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Sensitivity of Composite Structure with Directional Properties of Annular Three-Layered Plate Mechanically and Thermally Loaded

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Paper presents the sensitivity of the three-layered plate structure on the acting of mechanical and thermal loads. The cases of the annular plates, whose individual layers: facings and core have homogeneous building and/or heterogeneous one expressed by the variable material properties in radial direction have been examined. Numerical investigations have been carried out modelling the select examples of plate structure with the use of the finite element method. Plate is loaded in the plane of facings or is subjected to the flat temperature field. The evaluation of the structure sensitivity has been carried out analysing the values of critical loads or critical temperatures and corresponding with them buckling modes. Numerous results presented in diagrams create the image of plate behaviours, show responses of plate structure and indicate on the means of structure designs, which can fulfil the expected conditions of plate work.

Keywords: static stability, composite plate, mechanical, thermal loads, heterogeneous layers, finite element method.

1. Introduction

Dynamic development of the composite materials allows to design the structures having the suitable, expected properties customized to the work conditions. The control of the both macro and micro structure building can enable the element to fit to insignificant fluctuations of surroundings parameters. Such opportunity is disclosed by the responses of the examined, generally multi-parameter element. The reactions of the plate structure could be focused on the evaluation of its sensitivity to the acting loads. The disorder of the homogeneous structure of material by the participation of the additional components influences on the rate and means of the arrangement of heterogeneous material, whose responses in macro and micro scale can be different.

Such evaluation has been performed for the element of the annular plate composed of characteristic three layers: facings and core. The plate edges are loaded with the radially compressed forces or are subjected to the stationary temperature field calculating the values of the critical loads and temperatures, respectively. On account of the extensive area of the possible applications of annular plates: aerospace industry, mechanical and nuclear engineering, civil engineering or miniature mechanical systems the observation of their behaviours under mechanical or thermal loading is practically essential.

The investigations of layered annular plates are performed for many years. The specific analyses have been carried out for the axisymmetrical case of plates in dynamic or stability issues [1, 2]. The solution have been widened on the plate responses including the asymmetric plate modes. As the example the following works [3-8] can be. Besides the classical plates with the homogeneous facings, for example made of steel, the laminated composite plates have been examined [9-14]. The examples of the problem analyses, where the annular plate with the heterogeneous, different layout of material in structure are the works [15-16].

In this paper the approach to evaluate the rate of the sensitivity of the three-layered plate structure with variable heterogeneous material properties of both facings and core including the different buckling modes has been undertaken. The numerous results of analyses show the meaning of the thermal loading, as this one, which particularly influences on structure reaction of plate, whose support system is importance, too.

2. Problem formulation

The three-layered annular plate subjected to the loads acting in its plane is the object of the examination. The loss of plate static stability has been evaluated for plates mechanically or thermally loaded. In mechanical problem the plate is subjected to the axisymmetrical forces loaded in the plane of facings on inner or outer edge. The effect of the thermal loading is the result of the acting of the flat, axisymmetric field of temperature. The heat flow in the plate radial direction is steady expressed by the logarithmic function of temperature distribution [17]:

$$T_N = T_o + \frac{T_i - T_o}{\ln \rho_i} \ln \rho \tag{1}$$

where: T_i, T_o – temperatures of the inner and outer plate perimeters, respectively, ρ, ρ_i – dimensionless plate radius and inner plate radius expressed by $\rho = \frac{r}{r_o}$, $\rho_i = \frac{r_i}{r_o}$, respectively.

The exchange of the heat on the plate surfaces is neglected. The plate edges are clamed or slideably clamped. The scheme of the plate mechanically loaded is presented in Fig. 1.

The three-layered structure of plate is built of thin facings and thicker, soft core. Examined plate layers are homogeneous or composed of several material components creating the structure with properties variable in radial or transversal direction. Different cases of the portion of material components building the transversally symmetrical or asymmetrical plate structure have been analysed. The facings have been modelled as homogeneous layer, composed of two isotropic materials or built

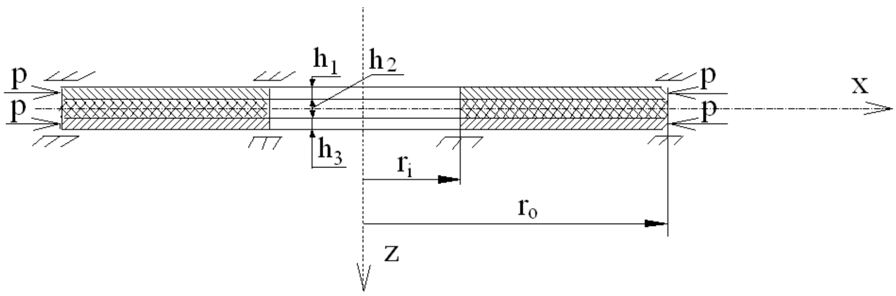

Figure 1 Scheme of analysed plate

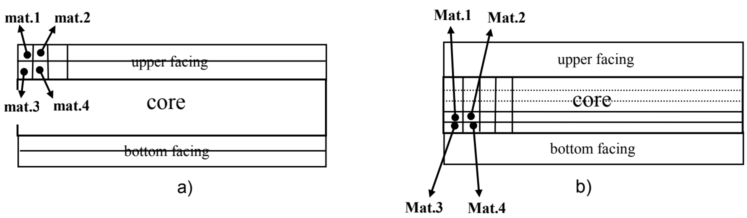

Figure 2 Scheme of plate structure focused on: a) facings, b) core

of two sublayers with maximum four components. Figure 2a explains the mean of facing building. The core consists with maximum four isotropic materials and ten sublayers. The scheme of the example material distribution is shown in Fig. 2b.
The critical static loads p_{cr} and the critical temperature differences T_{cr} between the outer and inner plate edges with the buckling modes expressed by the number m of circumferential waves are the results of the analyses.

3. Plate model

The FEM plate model is built in the form of full plate annulus. The model structure is composed of the 4-node shell and 8-node solid finite elements building the mesh of facings and the core, respectively. The outer surfaces of facings and core mesh elements are tied using the program option expressed as surface contact interaction. The mesh of facing elements composed of two subshells was modelled using the option Composite expressing the shell parameters.
The calculations were carried out at the ACC CYFRONET in placeCityCracow using the ABAQUS system (KBN/SGI_ORIGIN_2000/PŁódzka/ 030 /1999). Static stability problem has been analysed using the option Buckle of ABAQUS system.

4. Numerical investigations

Numerical investigations of plates concern the plates mechanically loaded or thermally loaded with the gradient directed to or from the middle of the plate. The examples of the plates, whose facings are in the form of single layers of homogeneous material or heterogeneous one are analysed. The heterogeneous material of the plate facings is built of two components or is composed of two sublayers built of homogeneous material or included two or four materials. The plates with heterogeneous core loaded mechanically are examined. In thermal analysis the influence of the support system was considered, too.

4.1. *Plate parameters*

Plate geometry is expressed by the following parameters: inner radius $r_i = 0.2$ m, outer radius $r_o = 0.5$ m, total thickness of facing $h' = 1.0$ mm, total thickness of the core $h_2 = 5$ mm. The plate structure cases composed of facing or core sublayers have the thickness of the each facing or core single layer equal to $h'' = 0.5$ mm. The facing can include two layers but the core consists of ten layers (see Fig. 2). In general four kinds of material parameters have been considered with the following values of Young's modulus E or Kirchhoff's modulus G, Poisson's ratio ν and expansion coefficient α (see Table 1). The effect of the thermal expansion of core materials is not included.

Table 1 Material parameters of plate layers

		mat.1	mat.2	mat.3	mat.4
facing	E [MPa]	2.1E5	0.7E5	1.078E5	0.98E5
	ν	0.3	0.34	0.375	0.4
	α [1/K]	12.0E-6	24.0E-6	16.8E-6	18.0E-6
		Mat.1	Mat.2	Mat.3	Mat.4
core	G [MPa]	5.0	2.5	1.25	10
	ν	0.3	0.3	0.3	0.3

The exemplary arrangement of the material parameters building the facing layer or plate core is shown in Fig. 2.

Presenting the results the following notation has been applied for characterization the examined cases of plate structures:

- the notation with the small letter, like: mat.1 for facing material,
- the notation with the big letter, like: Mat.1 for core material,
- the notation with the single word, like: material 1 describing the examined case of plate with homogeneous layers or sublayers,
- the notation, like: mat.1/mat.2 characterizing the facing layers built as single layer of two materials,

- the notation, like: mat.1/mat.2 mat.2/mat.1 characterizing the facing layer built as two layers of two materials, which are arranged in radial direction from the middle of the plate.

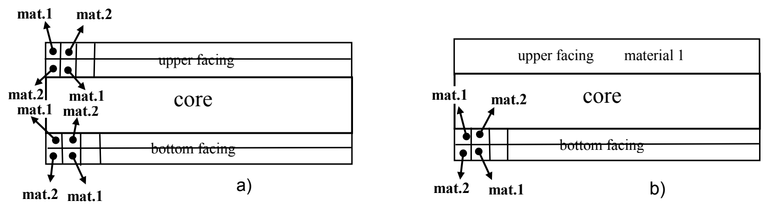

Figure 3 The exemplary description of material arrangement in: a) heterogeneous upper facing and bottom one, b) homogeneous upper facing and heterogeneous bottom one

For example the notation, like: mat.1/mat.2 mat.2/mat.1 mat.1/mat.2 mat.2/mat.1 means that transversal structure of plate is not symmetrical and two facings are built as shown in Fig. 3a.
For example the notation, like: material1 mat.1/mat.2 mat.2/mat.1 means that transversal structure of plate is not symmetrical and two facings are built as shown in Fig. 3b.
For example the notation, like: Mat.3/Mat.4 Mat.1/Mat.2 means that transversal structure of plate is not symmetrical due to the material distribution in ten sublayers of the core – see, Fig. 2b.

4.2. *Analysis of plates mechanically loaded*

Analysing the case of plate model loaded with forces in the plane of facings the influence of the changes of facing and core material parameters on values of static, critical loads p_{cr} and corresponding with them buckling modes has been examined. The plate slideably clamped and radially compressed on outer edge has been analysed. Figure 4 presents the critical load p_{cr} distribution dependent on modes of plates, whose facings are homogeneous made of material 1 or material 2 or are heterogeneous fulfilling the transversal symmetry of structure (example: mat.1/mat.2 mat.1/mat.2) or asymmetry (example: mat.1/mat.2 mat.2/mat.1). The core is homogeneous made of Material 1. The results show the meaning of heterogeneous facings in structure behaviour. Their application can increase or decrease the values of critical loads with relations to basic material 2 or material 1, respectively. The minimal value of critical load for the plates with heterogeneous facings with differently arranged material is comparable. The buckling mode m changes from $m = 5$ to $m = 4$ for transversally asymmetric structure.
Results shown in Fig. 5 complement the presented analysis. Different, selected combinations of material distribution in facings are examined. In the case of division of facing to two sublayers and mixed arrangement of material, which fulfils the transversal symmetry or asymmetry of plate structure, the relevant differences of critical parameters are not observed (see, the plate examples marked by ▲, △).

The values of critical loads correspond with those ones, which are obtained for the plate with facings built of four accepted materials. Similarly, the additional material component also located in the single facing layer can decrease or increase the values of critical loads. Then, the transversal plate structure is asymmetric.

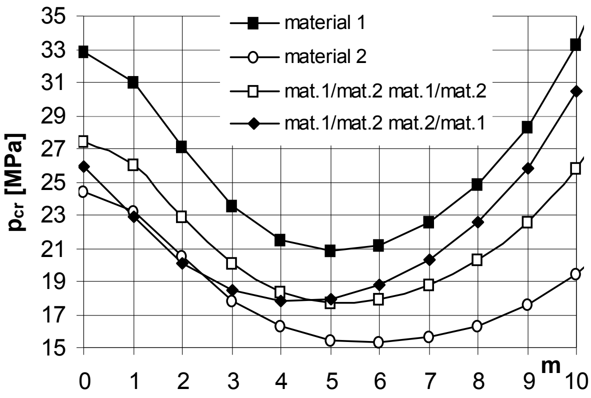

Figure 4 Critical load distribution versus buckling mode for plates with homogeneous and heterogeneous one-layer facings

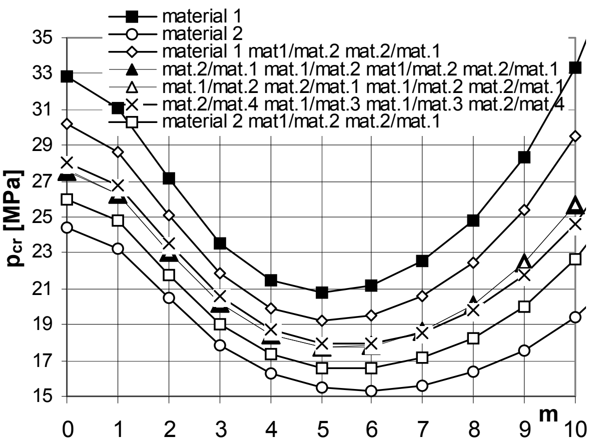

Figure 5 Critical load distribution versus buckling mode for plates with homogeneous and heterogeneous one-layer and two sublayers facings

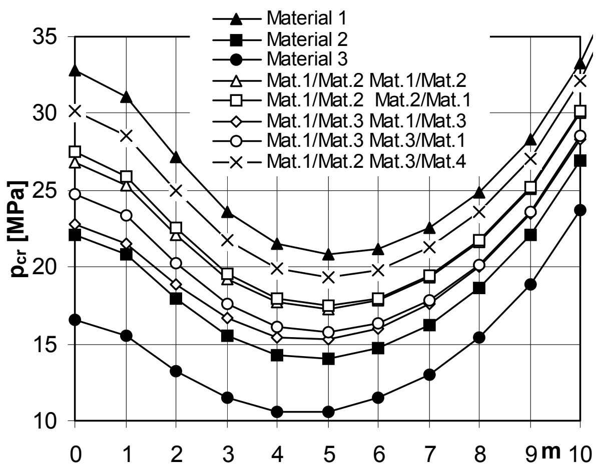

Figure 6 Critical load distribution versus buckling mode for plates with homogeneous and heterogeneous ten sublayers core

Whereas, Fig. 6 shows the buckling response of plates with homogeneous facings and homogeneous core or heterogeneous core layers. The results of plate models with homogeneous core are presented for three materials. The results indicate on the meaning of the stronger component of core material. Then, the values of critical load are higher. For example, adding of the fourth material component, which is much stronger than others increases the critical loads (see, plate example marked by $\times$). The composition of two materials: Mat.1 and Mat.2 or Mat.1 and Mat.3 building the transversal symmetry or asymmetry of plate structure does not influence on differences of minimal, critical loads values, significantly. A little greater differences of loads p_{cr} are observed for plates losing the static stability in the form of the several circumferential waves $m < 5$. The material arrangement of two components, which fulfil the patchwork composition, seems to be particularly advantageous.

4.3. *Analysis of plates thermally loaded*

Figures 7–11 shows the distribution of the critical temperatures dependent on the plate buckling modes. Figures 7, 8 and Fig. 11 concern the plate cases, whose edges are clamped (C-C) but Figs. 9, 10 present the results of plates with slideably clamped edges (SC-SC). Figure 7 shows the course of changes of critical temperatures for plate, whose temperature gradient is directed to the plate middle. The participation of the additional materials in facing layer influences on the temperature changes in the range of values, which are between the ones of basic materials. The influence of the arrangement of the material components fulfilling the transversal symmetry or asymmetry of plate structure on the values of the critical temperature is insignificant. The minimal critical temperature corresponds to waved plate mode with number m of circumferential waves equal to $m = 3$ or $m = 4$.

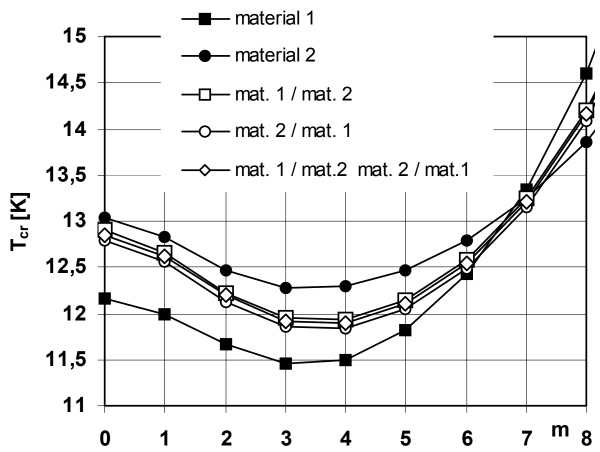

Figure 7 Critical temperature distribution versus buckling mode for plates (C-C) with homogeneous and heterogeneous one-layer facings, thermally loaded on outer edge

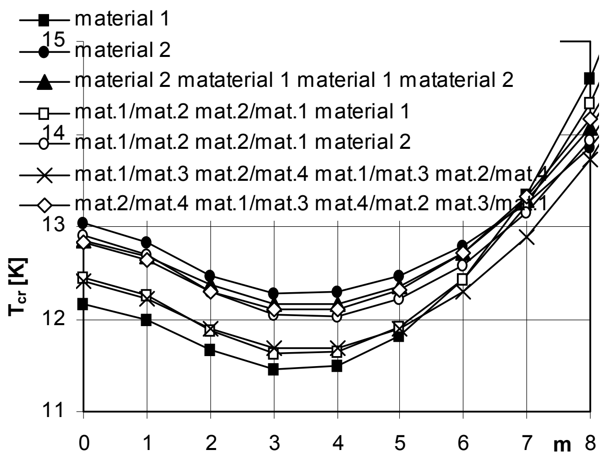

Figure 8 Critical temperature distribution versus buckling mode for plates (C-C) with homogeneous and heterogeneous two-layers facings, thermally loaded on outer edge

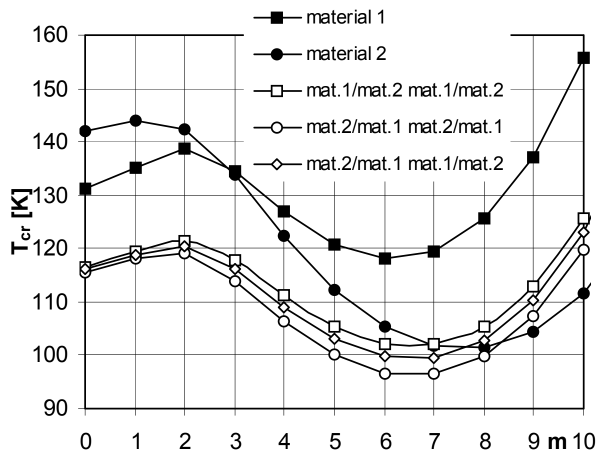

Figure 9 Critical temperature distribution versus buckling mode for plates (SC-SC) with homogeneous and heterogeneous one-layer facings, thermally loaded on inner edge

The influence of the other combinations of material layout is shown in Fig. 8. The participation of the additional material component, similarly like for plates mechanically loaded, increase or decrease the values of critical temperature – see, curves with marks ∘, □, respectively. Out of two presented examples of plate facings, whose two sublayers are built of four materials and are arranged fulfilling the structure asymmetry, more advantageous is this one, whose results are presented by mark ◊. Then, the layout of material components has the importance disclosing the different sensitivity of examined plate (see, the results marked by ×, ◊).

The examples of plates with slideably clamped edges present much higher values of critical temperatures. The results are shown in Fig. 9 and Fig. 10. The participation of the additional material component in the single layer of facing can significantly decrease the values of critical temperatures for the plates with temperature gradient directed from the plate middle (see, Fig. 9) or increase critical temperatures for the plates with temperature gradient directed to the plate middle (see, Fig. 10). The issue of the modelling of the facing layers has here the particular meaning and is connected with the plate support system. In reliance on work conditions and setting of requirements the structure building can be suitably fitted.

Comparing Fig. 9 and Fig. 11, it can be observed the influence of the support system on the response of the plate with temperature gradient directed from the plate middle. In the case of the plate with slideably clamped edges the minimal values of the critical temperature are for the circumferentially waved plates, whose number m is equal to $m = 6 \div 8$. Whereas, minimal critical temperature for plates with clamped edges corresponds with the axisymmetrical form of buckling $m = 0$. In this case the building of the transversally symmetrical structure with the participation of the four materials is noticed (see, results marked by ◊).

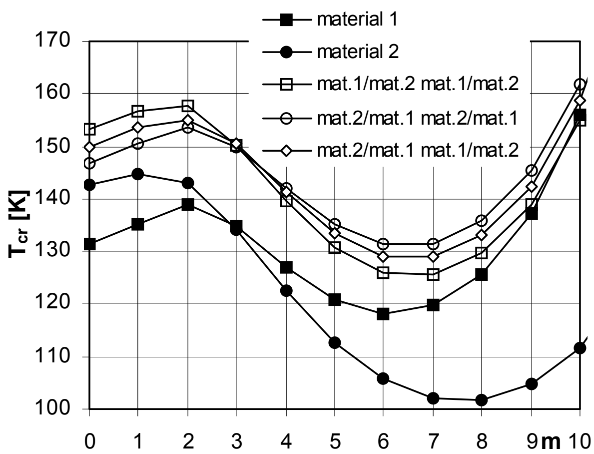

Figure 10 Critical temperature distribution versus buckling mode for plates (SC-SC) with homogeneous and heterogeneous one-layer facings, thermally loaded on outer edge

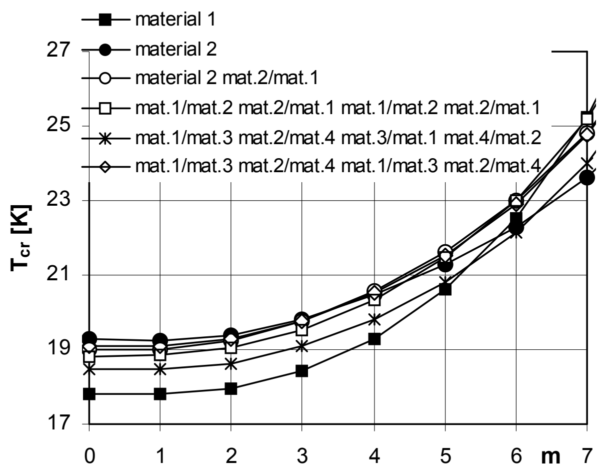

Figure 11 Critical temperature distribution versus buckling mode for plates (C-C) with homogeneous and heterogeneous two-layers facings, thermally loaded on inner edge

5. Conclusions

The paper presents the sensitivity of the layered, annular plates, whose structure is modelled by the layers of the homogeneous material and heterogeneous one composed of the several components. Plates are subjected to the acting of the mechanical or thermal loads. The numerous observations of the results show the meaning of the participation of the additional material component, which changes the values of the critical quantities. For the most of the analysed cases the minimal values of the critical quantities exist for the plates circumferentially waved. It indicates for the meaning of the such problem solution of plate defections, which is not limited to only axisymmetrical case. The use of the additional material component is particularly effective for the plates with slideably clamped edges (SC-SC) thermally loaded with the temperature gradient directed to the middle of the plate. For these plates the range of the values of critical temperatures is significantly wider than for the plates with clamped edges (C-C). The use of the heterogeneous building of layers for plates with clamped edges (C-C) is not practically necessary. The influence of the arrangement of two material components, which change the material properties in the plate radial direction is meaningless. Whereas, the adding of the additional, stronger component significantly improves the final results. In the compact, several-layers structure of plate material, like for example in the core layer, the patchwork layout of the material components is advantageous.

Presented results show the meaning of the modelling of the element structure, which requires to adjust to the influence of the many parameters, and requires the perceptive evaluation of the final results and the practical, also economical balance of the expected results in design process of the true object. Practically important direction of the further theoretical and numerical analyses could be the dynamic evaluation of the behaviours of the similar plate structures. Particularly valuable observation would be the analysis of the actual plates experimentally investigated.

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