Dispersion Waves in a Submerged Thermo-Piezoelectric Membrane

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Wave propagation in a thermo piezoelectric membrane immersed in an infinite fluid medium is discussed using three-dimensional linear theory of elasticity and thermos piezoelectricity. Three displacement potential functions are introduced to uncouple the equations of motion, heat and electric conduction equations. The frequency equations are obtained for longitudinal and flexural modes at the solid fluid interfacial boundary conditions. The numerical results are analyzed for PZT-4 material and the computed stress, strain, electric displacement and temperature distribution are presented in the form of dispersion curves and its characteristics are studied.

Keywords: Dispersion waves in piezoelectric plates, solid-fluid interaction, thermal cylinders/plates, temperature sensors.

1. Introduction

Piezoelectric and thermo piezo electric coupling form plays a major role in construction engineering as important component of structures. The intelligent structure system have thermo piezo component for self-monitoring and self-controlling process. The electro-elastic materials with thermal environment has many applications in sensors and actuators as magnetic probes, electric packing, hydrophones, medical, ultrasonic image processing due to the transition of energy in thermo-electro-mechanical conversion.

The propagation of compressional elastic waves along an anisotropic circular cylinder with hexagonal symmetry was first studied by Morse [1]. Bhimaraddi [2] developed a higher order theory for the free vibration analysis of circular cylindrical shell. Zhang [3] investigated the parametric analysis of frequency of rotating laminated composite cylindrical shell using wave propagation approach. The generalized theory of thermoelasticity was developed by Lord and Shulman [4] involving one relaxation time for isotropic homogeneous media, which is called the first generalization to the coupled theory of elasticity. These equations determine the finite

speeds of propagation of heat and displacement distributions, the corresponding equations for an isotropic case were obtained by Dhaliwal and Sheried [5].

Thermo-piezo-elastic field which will give piezo thermo elastic components give an idea to sense the thermo-mechanical disturbance coming from perturbed electric potential, to alter the response of the structure by applied electric fields. The thermo-piezo-electric theory was first proposed by Mindlin [6], later he derived the governing equations of a thermo-piezo-electric plate.

Thermo-piezo-electric materials and its laws for physical characters have been investigated by Nowacki [7, 8]. In order to achieve the finite thermal signal speed, Chandrasekhariah [9,10] analyzed the generalized theory of thermo piezo elasticity considering the thermal relaxation times in the analytical formulation [11]. Tang and Xu [12] developed a dynamical equation in general form for the plate composed of anisotropic material which have mechanical, thermal and electrical influence and they examined the forces acting on a thermo piezo elastic laminated plate and harmonic responses to distribution of temperature and electrical effect. The frequency shift of a vibration of linear piezo electric material with heat conduction by using perturbation methods were interpreted by Yang and Batra [13]. An analytical solution is achieved by Peyman et al. [14] for a piezolaminated rectangular plate with arbitrary clamped and simply supported boundary conditions under thermo-electro-mechanical loadings. The axisymmetric snap-through behavior of Piezo-FGM shallow clamped spherical shells under thermo-electro-mechanical loading was discussed by Boroujerdy and Eslami [15].

The natural frequencies of a thin clamped circular plate in an aperture of an infinite rigid plane wall coupled with water was investigated by Lamb [16] based on the assumption that the wet mode shapes are almost the same as those in vacuum. Lindholm et al. [17] reported the natural frequencies of cantilever plates in air and in water, whereas they applied strip theory to evaluate fluid actions. Added mass of thin rectangular plates in infinite fluid was obtained by Meyerhoff [18] and dipole singularities were employed to model the potential flow around a flat rectangular plate. A piecewise division is used by Kwak [19] to investigate the free vibrations of rectangular plates in contact with unbounded water on one side, whereas beam functions were used as admissible functions. Haddara and Cao [20] investigated dynamic responses of rectangular plates immersed in fluid. An approximate expression for the evaluation of the modal added mass was derived for cantilever and SFSF rectangular plates and the numerical results were verified by the experimental ones. The natural frequencies of annular plates in contact with a fluid on one side were theoretically obtained by Amabili et al. [21] using the added mass approach, whereas the coupled fluid–structure system was solved by adopting the Hankel transform. The wave propagation in a generalized thermo elastic plate immersed in fluid was analyzed by Selvamani and Ponnusamy [22]. Later, Selvamani and Ponnusamy [23] have studied the dynamic response of a solid Bar of cardioidal cross-sections immersed in an inviscid fluid using Fourier expansion collocation method. Recently, Selvamani [24] has studied the dispersion analysis in a fluid filled and immersed transversely isotropic thermo-electro-elastic hollow cylinder using the Bessel function in frequency equation.

In this paper, wave propagation in a anisotropic thermo piezo electric membrane immersed in an inviscid fluid is studied. The displacement functions to rep-

resent three displacement components on the basis of three-dimensional generalized piezothermo elasticity are considered. The frequency equations are obtained for longitudinal and flexural modes at the solid fluid interfacial boundary conditions. The numerical results are analyzed for PZT-4 material and the computed stress, strain, electric displacement and temperature distribution are presented in the form of dispersion curves.

2. Model of the Problem

We consider a rectangular thermo piezo electric membrane of thickness $2h$. In Cartesian coordinates, the motion takes place in XZ plane in which the mid plane is the origin of the membrane and Z axis is perpendicular to the mid plane. The complete governing equations that explains the behavior of thermo piezoelectric membrane have been considered from Mindlin [6].

$$S_{xx} = c_{11}u_{,x} + c_{13}w_{,z} + e_{31}\phi_{,z} - \beta_1 T$$
$$S_{yy} = c_{12}u_{,x} + c_{13}w_{,z} + e_{31}\phi_{,z} - \beta_1 T$$
$$S_{zz} = c_{13}u_{,x} + c_{33}w_{,z} + e_{33}\phi_{,z} - \beta_3 T$$
$$S_{xy} = 0, S_{yz} = 0, S_{xz} = c_{44}(w_{,x} + u_{,z}) + e_{15}\phi_{,x} \quad (1)$$
$$D_x = e_{15}(w_{,x} + u_{,z}) - \in_{11} \phi_{,x}$$
$$D_y = 0 D_z = e_{31}u_{,x} + e_{33}w_{,z} - \varepsilon_{33}\phi_{,z} + p_3 T \sigma = \beta_1 u_{,x} + \beta_3 w_{,z} - p_3\phi_{,z} + dT$$

where S_{xx}, S_{yy}, S_{zz}, S_{xy}, S_{yz}, S_{xz} are the strain components, u, w are the displacement components, c_{11}, c_{12}, c_{13}, c_{33} are the elastic constants, e_{31}, e_{33} are the piezoelectric constants, $\in_{11}$, $\in_{33}$ are the dielectric constants, T is the temperature change about the equilibrium temperature T_0, p_3 is the pyroelectric constant, β_1, β_3 are the thermal expansion coefficients, K_1, K_3 are the thermal conductivity, ρ is the mass density. The equations of motion for hexagonal (6 mm) class are derived as follows

$$c_{11}u_{xx} + (c_{13}c_{44})w_{xz} + c_{44}u_{zz} + (e_{31} + e_{15})\phi_{xz} + (e_{31} + e_{15})V_{rz} = \rho u_{tt}$$
$$c_{44}w_{xx} + c_{33}w_{xz} + (c_{13} + c_{44})u_{zx} + e_{15}\phi_{xx} + e_{33}\phi_{zz} = \rho w_{tt}$$
$$e_{15}w_{xx} + (e_{15} + e_{31})u_{zx} + \varepsilon_{11}\phi_{xx} + e_{33}w_{zz} - \varepsilon_{33}\phi_{zz} = 0 \quad (2)$$
$$K_{11}T_{xx} + K_{33}T_{zz} = T_0 \left(\beta_1 u_{xt} + \beta_3 w_{xt} - P_3\phi_{zt} + dt_t \right)$$

3. Solutions of the Field Equation

The harmonic wave solution of the above equation has been arrived by taking the displacement component with respect to the potential derivatives and are considered from Paul [24] as follows

$$u = U(z) \sin kx \, e^{i\omega t}$$
$$w = W(z) \cos kx \, e^{i\omega t}$$
$$\varphi = (c_{44}/e_{33}) \, \varphi(z) \cos kx \, e^{i\omega t}, \quad (3)$$
$$T = \left(c_{44}//\beta_3 \right) kT(z) \cos kx e^{i\omega t}$$

where $i = \sqrt{-1}$, k is the wave number, ω is the angular frequency, $U(z)$, $W(z)$, $\phi(z)$ and $T(z)$ are the displacement potentials. By introducing the dimensionless quantities such as $x = rh$, $\varepsilon = kh$, $\overline{c}_{ij} = \frac{c_{ij}}{c_{44}}$, $\overline{c}_{ij} = \frac{e_{ij}}{e_{33}}$, $\varepsilon_{ij} = \frac{\varepsilon_{11}}{e_{33}}$, $\overline{p} = \frac{p_1 c_{44}}{\beta_3 e_{33}}$, $k_\beta^{-2} = \frac{(\varepsilon_{ij})c_{44}}{\varepsilon_{33}^2}$, $k_j = \frac{1}{h^2}\frac{c_{44}}{\beta_3^2 pT_0}k_{ij}$ and substituting Eq. (3) in Eq. (2), we obtain

$$\frac{d^2U}{dr^2} - \varepsilon^2 c_{11}u - (1+\overline{c}_{13})\varepsilon\frac{dW}{dr} - (\overline{e}_{31}+\overline{e}_{15})\varepsilon\frac{d\varphi}{dr} = -(ch)^2 u$$

$$(1+\overline{c}_{13})\varepsilon\frac{dU}{dr} + \overline{c}_{13}\frac{d^2W}{dr^2} - \varepsilon^2 W - \varepsilon_{15}^2\varepsilon^2 + \frac{d^2\varphi}{dr^2} - \varepsilon\frac{dT}{dr} = -(ch)^2 W,$$

$$(\overline{e}_{31}+\overline{e}_{15})\varepsilon\frac{dU}{dr} - \varepsilon^2\overline{e}_{31}W + \frac{d^2W}{dr^2} - k_{33}^{-2}\frac{d^2\varphi}{dr^2} + k_{13}^{-2}\varepsilon^2\varphi = 0 \qquad (4)$$

$$\beta\varepsilon U + \frac{dW}{dr} - \overline{p}\frac{d\varphi}{dr} + \varepsilon\left[d + (k_3\frac{d^2}{dr^2} - k_1\varepsilon^2)i\right]T = 0,$$

Eq. (4) can be written as the vanishing determinant form

$$\begin{vmatrix} \frac{d^2}{dr^2}(ch^2 - \varepsilon^2 c_{11}) & -(1+\overline{c}_{13})\varepsilon\frac{d}{dr} & (\overline{e}_{31}+\overline{e}_{15})\varepsilon\frac{du}{dr} & \beta\varepsilon^2 \\ (1+\overline{c}_{13})\varepsilon\frac{d}{dr} & \overline{c}_{33}\frac{d^2}{dr^2}(ch^2-\varepsilon^2) & \frac{d^2}{dr^2}-\overline{e}_{15}\varepsilon^2 & \varepsilon\frac{d}{dr} \\ (\overline{e}_{31}+\overline{e}_{15})\varepsilon\frac{d}{dr} & \frac{d^2}{dr^2}\left(-\overline{e}_{15}\varepsilon^2\right) & k_{33}^{-2}\left(\overline{e}_{11}\varepsilon^2-\frac{d^2}{dr^2}\right) & \varepsilon\overline{p}\frac{d}{dr} \\ \beta\varepsilon & \frac{d}{dr} & \overline{p}\frac{d}{dr} & \varepsilon\left(d+i\left(k_3\frac{d^2}{dr^2}-k_1\varepsilon^2\right)\right) \end{vmatrix}$$

$$\cdot(U,W,\varphi,T) = 0 \qquad (5)$$

Evaluating the determinant given in Eq. (5), we obtain a differential equation of the form

$$\left(\frac{d^8}{dr^8} + A\frac{d^6}{dr^6} + B\frac{d^4}{dr^4} + C\frac{d^2}{dr^2} + D\right)(U,W,\varphi,T) = 0 \qquad (6)$$

where

$$A = g_9\left(g_7\overline{c}_{33} + 2g_6 - g_1g_{10}\overline{c}_{33} - g_1 - g_2^2g_{10} - 2g_2g_3 - g_3^2\overline{c}_{33} - g_9g_5g_{10}\right)$$
$$+ \left[g_8(-g_{10}\overline{c}_{33}-1) + \overline{p}^{-2}\overline{c}_{33} - g_7g_5g_{10} + 2\overline{p} - g_{10}\right]/\left[-g_9(g_{10}\overline{c}_{33}+1)\right]$$
$$B = \left[g_8\left(g_7\overline{c}_{33} + 2g_6 - g_1g_{10}\overline{c}_{33} - g_1 - g_2^2g_{10} - 2g_2g_3 - g_3^2\overline{c}_{33}\right)\right.$$
$$+g_9\left(g_5g_7 - g_6^2 + g_1g_7\overline{c}_{33} - g_1g_{10}g_5 + 2g_1g_6 + g_2 2g_7\right.$$
$$\left.+2g_2g_6g_3 + g_3^2g_5\right) + \overline{p}\left(g_5\overline{p} + g_1\overline{p}\overline{c}_{33} + 2g_1 - 2g_6\right.$$
$$+g_2^2\overline{p} + 2g_2g_4 + 2g_2g_3 - 2g_3g_4\overline{c}_{33}\right)g_7 - g_1g_{10} - 2g_2g_4g_{10}$$
$$\left.+g_3^2 - 2g_3g_4 + g_4^2g_{10}\overline{c}_{33} + g_4^2\right]/\left[-g_9(g_{10}\overline{c}_{33}+1)\right] \qquad (7)$$
$$C = \left[g_8\left(g_5g_7 - g_6^2 + g_1g_7\overline{c}_{33} - g_1g_5g_{10} + 2g_1g_6 + g_2^2g_7 + 2g_2g_3g_6 + g_3^2g_5\right.\right.$$
$$+g_9(g_1g_5g_7 - g_1g_6^2 + g_1(g_5)\overline{p}^{-2} + g_7 - 2\overline{p}g_6$$
$$+g_3(-g_4g_5\overline{p} + 2g_4g_6 - \overline{p}g_4g_5) + g_4(-g_2g_6\overline{p} + 2g_1g_6 + 2g_2g_7$$
$$\left.\left.-\overline{p}g_2g_6 - g_4g_5g_{10} - 2g_4g_6)\right]/\left[-g_9(g_{10}\overline{c}_{33}+1)\right],\right.$$
$$D = (g_1g_8 - g_4^2)(g_5g_7 - g_6^2)/\left[-g_9(g_{10}\overline{c}_{33}+1)\right]$$

in which

$$g_1 = (ch)^2 - \varepsilon^2 \bar{c}_{11} \quad g_2 = (1 + \bar{c}_{13})\varepsilon \quad g_3 = (\bar{e}_{31} + \bar{e}_{15})\varepsilon$$

$$g_4 = \beta\varepsilon^2 \quad g_5 = (ch)^2 - \varepsilon^2 \quad g_6 = \varepsilon^2 \bar{e}_{11} \quad g_7 = \frac{\varepsilon^2 \bar{e}_{11}}{k_{33}^2} \tag{8}$$

$$g_8 = \frac{\rho c_u c_{44}}{\beta_3^2 T_0 - i k_1 \varepsilon^2} \quad g_9 = i k_3 \quad g_{10} = \frac{1}{k_{33}^2}$$

Solving the Eq. (6), we get solutions for a symmetric mode as

$$U = \sum_{i=1}^{4} A_i \cosh(p_i r),$$

$$W = \sum_{i=1}^{4} A_i q_i \sinh(p_i r), \tag{9}$$

$$\varphi = \sum_{i=1}^{4} A_i e_i \cosh(p_i r)$$

$$T = \sum_{i=1}^{4} A_i r_i \sinh(p_i r), \tag{10}$$

Here $(\alpha_i a)^2 = 0$, $p_i^2 > 0$, $(i = 1, 2, 3, 4)$ are the roots of the algebraic equation

$$p^8 + Ap^6 + Bp^4 + Cp^2 + D = 0 \tag{11}$$

The constants q_i, e_i, r_i defined in the Eq. (12) can be calculated from the equations

$$(1 + \bar{c}13)\varepsilon p_i q_i - (\bar{e}_{31} + \bar{e}_{15})\varepsilon p_i e_i + \beta\varepsilon^2 r_i + \left\{ p_i^2 \left[(ch)^2 - \varepsilon^2 \bar{c}_{11} \right] \right\} = 0$$
$$\left\{ \bar{c}_{33} p_i^2 + \left[(ch)^2 - \varepsilon^2 \right] \right\} q_i + \left(p_i^2 - \bar{e}_{15}\varepsilon^2 \right) e_i - \varepsilon p_i r_i \left(1 + \bar{c}_{13} \right) \varepsilon p_i = 0 \tag{12}$$
$$\left(p_i^2 - \bar{e}_{11}\varepsilon^2 \right) q_i + \left[\varepsilon^2 \left(\bar{e}_{11} - p_i^2 \right) / k_{33}^2 \right] e_i + \varepsilon \bar{p} p_i r_i + (\overline{(e}_{31} + \bar{e}_{15})\varepsilon p_i = 0.$$

4. Formulation of the Fluid

The fluid pressure acting upon the structure can be expressed as a function of acceleration. The fluid force matrices are superimposed onto the structural matrices to form the dynamic equations of a coupled fluid-structure system. Linear potential flow is applied to describe the fluid effect that leads to the fluid dynamic forces. The mathematical model is based on the following assumptions:

 (i) the fluid flow is potential;

 (ii) vibration is linear;

 (iii) since the flow is inviscid, there is no shear and the fluid pressure is purely normal to the plate wall;

 (iv) the fluid is incompressible.

Based on the aforementioned hypothesis the potential function, which satisfies the Laplace equation, is expressed in the Cartesian coordinate system as:

$$\frac{\partial \phi}{\partial z} = -\frac{1}{g}\frac{\partial^2 \phi}{\partial t^2} \tag{13}$$

The fluid pressure at solid fluid interface is given by

$$p = -p^f \frac{\partial \phi}{\partial t} \tag{14}$$

The impermeability condition of the structure surface requires that the out-of-plane velocity component of the fluid on the plate surface should match the instantaneous rate of change of the plate displacement in the transversal direction.

$$\left(\frac{\partial \phi}{\partial z}\right)_{z=0} = \frac{\partial W}{\partial t} \tag{15}$$

Substituting Eq. (15) in to Eq. (14) we can get the following potential function

$$\phi(x,y,z,t) = \frac{1}{\mu}\left(\frac{e^{\mu z} + Ce^{-\mu(z-2h)}}{1 - Ce^{2\mu h}}\right) \tag{16}$$

where

$$C = \frac{(g\mu - \omega^2)}{(g\mu + \omega^2)}$$

Here, g is the acceleration due to gravity, By substituting Eq. (16) in (14) along with (15), the acoustic pressure for the fluid at $z = h_1$, h_2 can be expressed as

$$p = -\frac{\rho^f}{\mu}\left(\frac{1 + Ce^{2\mu z}}{1 - Ce^{2\mu z}}\right)\frac{\partial^2 W}{\partial t^2} \tag{17}$$

5. Boundary Conditions and Frequency Equations

In this problem, the free vibration of anisotropic thermo piezoelectric membrane immersed in fluid is considered. Interaction between the fluid and the structure can be discussed by the coupling of the equation of motion of the structure and the equation of motion of the fluid. The boundary conditions can be written as

$$\begin{aligned}\bar{c}_{33}\varepsilon U_x + \bar{c}_{33}W, r + \varphi, r - T &= p\\ U, r - \varepsilon W - \bar{e}_{15}\varepsilon\varphi &= u^f\\ \varphi = 0, \quad T &= 0\end{aligned} \tag{18}$$

Substituting the solutions given in the Eqs. (9), (10) and (17) in the boundary condition in the Eq. (18), we obtain a system of five linear algebraic equation as follows:

$$[A]\{X\} = \{0\} \tag{19}$$

where $[A]$ is a 4×4 matrix of unknown wave amplitudes, and $\{X\}$ is an 4×1 column vector of the unknown amplitude coefficients A_1, A_2, A_3, A_4. The solution of Eq.

(19) is nontrivial when the determinant of the coefficient of the wave amplitudes $\{X\}$ vanishes, that is

$$|A| = 0 \qquad (20)$$

The components in the above determinant is defined as follows

$$|a_{ij}| = 0, \quad i, j = 1, 2, 3, 4, \qquad (21)$$

where

$$a_{ij} = (\bar{c}_{33}\varepsilon + \bar{c}_{33}s_i q_j + s_i e_j - r_j)f(s_j)$$
$$-\omega^2 \frac{\rho^f}{\mu}\left(\frac{g\mu + \omega^2 + (g\mu - \omega^2)e^{2\mu z}}{g\mu + \omega^2 - (g\mu - \omega^2)e^{2\mu z}}\right)g(s_j)$$

$$a_{2j} = s_j - \varepsilon q_j \bar{e}_{15}\varepsilon e_j)g(s_j),$$

$$a_{3j} = e_j g(s_j),$$

$$a_{4j} = r_j f(s_j).$$

6.   Numerical Results and Discussion

The frequency equation given in Eq. (20) is transcendental in nature with unknown frequency and wavenumber. The solutions of the frequency equation are obtained numerically by fixing the wave number. The material chosen for the numerical calculation is PZT-4. The material properties of PZT-4 is taken from Sharma et al. [25].

$$c_{11} = 13.9 \times 10^{10}\text{Nm}^{-2}, \quad c_{12} = 7.78 \times 10^{10}\text{Nm}^{-2}, \quad c_{13} = 7.43 \times 10^{10}\text{Nm}^{-2},$$

$$c_{33} = 11.5 \times 10^{10}\text{Nm}^{-2}, \quad \beta_1 = 1.52 \times 10^6\text{NK}^{-1}\text{m}^{-2}, \quad \beta_3 = 1.53 \times 10^6\text{NK}^{-1}\text{m}^{-2},$$

$$T_0 = 298\text{K}, \quad c_\nu = 420\text{Jkg}^{-1}\text{K}^{-1}, \quad p_3 = -452 \times 10^{-6}\text{CK}^{-1}\text{m}^{-2},$$

$$K_1 = K_3 = 1.5\text{Wm}^{-1}\text{K}^{-1}, \ e_{31} = -5.2\text{Cm}^{-2}, \ e_{33} = 15.1\text{Cm}^{-2}, \ e_{15} = 12.7\text{Cm}^{-2},$$

$$\varepsilon_{11} = 6.46 \times 10^{-9}\text{C}^2\text{N}^{-1}\text{m}^{-2}, \quad \varepsilon_{33} = 5.62 \times 10^{-9}\text{C}^2\text{N}^{-1}\text{m}^{-2}, \quad \rho = 7500\text{kgm}^{-2}.$$

and for fluid the density $\rho^f = 1000$ kgm^{-3}, phase velocity $c = 1500$ ms^{-1} and used for the numerical calculations. Here, the longitudinal and flexural modes of the vibration have been considered by choosing respectively $n = 0$ and $n = 1$.

6.1.   *Dispersion Curves*

The results of longitudinal and flexural modes are plotted in the form of dispersion curves. The notation used in the figures, namely L_m, F_{sm} and F_{asm} respectively denotes the longitudinal mode, flexural symmetric mode and flexural anti-symmetric mode.

The dispersion curves are drawn in Figs. 1 and 2 for variation of normal stress S_{zz} versus the thickness of the thermo piezo electric membrane in space and water. From the Fig. 1, it is observed that the normal stress is increasing with respect to its thickness for the different vibration modes such as longitudinal, flexural symmetric and flexural anti-symmetric. Fig. 2 experiences some oscillation in the wave trend due to the added mass effect of fluid. A comparison is made between the normal
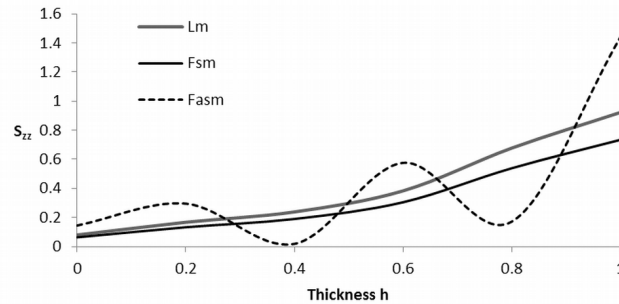
Figure 1 Variation of normal stress S_{zz} vs. thickness h of a thermo piezoelectric membrane in space
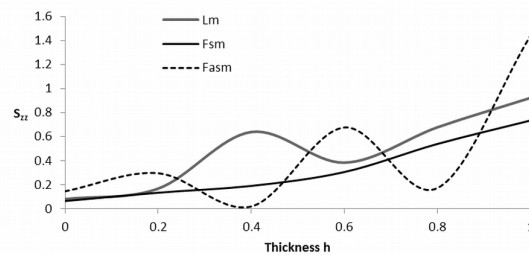

Figure 2 Variation of normal stress S_{zz} vs. thickness h of a thermo piezoelectric membrane in water

strain and the thickness of the membrane for longitudinal and flexural modes of vibration is respectively shown in the Figs. 3 and 4. From the Figs. 3 and 4, it is clear that, the lower range of thickness the normal strain attain maximum in both cases of membrane in space and in water, after that, it starts decreases. The merging of points between the vibration modes of strain shows that, there is a energy transfer between the longitudinal and flexural modes.

Figures 5 and 6 demonstrate the dispersion curves for the electric displacement of thermo piezo electric membrane with the thickness in case of with fluid (immersed in water) and without fluid (in space). From the Figs. 5 and 6, it is observed that the electric displacement having oscillating nature as the increase in thickness in three vibration modes. In Fig. 6, the electric displacement is in wave propagation trend in all the modes of vibration.

A dispersion curve is drawn to compare the temperature distribution of longitudinal and flexural symmetric and anti-symmetric modes of vibration for a thermo-piezoelectric membrane in space and water is shown respectively in the Figs. 7 and 8. From the Figs. 7 and 8, it is clear that the temperature distributions are varies linearly and having peak values in some range of thickness. In Fig. 8, the temperature distribution is getting peak values in the lower range of thickness.
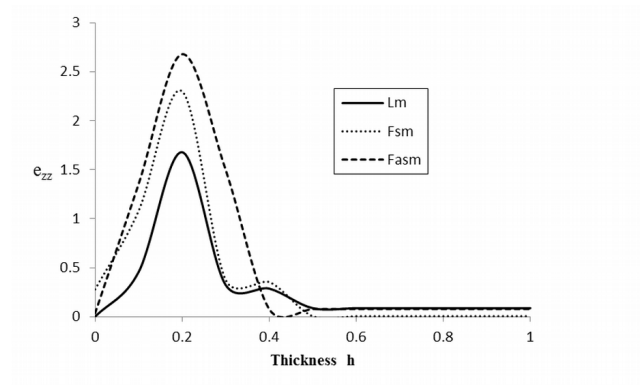
Figure 3 Variation of normal strain e_{zz} versus thickness h of a thermo piezoelectric membrane in space
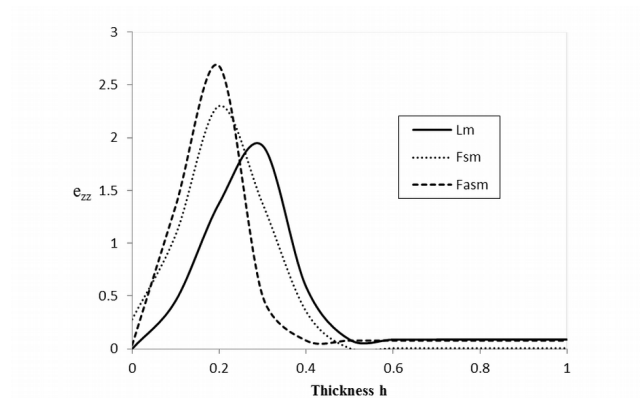

Figure 4 Variation of normal strain e_{zz} versus thickness h of a thermo piezoelectric membrane in water
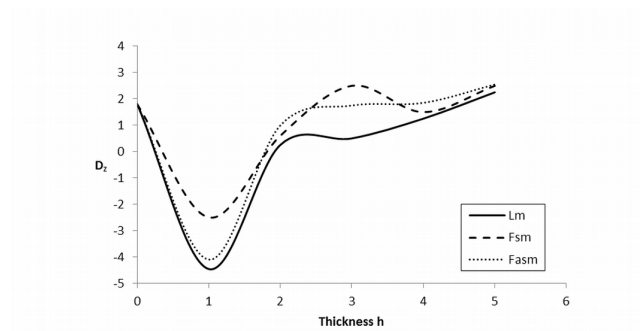

Figure 5 Variation of electric displacement D_z versus thickness h of a thermo piezoelectric membrane in space
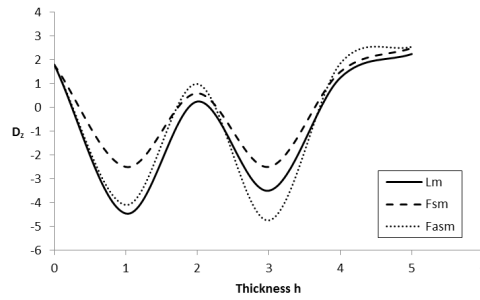
Figure 6 Variation of electric displacement D_z versus thickness h of a thermo piezoelectric membrane in water
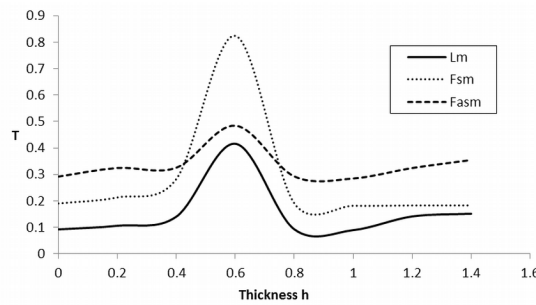

Figure 7 Variation of temperature distribution T versus thickness h of a thermo piezoelectric membrane in space
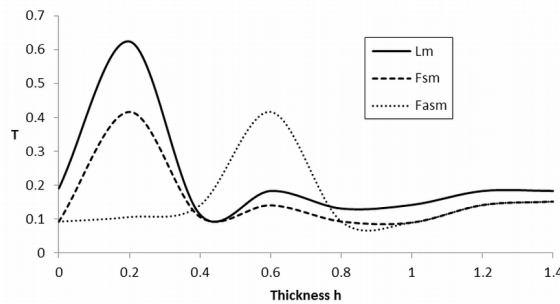

Figure 8 Variation of temperature distribution T versus thickness h of a thermo piezoelectric membrane in water

7.   Conclusions

This study demonstrates the wave propagation model in an anisotropic thermo piezo electric membrane immersed in an infinite inviscid fluid. The displacement functions to represent three displacement components on the basis of three-dimensional gene-ralized thermo piezo elasticity are considered. The frequency equations are obtained

for longitudinal and flexural modes at the solid fluid interfacial boundary conditions. The numerical results are analyzed for PZT-4 material and the computed stress, strain, electric displacement and temperature distribution are presented in the form of dispersion curves. The values of all physical quantity that are discussed here having significant variations due to the surrounding fluid medium and also the further interacting field. This method is applicable to wide range of problems in hydrodynamics, thermo piezo elasticity.

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