosserat continuum. For the solution of the non-stationary problem, the Fourier transform for time variable was used. The potential representations of Fourier transform displacements and microrotations are written. The fundamental functions of displacements and microrotations for the two-dimensional case of Cosserat continuum are built. Thus, the fundamental functions of displacement for the time-domain problem are derived as the functions of the two-dimensional isotropic continuum and the functions, which are responsible for the effect of shear-rotation deformations. The method of mechanical quadrature is applied for numerical calculations. Numerical example shows the comparison of distribution of dynamic stresses in the foam medium with the cavity under the action of impulse load accounting for the shear-rotation deformations effect and without accounting for this effect.

Keywords: Cosserat elasticity, fundamental functions, time-domain problem, stresses concentration.

1. Introduction

Many modern studies are devoted to the creation of new granular and foam composite materials that are widely used. The main advantages of such materials include low thermal conductivity, low density (up to 50 kg/m^3), long service life, and resistance to aggressive environments.

The using of the classical theory of elasticity for studying of the stress state of such materials, especially under the dynamic loads, leads to significant differences between theoretical and experimental results. This is explained by the fact that the solid model, which forms the basis of the classical theory of elasticity, does not allow to show such properties of real bodies, which are determined by their discrete structure.

Accounting for such properties it is necessary to use other models, where the properties determined by the discrete structure would be clearly reflected. Investigation of the stress state for these models is carried out not only accounting for the force stresses, but also couple stresses.

The existence of couple stresses in materials was initially postulated by Voyght in 1887. However, in 1909 the brothers Cosserat were the first developers of the mathematical model for the analysis of materials with couple stresses. In the original Cosserat theory, the kinematic quantities were displacements and microrotations in the medium. Hypothetically thought that the microrotation could be independent of the mechanical rotation of the whole medium, called macrorotation.

Due to the complexity of the solution of equilibrium or motion equations in the Cosserat elasticity, analytical solutions were constructed only for some classes of problems. Pal'mov V.A. solved the problem of the stress concentration near a circular hole [2]. Sandru N. and Mindlin R.D. obtained a solution to the problem of the action of concentrated force and the concentrated moment in an infinite elastic space [3].

The wave processes in micropolar continua are investigated in the monographs of Eringen A.C. [4], Erofeev V.I. [5], Maugin G.A. [6] and Nowacki W. [7].

The model of Cosserat continuum is used to describe polycrystalline and composite materials, granular and powder-like materials, porous media and foams, and even bones.

Many works are devoted to the development of experimental methods for determining the elastic characteristics of such materials within the framework of the Cosserat continuum: material of human bone, foam and fibrous materials. Among such works should be noted a significant number of works Lakes R. S., among which [8-10] and other authors.

Therefore, in the paper the Cosserat continuum [1] is used to study the dynamic be-havior of micropolar continuum.

2. Constructive relations

According to [1, 4], the motion equations of Cosserat continuum are described as:

$$\sigma_{ji,j} + X_i = \rho \ddot{u}_i \tag{1}$$

$$\epsilon_{kji}\sigma_{ij} + \mu_{jk,j} + Y_k = J\ddot{\phi}_k \tag{2}$$

where σ_{ji} is the force stress, μ_{ji} is the couple stress, ρ is the material density, $\mathbf{X} = \{X_i\}$ is the mass forces vector, $\mathbf{Y} = \{Y_i\}$ is the couple forces vector, J is the inertia of unit volume rotation, ϵ_{klm} is the permutation symbol, $\boldsymbol{\phi} = \{\phi_i\}$ is the displacement vector, $\mathbf{X} = \{X_i\}$ is the rotation vector. Functions $\mathbf{u}$ and $\boldsymbol{\phi}$ are continuous functions.

Here and further the Einstein summation convention is used. A comma at subscript denotes differentiation with respect to a coordinate indexed after the comma, i.e. $u_{j,i} = \partial u_j/\partial x_i$.

Under the condition of plane strain indices vary from 1 to 2, and $k = 3$.

According to [1], the dependencies for determining force and couple stresses are written as:

$$\begin{aligned} \sigma_{ji} &= (\mu+\alpha)\gamma_{ji} + (\mu-\alpha)\gamma_{ij} + \lambda\gamma_{kk}\delta_{ij} \\ \mu_{ji} &= (\gamma+\varepsilon)\kappa_{ji} + (\gamma-\varepsilon)\kappa_{ij} + \beta\kappa_{kk}\delta_{ij} \end{aligned} \tag{3}$$

where $\alpha,\ \beta,\ \gamma,\ \kappa$ are the elastic constant required to describe an isotropic constrained Cosserat elastic solid, $\lambda,\ \mu$ are Lame parameters, $\gamma_{ij} = u_{i,j} - \epsilon kji\phi_k$ is the asymmetric deformation tensor, $\kappa_{ij} = \phi_{i,j}$ is the torsion bending tensor.

3. Boundary integral formulation

Let's consider a micropolar elastic medium with tunnel cavity of sufficiently small diameter, that a plane strain condition is satisfied (Fig. 1). We denote a configuration of micropolar elastic medium by Ω and the boundary of tunnel cavity with constant cross-section by L. The center of gravity is placed at the origin of Cartesian coordinate system $x_1Ox_2x_3$.

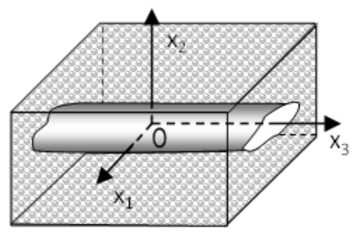

Figure 1 Model of the research object

The boundary conditions for the second exterior problem in Cosserat elasticity are written as:

$$\sigma_n|_L = -\sigma_0\varphi(t), \quad \tau_{sn}|_L = 0, \quad \mu_n|_L = 0 \tag{4}$$

where $\varphi(t)$ is the function that describes an impulse load, which is applied to the cavity's boundary in radial direction, **n** is the normal to the boundary of the cavity, σ_0 is the constant, which dependents on the intensity of the applied load. The function $\varphi(t)$ is given at the boundary L.

The impulse load over time is given as:

$$\varphi(\tilde{t}) = p_*\tilde{t}^{n_*}e^{-\alpha_*\tilde{t}},\ \tilde{t} > 0,\ n_* \geq 0 \tag{5}$$

where $\tilde{t} = t \cdot c_l/a$ is a dimensionless time parameter, $c_l = \sqrt{(\lambda+2\mu)/\rho}$ is the speed of expansion wave, $p_*,\ n_*,\ \alpha_*$ are the constants, a is some characteristic scale. For numerical calculations the value of characteristic scale is chosen as $a = a_1 \cdot 10^3 m$.

3.1. *Potential representation of displacements and microrotations*

Applying the Fourier transform to the motion equations (1) and (2), we obtained the equations, which are equivalent to the equations of time-harmonic motion with cyclic frequency ω:

$$\hat{\sigma}_{ji,j} + \hat{X}_i + \omega^2 \rho \hat{u}_i = 0 \tag{6}$$

$$\in_{ijk} \hat{\sigma}_{ij} + \hat{\mu}_{mk,m} + \hat{Y}_k + \omega^2 J \hat{\phi}_k = 0 \tag{7}$$

Here $\hat{\sigma}_{ji},\ \hat{\mu}_{mk},\ \hat{X}_i,\ \hat{Y}_k,\ \hat{u}_i,\ \hat{\phi}_k$ are Fourier transforms of force and couple stresses, mass and force forces, displacements and microrotations accordantly.

According to the boundary element method [11] the Fourier transform of the boundary conditions for the second exterior problem in Cosserat elasticity are written as:

$$\begin{array}{l} p_i|_L = \hat{\sigma}_{ji} n_j|_L = \overline{\overline{p}}_i \\ m_k|_L = \hat{\mu}_{jk} n_j|_L = \overline{\overline{m}}_k \end{array} \tag{8}$$

According to weighted residual approach [12] we are interested in minimizing equations (6) and (7). To this end one can weight equation (6) by displacement type function u_i^* and weight equation (7) by microrotation type function ϕ_k^* with accounting for the boundary conditions (8):

$$\int_\Omega \left(\hat{\sigma}_{ji,j} + \hat{X}_i + \omega^2 \rho \hat{u}_i \right) u_i^* d\Omega = \int_L (p_i - \overline{\overline{p}}_i) u_i^* dL \tag{9}$$

$$\int_\Omega \left(\in_{ijk} \hat{\sigma}_{ij} + \hat{\mu}_{mk,m} + \hat{Y}_k + \omega^2 J \hat{\varphi}_k \right) \phi_k^* d\Omega = \int_L (m_k - \overline{\overline{m}}_k) \phi_k^* dL \tag{10}$$

If we carry out integrated by parts on the first term of equation (9) and the second term of equation (10), and group the corresponding terms together, and accounting for the asymmetric deformation tensor, we can write the potential representation for transforms of displacements and microrotations:

$$\hat{u}_i = \int_L p_j \cdot U_{ij}^* \, dL + \int_L m_k \cdot \Phi_{kj}^* \, dL + \int_\Omega X_j \cdot U_{ij}^* \, d\Omega + \int_\Omega Y_k \cdot \Phi_{kj}^* \, d\Omega \tag{11}$$

$$\hat{\phi}_k = \int_L p_j \cdot U_{kj}^{**} \, dL + \int_L m_k \cdot \Phi_{kk}^{**} \, dL + \int_\Omega X_j \cdot U_{kj}^{**} \, d\Omega + \int_\Omega Y_k \cdot \Phi_{kk}^{**} \, d\Omega \tag{12}$$

where $U_{ij}^*,\ U_{kj}^{**},\ \Phi_{kj}^*,\ \Phi_{kk}^{**}$ are the fundamental functions for displacements and mictrorotations, $p_j,\ m_k$ are unknown potential functions.

3.2. *Fundamental functions for displacements and mictrorotations*

The fundamental functions $U_{ij}^*,\ U_{kj}^{**},\ \Phi_{kj}^*,\ \Phi_{kk}^{**}$ for Cosserat continuum are built regarding Sommerfield radiation condition. We use methods of potential theory [13] and collocation methods [12]. For the plane stain the motion equations (6) and (7) via displacement and microrotations are written as:

$$(\lambda + \mu)\partial_1 \hat{\theta} + \mu \Delta \hat{u}_1 - \alpha \partial_2 (\partial_1 \hat{u}_2 - \partial_2 \hat{u}_1) + 2\alpha \partial_2 \hat{\phi}_3 + \hat{X}_1 + \rho \omega^2 \hat{u}_1 = 0 \tag{13}$$

$$(\lambda + \mu)\partial_2 \hat{\theta} + \mu \Delta \hat{u}_2 + \alpha \partial_1 (\partial_1 \hat{u}_2 - \partial_2 \hat{u}_1) - 2\alpha \partial_1 \hat{\phi}_3 + \hat{X}_2 + \rho \omega^2 \hat{u}_2 = 0 \tag{14}$$

$$(\gamma+\varepsilon)\Delta\hat{\phi}_3+2\alpha(\partial_1\hat{u}_2-\partial_2\hat{u}_1)-4\alpha\hat{\phi}_3+\hat{Y}_3+J\omega^2\hat{\phi}_3=0 \tag{15}$$

We differentiate the equations (14) by x_1 and the equation (13) by x_2 and subtract them. Then we substitute the obtained equation in the equation (15). The equation for determining the microrotations in the Cosserat continuum is obtained in form:

$$\Delta\Delta\hat{\phi}_3-p\Delta\hat{\phi}_3-q\hat{\phi}_3=-\frac{1}{l_3^2}\left(\frac{-\partial_1\hat{X}_2+\partial_2\hat{X}_1}{\rho c_2^2}+\frac{(\Delta-\kappa_2^2)}{2\alpha}\hat{Y}_3\right) \tag{16}$$

where:

$l_3=\sqrt{(\gamma+\varepsilon)/2\mu}$ is the scale parameter in Cosserat elasticity,

$c_2=\sqrt{(\mu+\alpha)/\rho}$ is the speed of shear wave in Cosserat elasticity.

Applying the collocation method [12] to solving equation (16) and inserting the obtained solutions into the equations (13) – (15) we can write the fundamental functions for displacements and mictrorotations as:

$$\begin{gathered}U_{ij}^{*}=A_0K_0(\kappa_\tau r)\delta_{ij}+A_1\partial_i\partial_j\left(K_0(\kappa_l r)-K_0(\kappa_\tau r)\right)+\\ (\Delta\delta_{ij}-\partial_i\partial_j)\left(a_0K_0(\kappa_\tau r)+a_1K_0(\kappa_1 r)+a_2K_0(\kappa_2 r)\right)\end{gathered} \tag{17}$$

$$U_{kj}^{**}=(-1)^{j+1}\partial_{j+1}\left(\alpha_0K_0(\kappa_\tau r)+\alpha_1K_0(\kappa_1 r)+\alpha_2K_0(\kappa_2 r)\right) \tag{18}$$

$$\Phi_{kj}^{**}=A_0A_1(-1)^{j+1}\partial_{j+1}\left(K_0(\kappa_1 r)-K_0(\kappa_2 r)\right) \tag{19}$$

$$\Phi_{kk}^{**}=A_0A_2\left(b_3K_0(\kappa_1 r)-b_4K_0(\kappa_2 r)\right) \tag{20}$$

where $\kappa_l=I\omega/c_l$,$\kappa_\tau=I\omega/c_\tau$,$\kappa_1=I\omega/v_1$, $\kappa_2=\omega/v_2$ are the wave numbers, $c_\tau=\sqrt{\mu/\rho}$ is the speed of shear wave in classical elasticity, $v_1,\ v_2$ are the wave speeds in Cosserat elasticity, which are obtained as a solutions of characteristic equation of (16), $K_m(r)$ are Bessel functions of the third kinds, $r=\sqrt{(x_1-x_1^0)^2+(x_2-x_2^0)^2}$is the distance, I is imaginary number, $A_k,\ a_k,\ \alpha_k,\ b_m$ are the known constants, *k=1, 2, 3; m=3, 4.*

Performing the analysis of the obtained fundamental function (17) for the displacements one can write:

$$U_{ij}^{*}=U_{ij}^{*(class)}+U_{ij}^{*(coupl)}$$

where $U_{ij}^{*(class)}$ are the fundamental functions of the classical elasticity. It is selected for non-stationary loads as [14]:

$$U_{ij}^{*(class)}=\frac{1}{2\pi\mu}\left(K_0(\kappa_\tau r)\delta_{ij}+\frac{1}{\kappa_\tau^2}\partial_i\partial_j\left(K_0(\kappa_l r)-K_0(\kappa_\tau r)\right)\right)$$

According to formula (17), the expression for part of the fundamental functions in the Cosserat elasticity $U_{ij}^{*(coupl)}$, which accounting for the influence of shear rotation deformations, we write as:

$$U_{ij}^{*(coupl)}=(\Delta\delta_{ij}-\partial_i\partial_j)\left(a_0K_0(\kappa_\tau r)+a_1K_0(\kappa_1 r)+a_2K_0(\kappa_2 r)\right)$$

From the above it is clear that from the formulas (17) - (20) for zero values of some constant, one can obtain solutions for the classical elasticity as a partial case. This will enable to verify the reliability of the results, which are obtained on the basis of the developed method, with known results in the literature.

3.3. *Novel integral equations for the second exterior problem in Cosserat elasticity*

For the solving of the problem one must determine unknown functions $p_1,\ p_2,\ m_3$. The calculation of the force and couple stresses at an arbitrary point of the plate is performed by the formulas [15]:

$$\begin{aligned}&\hat{\sigma}_n=\frac{\hat{\sigma}_{11}+\hat{\sigma}_{22}}{2}+\frac{1}{4}\left(e^{-2I\alpha}\left(\hat{\sigma}_{11}-\hat{\sigma}_{22}+I(\hat{\tau}_{12}+\hat{\tau}_{21})\right)\right.\\&\left.+\,e^{2I\alpha}\left(\hat{\sigma}_{11}-\hat{\sigma}_{22}-I(\hat{\tau}_{12}+\hat{\tau}_{21})\right)\right)\\&\hat{\tau}_{sn}=\frac{\hat{\tau}_{12}-\hat{\tau}_{21}}{2}+\frac{i}{4}\left(e^{2I\alpha}\left(\hat{\sigma}_{11}-\hat{\sigma}_{22}-I(\hat{\tau}_{12}+\hat{\tau}_{21})\right)\right.\\&\left.-\,e^{-2I\alpha}\left(\hat{\sigma}_{11}-\hat{\sigma}_{22}+I(\hat{\tau}_{12}+\hat{\tau}_{21})\right)\right)\\&\hat{\mu}_n=\frac{1}{2}\left(\left(\hat{\mu}_{31}-I\hat{\mu}_{32}\right)e^{I\alpha}+\left(\hat{\mu}_{31}+I\hat{\mu}_{32}\right)e^{-I\alpha}\right)\end{aligned}\tag{21}$$

where α is the angle between the normal $\mathbf{n}$ to the boundary of the plate and the axis Ox_1.

For plane strain the components of force and couple stresses are defined by the formulas, which are the analogies of Hooke's law in Cosserat elasticity [1].

Inserting the potential representation of displacements (11) and microrotations (12) to the force and couple stress formulas (21) with accounting for (17) – (20) we obtain the integral dependencies for absent of mass and couple forces:

$$\begin{aligned}\hat{\sigma}_n&=\int_L\left(f_1\left(\mathbf{x},\mathbf{x}^0\right)p_1+f_2\left(\mathbf{x},\mathbf{x}^0\right)p_2+f_3\left(\mathbf{x},\mathbf{x}^0\right)m_3\right)dL(\mathbf{x}^0)\\\hat{\tau}_n&=\int_L\left(g_1\left(\mathbf{x},\mathbf{x}^0\right)p_1+g_2\left(\mathbf{x},\mathbf{x}^0\right)p_2+g_3\left(\mathbf{x},\mathbf{x}^0\right)m_3\right)dL(\mathbf{x}^0)\\\hat{\mu}_n&=\int_L\left(G_1\left(\mathbf{x},\mathbf{x}^0\right)p_1+G_2\left(\mathbf{x},\mathbf{x}^0\right)p_2+G_3\left(\mathbf{x},\mathbf{x}^0\right)m_3\right)dL(\mathbf{x}^0)\end{aligned}\tag{22}$$

where $f_m,\ g_m,\ G_m$ are known functions, which contain Bessel function of third kind.

Integration of functions $f_m,\ g_m,\ G_m$ for the small value of argument leads to the singularity. To establish their characteristic we used the asymptotic expressions for Bessel function of the third kind for small values of the argument.

Let's apply the approach, which is developed in [16] for the time-domain problem of classical theory of elasticity. For the determination of the unknown functions $p_1.\ p_2,\ m_3$ we satisfy the Fourier transforms of boundary condition (4) and apply Plemelj-Sokhotski formulas for the limits, when internal point tends to the boundary. We obtain the system of integral equations:

$$\begin{aligned}&\frac{Re\,(q)}{2}+\mathbf{v.p.}\int_L\left(f_1\left(\mathbf{x},\mathbf{x}^0\right)qd\zeta+f_2\left(\mathbf{x},\mathbf{x}^0\right)\bar{q}d\bar{\zeta}+f_3\left(\mathbf{x},\mathbf{x}^0\right)m_3dL\right)\\&=-\sigma_0\hat{\phi}(\omega)\end{aligned}\tag{23}$$

$$\frac{Im(q)}{2}\vartheta_1+\mathbf{v.p.}\int_L\left(g_1\left(\mathbf{x},\mathbf{x}^0\right)qd\zeta+g_2\left(\mathbf{x},\mathbf{x}^0\right)\bar{q}d\bar{\zeta}+g_3\left(\mathbf{x},\mathbf{x}^0\right)m_3dL\right)=0\tag{24}$$

$$\frac{m_3}{2} I + \mathbf{v.p.} \int_L \left(G_1 \left(\mathbf{x}, \, \mathbf{x}^0 \right) q d\zeta + G_2 \left(\mathbf{x}, \, \mathbf{x}^0 \right) \bar{q} d\bar{\zeta} + G_3 \left(\mathbf{x}, \, \mathbf{x}^0 \right) m_3 dL \right) = 0 \qquad (25)$$

where $pdL = -Iqd\zeta$, $p = p_1 + Ip_2$ is the unknown complex function, $\zeta = x_1^0 + Ix_2^0$, $\vartheta_1 = \left(1 - (\alpha/\mu)^2\right)$ is the constants. Here the integrals are understood in the sense of Cauchy principal value.

The algorithm [16] was applied to the numerical solving of the system of integral equations (23) – (25). It is used method of mechanical quadrature method.

4. Dynamic stress calculation

Calculations of the hoop stress transforms on the boundary and the radial stress transforms in the medium are performed by formulas [1]. Substituting in these formulas the potential representations for displacements (11) and microrotations (12), selecting irregular parts and completing limit transition the transforms of the hoop stresses on the boundary and the radial stresses inside the medium are received:

$$\hat{\sigma}_\theta = \frac{Re(q)}{2} \vartheta_2 + \mathbf{v.p.} \int_L \left(h_1 \left(\mathbf{x}, \, \mathbf{x}^0 \right) q d\zeta + h_2 \left(\mathbf{x}, \, \mathbf{x}^0 \right) \bar{q} d\bar{\zeta} + h_3 \left(\mathbf{x}, \, \mathbf{x}^0 \right) m_3 dL \right)$$

$$\hat{\sigma}_r = \mathbf{v.p.} \int_L \left(h_4 \left(\mathbf{x}, \, \mathbf{x}^0 \right) q d\zeta + h_5 \left(\mathbf{x}, \, \mathbf{x}^0 \right) \bar{q} d\bar{\zeta} + h_6 \left(\mathbf{x}, \, \mathbf{x}^0 \right) m_3 dL \right)$$

where $h_k = h_k(\mathbf{x}, \, \mathbf{x}^0)$ are known functions, ϑ_2 is the constant ($\vartheta_2 = \nu/(1-\nu)$ for plane strain).

Modified discrete Fourier transform is used for calculation of originals of the dynamic hoop and radial stresses [17].

5. Numerical example

Using the developed method, we investigate the dynamic stress state of a the medium with the tunnel cavity under the action of the impulse load, which is applied to the cavity's boundary in radial direction. The center of gravity is placed at the origin of Cartesian coordinate system $x_1 O x_2$.

The impulse load over time is given in form (6) for $p_* = 272$, $n_* = 2$, $\alpha_* = 10$. The calculations are performed for the dimensionless time parameter $\tilde{t} \in [0, \; 8]$.

In [8] it was shown the experimental results of studying of the following material properties of closed-cell polymethacrylimide foam: Young's moduli, Poisson's ratios, yield strengths, and characteristic lengths associated with inhomogeneous deformation.

For numerical calculations we use the elastic constants, which is obtained [8], required to describe isotropic Cosserat elastic solid. For polymer foam the values of elastic constants has chosen as: the initial density is $\rho = 380$ kg/m^3, Poisson's ratio is $\nu = 0.13$, Young's modulus is E = $637 \cdot 10^6$ Pa, elastic characteristic of Cosserat continuum: $\alpha = 2.85 \cdot 10^6$ Pa, $\epsilon = 494$ N, $\gamma = 182$ N, $l_3 = 10.8$ mm, $a_1 = 2.02$, J = $9.12 \cdot 10^{-4}$ g/cm.

Distribution of the relative hoop stresses on the boundary of the circular cavity of radius $R = 10$ mm. A curve 1 is plotted without accounting for the effect of shear-rotation deformations (basic on motion equations of classical elasticity) by the method [14]. Curve 2 is plotted for Cosserat elasticity with constrained microrotations (couple stress elasticity) by the method [18]. Curve 3 is plotted by the modified in this work method. Curves 2 and 3 are plotted with accounting for the effect of the shear-rotation deformations, which arise in the medium under the action of the impulse load.

Numerical calculations for the case of couple stress elasticity, which are based on method [18], are performed for the same values of elastic characteristics. The value of characteristic length l in couple stress elasticity for polymer foam is chosen as $l = l_t = 0.78$ mm [8].

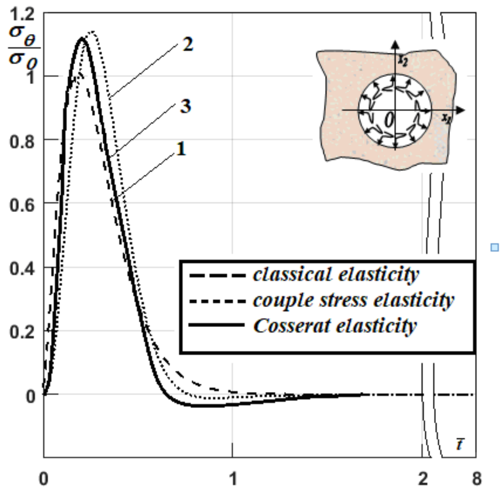

Figure 2 Relative hoop stresses on the boundary of the cavity in polymer foam

Fig. 2 shows that the maximum value of the dynamic hoop stresses on the cavity's boundary in the foam medium is higher by 11.5% for the case of Cosserat elasticity and by 13% for the case of couple stress elasticity. Therefore, dynamic hoop stresses are higher with accounting for the influence of the shear-rotation effect than without accounting for this effect.

Numerical results, which are obtained for Cosserat elasticity and couple stress elasticity, practically coincide. This is explained by the fact that the values of the microrotations of the medium points at the boundary of the cavity practically coincide with the values of the macrorotations of the medium for the case of distribution non-stationary load.

In addition, accounting for the influence of shear-rotation deformation allows describing more accurately the dynamic stress state of the foam medium behind the propagating wave. So, in the medium, there are also fields of compressive stress that arise behind the propagating wave. Such results can not be described on the basis of the classical theory of elasticity. But they are consistent with the basic principles of wave mechanics.

For the case of polymer foam media with smaller values of elastic characteristics behind the propagating impulse wave, the hoop dynamic stresses on the boundary of foam medium have wave-shaped character [18].

The numerical calculation results of relative radial stresses at distances of $\delta = 2.5$ R (curve 1) $\delta = 5$ R (curve 2), $\delta = 10$ R (curve 3), $\delta = 15$ R (curve 4) to the cavity center in a polymethacrylimide foam medium are shown in Fig. 3. Here the dashed curves correspond to the radial stresses in the foam medium without accounting for the influence of the shear-rotation deformations (which are obtained basic on motion equations of classical elasticity), and the continuous curves correspond to the stresses, which are calculated including of the influence of the shear-rotation deformations (which are obtained basic on motion equations of Cosserat elasticity). Numerical calculations of radial stresses without consideration of couple stresses are performed based on [14].

Fig. 3 shows that the maximum value of dynamic radial stresses in the foam medium are higher by 8-14% with accounting for the influence of the shear-rotation deformations (for Cosserat elasticity) than without accounting for this effect (for classical elasticity). Same results were obtained for the case of couple stress elasticity in [18].

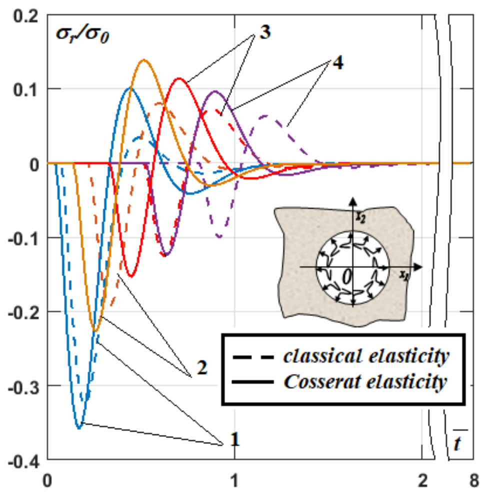

Figure 3 Relative radial stresses in the polymer foam with tunnel cavity

The accuracy of the proposed approach is ensured by the agreement of the results with the basic principles of wave mechanics. Thus, the values of the relative radial stresses are zero until a wave of the corresponding cross section is reached.

6. Discussion

Numerical calculations confirm that using of the equations of the Cosserat elasticity for the study of the dynamic stress state of foam materials allows accounting for not only the influence of shear-rotation deformations but also more accurately describe the dynamic stress state of the medium.

The displacement and microrotations in the medium can be calculated by the formulas (11) and (12) using the modified approach. Based on the values of dynamic stresses and displacements, the analysis of the propagation of non-stationary process in the foam materials can be carried out.

Thus, the approach, which is modified in the work, allows investigating of non-stationary wave processes in structurally inhomogeneous media.

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