

Dynamic Behavior of Sandwich FGM Beams

Mohamed BOUAMAMA, Kaddour REFASSI, Abbas ELMEICHE and Abdelkader MEGUENI
Mechanics of Structures and Solids Laboratory
Department of Mechanics, Faculty of Technology
BP 89, Cité Ben M'hidi, University of Sidi Bel Abbès, Sidi Bel Abbès 22000, Algeria
e-mail: bouamamamohameddoc@gmail.com

Received (7 September 2017)

Revised (5 July 2018)

Accepted (7 September 2018)

This work is consisted to investigate the vibration behavior of FGM beams under different boundary conditions with diverse volume fraction. The main objective in this paper is to study the thickness influence of the sandwich beams skin on the frequencies of the structures. The classical Euler-Bernoulli theory (CLBT) with assuming that the material properties of the FGM layer will evaluated continuously in the thickness direction according to the power law (P-FGM) is used to derived the equation of motion. The frequencies obtained are compared with the natural frequencies of a two-material and those of the base materials.

Keywords: Vibration, FGM, sandwich, natural frequencies.

1. Introduction

Functionally gradient material (FGM) characterized by shifting without interruption property due to a continuous change of the composition, morphology and effectively in the structure, represent a rapid technological leap across the different areas of potential application; research was established to prepare efficient thermal barrier materials for unlimited durable. Their main purpose is to improve the wear resistance or oxidation, have a light armored material with high ballistic efficiency (aerospace shuttles) and can withstand the high temperature environment (aeronautics, terrestrial or turbo-machinery).

Currently, most research activities and developments in structural applications focused primarily on sectors to join two basic components, ceramic and metal. Many benefits are expected in the using of this class of FGM, for example, the face with the content is high ceramic can provide high wear resistance, while the opposite side where the content is high in the metal provides high hardness and strength. Thus, such materials are very desirable for tribological applications where the wear resistance and high hardness are required simultaneously.

The beam is considered as the structural element most responded, as an integral part in most of the construction works or machinery parts [1], which made necessary the study of their static / dynamic behavior. The beams are used as structural component in several construction applications and a large number of studies can be found in the literature about the transverse vibration of the uniform isotropic beams [2]. The importance of developing an analysis of the vibration behavior of the beams is related to the use of the beams as a basic element in achieving the structures, and to determine the effects of changes in boundary conditions, and changes of the material on the natural frequencies and natural modes of vibration.

In the mid 60's, the sandwich construction underwent several searches. For a thorough review of the literature for the analysis of structures sandwiches the researcher should consult articles Plantema [3], Allen [4], Whitney [5], Zenkert [6], Vinson [7]. The methods to analysis sandwich structures and digital solutions for standard problems are grouped in [3,4]. The structural analysis of composite sandwich structures with constant thickness are discussed in [5,7], where they showed the importance of the introduction of the shear flexibility of the heart. More functionally graded materials (FGM) Koizumi [8], Suresh and Mortensen [9], a new generation advanced composite materials primarily homogeneous proposed for thermal barriers [8], have been increasingly applied to the structures of modern engineering in a high temperature environment.

Several researches have been done on the mechanical and thermal behavior of FGM [9], Tanigawa [10]. The simplest FGM, two different materials gradually change from one to the other.

Most of the families of FGM are gradually made of refractory ceramic to metal. Typically, the FGM is constructed from a mixture of ceramic and metal or a combination of different materials. The ceramic in an FGM as a barrier to thermal effects and protects the metal against corrosion and oxidation, and the FGM is hard and reinforced by the metal composition. Currently FGM are developed for general use as structural elements in extremely high temperature environments and different applications. Due to the wide FGM applications, several studies were performed to analyze the behavior and understand the mechanical and mechanisms of FGM structures. Theoretical and experimental depth studies have been conducted and published on the fracture mechanics Bao and Wang [11], Marur [12]. the distribution of thermal stress Williamsson and Drake [13]. Free vibration analysis of FG nano-beams using Ritz method Elmeiche et al. [14]. These FGM structures, beams and shells are always interests researchers for their applications. Approaches such as the use of the deformation of the shear beam theory, the energy method, and the finite element method, were performed.

FGM sandwich can mitigate a concentration of shear stress between the faces because of the gradual change in material properties at the interface Coeur skins. The effects of FGM nucleus have been studied by Venkataraman and Sankar [15] and Anderson [16] on the shear stresses at the skin interface of a beam Coeur sandwich FGM. Pan and Han [17] analyzed the static response of the rectangular plate made of several functionally graded layers, anisotropic, Shen [18] studied two types two types laminated hybrid plates FGM, one with FGM core and piezoelectric skins ceramic and the other is with FGMs skins and piezoelectric ceramic core. There have been also studies dealt with active vibration control of FGM structures the

reader can be referred to Bendine et al. [19], Bendine and Wankhade [20], He et al. [21], Liew et al. [22].

The FGM sandwich is generally made of two types: skin FGM – uniform core and homogeneous skins - FGM core. In the present paper the second type (sandwich beam with homogeneous skin – core FGM) is considered and analyzed to investigate the frequency response of the sandwich FGM beam with different boundaries conditions and volume fraction.

2. Material Properties of the FGM Beam

Consider the case of a sandwich beam FGM with uniform thickness composed of three heterogeneous layers relative to a Cartesian coordinate system (x, y, z) . Upper and lower faces are located at $Z = \pm h/2$ and the sides of the beam are parallel to the x and y axes

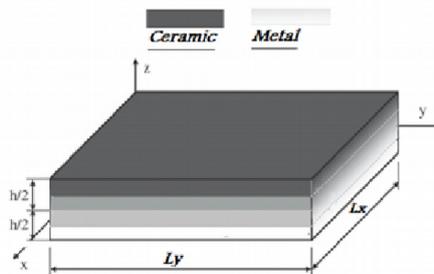


Figure 1 FGM rectangular sandwich beam in Cartesian coordinates

The sandwich beam is composed of three elastic layers, named; layer "1", layer "2" and layer "3" on the front ,lower and the top of the beam. The vertical coordinates of the lower face, the two interfaces, and the upper sides are denoted by $h_1 = -h/2, h_2, h_3,$ and $h_4 = +h/2,$ respectively. The thickness ratio in each layer from bottom to top is denoted briefly by combinations of three digits "1-0-1", "2-1-2" as shown in Fig. 2.

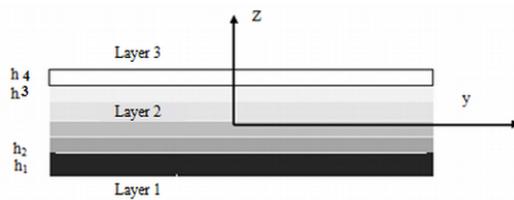


Figure 2 Geometry of FGM rectangular sandwich beam in Cartesian coordinates

The properties of the FGM vary continuously as a function of the volume fraction of the material in the direction of the thickness. A function of power law is commonly used to describe this variation in material properties. The sandwich structures FGMs is discussed as follows.

The volume fraction in the P-FGM is provided by a power law depending on the thickness:

$$V^{(1)} = 0 \quad z \in [h_1, h_2] \tag{1}$$

$$V^{(2)} = \left(\frac{z - h_2}{h_3 - h_2} \right)^p \quad z \in [h_2, h_3] \tag{2}$$

$$V^{(3)} = 1 \quad z \in [h_3, h_4] \tag{3}$$

where $V^{(n)}$, ($n = 1, 2, 3$), denotes a function of the volume fraction of the layer n ; p is the index of the volume fraction ($0 \leq p \leq +\infty$), which indicates the physical change through thickness as displayed in Fig. 3.

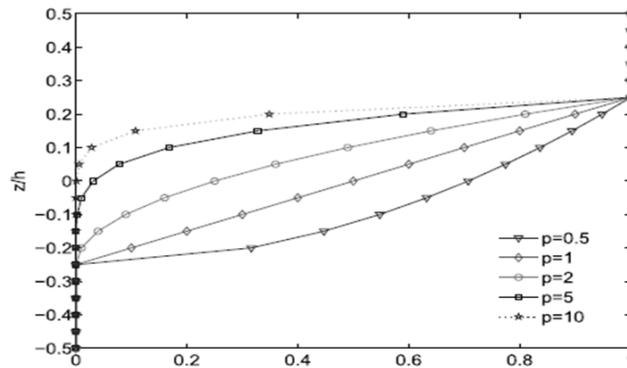


Figure 3 Volume fraction distribution of the sandwich beam according to P-FGM law with different index p

Actual material properties such as Young’s modulus E , and Poisson’s ratio ν and bulk density ρ can be expressed by the law of mixtures Marur [12] as:

$$P^{(n)}(z) = P_2 + (P_1 - P_2) V^{(n)} \tag{4}$$

or $P^{(n)}$ is the actual physical ownership of FGM layer n . P_1 and P_2 are the physical properties of the upper and lower surfaces of the layer 3 and 1, respectively, in this study, it is assumed that the Poisson’s ratio is constant [24].

3. Mathematical formulations

In the present formulation, classical Bernoulli beam theory description of the displacement field is adopted

$$\begin{aligned} u(x, z, t) &= u(x, t) - z \frac{\partial w(x, t)}{\partial x} \\ w(x, z, t) &= w(x, t) \end{aligned} \tag{5}$$

where u , w are the displacement components of the reference plane of the plate, respectively. According to the hooks low, the stress–strain relations for an FGM

beam are given as follows

$$\begin{Bmatrix} \tau_{xx} \\ \tau_{zz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} \frac{E(z)}{1-\nu(z)^2} & 0 & 0 \\ 0 & \frac{E(z)}{1-\nu(z)^2} & 0 \\ 0 & 0 & \frac{E(z)}{2(1+\nu(z))} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \gamma_{xz} \end{Bmatrix} \quad (6)$$

where $\tau_{xx}, \tau_{zz}, \tau_{xz}$ and $\varepsilon_{xx}, \varepsilon_{zz}, \gamma_{xz}$ are the stress and strain components, respectively. The strain–displacement relations are given by

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u(x,z,t)}{\partial x} = \frac{\partial u(x,t)}{\partial x} - z \frac{\partial^2 w(x,t)}{\partial x^2} \\ \varepsilon_{zz} &= \frac{\partial w(x,z,t)}{\partial z} = \frac{\partial w(x,t)}{\partial z} \\ \gamma_{xz} &= \frac{\partial u(x,z,t)}{\partial z} + \frac{\partial w(x,z,t)}{\partial x} \end{aligned} \quad (7)$$

Using the virtual work principle, the following motion equations of the FGM beam can be obtained.

$$A_{11} \frac{\partial^2 u_0(x,t)}{\partial x^2} - B_{11} \frac{\partial^3 w(x,t)}{\partial x^3} = 0 \quad (8a)$$

$$B_{11} \frac{\partial^3 u_0(x,t)}{\partial x^3} - D_{11} \frac{\partial^4 w(x,t)}{\partial x^4} + I_1 \ddot{w}(x,t) = 0 \quad (8b)$$

where the following definitions apply

$$(A_{11}, B_{11}, D_{11}) = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \frac{E(z)}{1-\nu(z)^2} (1, z, z^2) dz \quad (9)$$

$$I_1 = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \rho(z) dz \quad (10)$$

where A_{11}, B_{11}, D_{11} denoted as the FGM beam rigidities and I_1 is the density. The general solution of the dynamic Eqs. (8a) and (8b) is given in the following form:

$$w(x,t) = w_n(x) \cdot e^{i_n t} \quad (11)$$

where w_n is the natural frequency and w_n is the mode shape of the FGM beam

$$w_n(\tau) = A_1 \cos(\beta_n L \tau) + A_2 \sin(\beta_n L \tau) + A_3 \cosh(\beta_n L \tau) + A_4 \sinh(\beta_n L \tau)$$

where

$$\tau = \frac{x}{L} \tau \in [0, 1] \quad (12)$$

4. Numerical Results and Discussion

Considering the case of a homogeneous rigid FGM heart and skin wherein the Young’s modulus and mass density of the layer "1" are $E_c = 380$ GPa and $\rho_c = 3800$ kg/m³ (P1, Alumina) in the upper face and $E_M = 70$ Gpa, $\rho_m = 2708$ kg/m³ (P2, Aluminum) in the underside. For various boundary conditions we obtain results shown in Tab. 1.

Table 1 The variation of non dimensional frequencies for different boundary conditions with the power low index

Boundary condition	Type	$\bar{\omega}_1$	$p = 0$	$p = 0.5$	$p = 1$	$p = 2$	$p = 5$	$p = 10$
C-C	0-1-0	Exact solution (present)	12.4302	10.5840	9.5685	8.7317	8.3138	8.0677
		M. Simsek	12.4142	10.5713	9.5554	8.7186	8.3006	8.0556
		Nguyen et al. [25]	12.2243	10.4269	9.4319	8.5977	8.1446	7.8860
	1-2-1	Exact solution (present)	9.6507	9.0283	8.7312	8.5100	8.4430	8.4771
		Nguyen et al. [25]	9.5451	8.9243	8.6264	8.3959	8.3047	8.3205
		---	---	---	---	---	---	---
C-S	0-1-0	Exact solution (present)	8.5661	7.2938	6.5940	6.0173	5.7293	5.5597
		---	---	---	---	---	---	---
		---	---	---	---	---	---	---
	1-2-1	Exact solution (present)	6.6507	6.2217	6.0170	5.8645	5.8183	5.8418
		---	---	---	---	---	---	---
		---	---	---	---	---	---	---
S-S	0-1-0	Exact solution (present)	5.4834	4.6690	4.2210	3.8518	3.6675	3.5589
		M. Simsek [24]	5.4777	4.6646	4.2163	3.8472	3.6628	3.5547
	1-2-1	Nguyen et al. [25]	5.4603	4.6506	4.2051	3.8361	3.6485	3.5390
		Exact solution (present)	4.2573	3.9827	3.8516	3.7541	3.7245	3.7395
		Nguyen et al. [25]	4.2445	3.9695	3.8387	3.7402	3.7081	3.7214
C-F	0-1-0	Exact solution (present)	1.9534	1.6633	1.5037	1.3722	1.3065	1.2679
		M. Simsek [24]	1.9525	1.6627	1.5029	1.3714	1.3057	1.2671
		Nguyen et al. [25]	1.9496	1.6602	1.5011	1.3696	1.3034	1.2646
	1-2-1	Exact solution (present)	1.5166	1.4188	1.3721	1.3374	1.3268	1.3322
		Nguyen et al. [25]	1.5145	1.4165	1.3700	1.3350	1.3241	1.3292

Table 2 The natural frequencies for clamped-clamped (C-C) beam with the power low index

p	$\bar{\omega}_n$	2-1-2	1-1-1	1-2-1	1-3-1	1-8-1	0-1-0
0.1	$\bar{\omega}_1$	29.425	29.635	30.048	30.216	29.763	26.368
	$\bar{\omega}_2$	81.112	81.707	82.828	83.291	82.042	72.684
	$\bar{\omega}_3$	153.01	160.14	162.37	163.28	160.83	142.49
1	$\bar{\omega}_1$	29.807	30.178	30.869	31.381	32.558	33.380
	$\bar{\omega}_2$	82.164	83.160	85.093	86.504	89.750	93.254
	$\bar{\omega}_3$	161.07	163.02	166.81	169.58	175.94	182.82
10	$\bar{\omega}_1$	30.488	31.466	33.217	34.484	37.500	41.073
	$\bar{\omega}_2$	84.043	86.739	91.564	95.059	103.37	113.22
	$\bar{\omega}_3$	164.75	170.04	179.50	186.35	202.64	221.95

Table 3 The natural frequencies for clamped-supported (C-S) with the power low index

p	$\bar{\omega}_n$	2-1-2	1-1-1	1-2-1	1-3-1	1-8-1	0-1-0
0.1	$\bar{\omega}_1$	12.980	13.072	13.255	13.329	13.129	11.631
	$\bar{\omega}_2$	51.922	52.291	53.020	53.317	52.517	46.527
	$\bar{\omega}_3$	116.82	117.65	119.29	119.96	118.16	104.69
1	$\bar{\omega}_1$	13.148	13.308	13.617	13.843	14.362	14.922
	$\bar{\omega}_2$	52.595	53.233	54.470	55.374	57.451	59.694
	$\bar{\omega}_3$	118.34	119.77	122.55	124.59	129.26	134.31
10	$\bar{\omega}_1$	13.449	13.881	14.653	15.212	16.542	18.119
	$\bar{\omega}_2$	53.798	55.524	58.612	60.849	66.170	72.744
	$\bar{\omega}_3$	121.04	124.92	131.87	136.91	148.88	163.07

Table 4 The natural frequencies for supported-supported (S-S) beam with the power low index

p	$\bar{\omega}_n$	2-1-2	1-1-1	1-2-1	1-3-1	1-8-1	0-1-0
0.1	$\bar{\omega}_1$	20.278	20.422	20.707	20.510	20.510	18.171
	$\bar{\omega}_2$	65.713	66.181	67.104	67.499	66.467	58.886
	$\bar{\omega}_3$	137.10	138.08	140.00	140.79	138.67	122.86
1	$\bar{\omega}_1$	20.541	20.790	21.273	21.626	22.437	23.314
	$\bar{\omega}_2$	66.566	67.373	68.939	70.082	72.711	75.551
	$\bar{\omega}_3$	138.88	140.56	143.83	146.22	151.70	157.63
10	$\bar{\omega}_1$	21.010	21.684	22.891	23.764	25.842	28.305
	$\bar{\omega}_2$	68.088	70.272	74.181	77.013	83.746	91.725
	$\bar{\omega}_3$	142.06	146.61	154.77	160.68	174.73	191.38

Table 5 The natural frequencies for clamped-free (C-F) with the power low index

p	$\bar{\omega}_n$	2-1-2	1-1-1	1-2-1	1-3-1	1-8-1	0-1-0
0.1	$\bar{\omega}_1$	4.624	4.657	4.722	4.748	4.677	4.144
	$\bar{\omega}_2$	28.979	29.186	29.593	29.758	29.312	25.969
	$\bar{\omega}_3$	81.144	81.721	82.861	83.324	82.075	72.713
1	$\bar{\omega}_1$	4.684	4.741	4.851	4.931	5.116	5.316
	$\bar{\omega}_2$	29.355	29.711	30.402	30.906	32.065	33.318
	$\bar{\omega}_3$	82.196	83.193	85.127	86.539	89.785	93.291
10	$\bar{\omega}_1$	4.791	4.945	5.220	5.419	5.893	6.455
	$\bar{\omega}_2$	30.026	30.990	32.714	33.962	36.932	40.451
	$\bar{\omega}_3$	84.076	86.773	91.600	95.096	103.41	113.26

For checking the accuracy of the present method, the non dimensional frequencies of the sandwich FGM beam with different boundary condition and power low index were compared with those obtained in [24,25]. The results are depicted in Tab. 1. It can be noticed that the results were in a good agreement which demonstrated the precision of our model.

Tables 2–5 show the variation of the first three natural frequencies of four boundary condition (C–C, S–S, C–S and C–F) of the beam with the power-law exponents (0.1, 1 and 10) for verses materials combinations. The first observation is that the natural frequencies is proportional to the power low index p , that’s means that the frequencies of the structure are important when it’s have a higher ceramic ratio; it can be also remarked that the thickness ratio getting more significant when the variation of the natural frequencies is in higher vibration modes.

The natural frequencies of the sandwich beam versus the FGM thickness ratio for the power low index 0.1, 1 and 10 are detected in Fig 4 to 6, respectively, its noticed that the variation ratio of frequency under clamped-clamped boundary

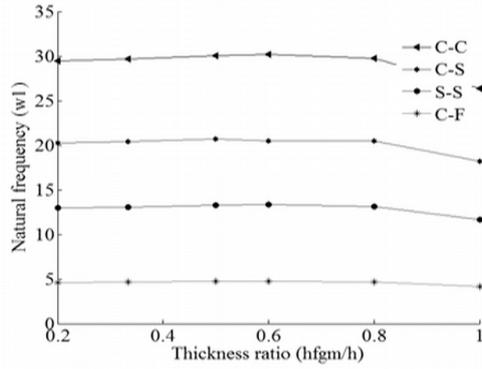


Figure 4 Natural frequencies of a sandwich beam for $p = 0.1$

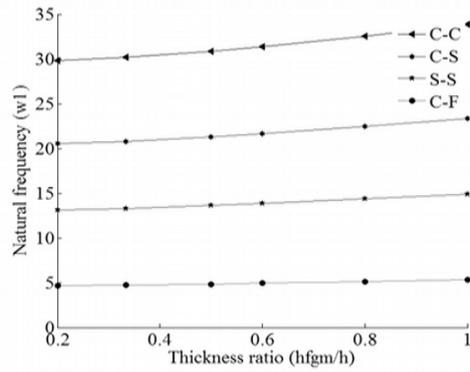


Figure 5 Natural frequencies of a sandwich beam for $p = 1$

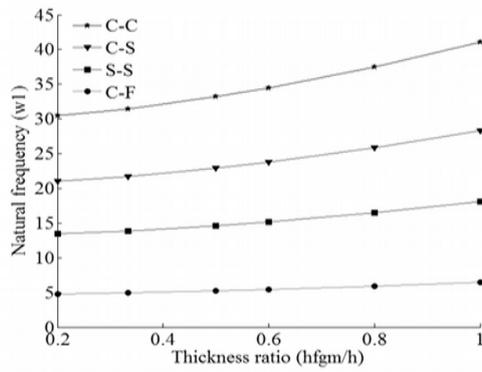


Figure 6 Natural frequencies of a sandwich beam for $p = 10$

conditions is important to the other sets (C-S, S-S and C-F), This importance is lessened when the stiffness on the beam decreases. In the same figures, it is found that for the weak voluminal fraction index ($p < 1$), natural frequencies are greater when the thickness of the FGM layer (h -FGM) reaches the margins of 60% in the total thickness (h -total) and low when the h -FGM equal 100% of h -total, This reduction is rapid when the thickness ratio between 0.8 and 1. For power low index superior $p \geq 1$, the natural frequencies are maximal when the beam thickness is purely FGM and minimal when h -FGM assumes a low values, it is well observed in the Figs. 5 and 6 that the natural frequencies is proportional to the FGM thickness and take more important value when the FGM thickness is superior.

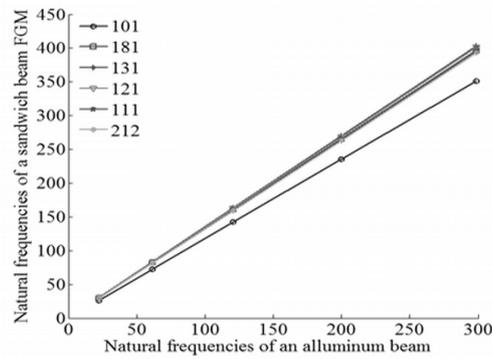


Figure 7 Natural frequencies of a clamped - clamped In sandwich beam compared to the natural frequencies of an aluminum beam for $p = 0.1$

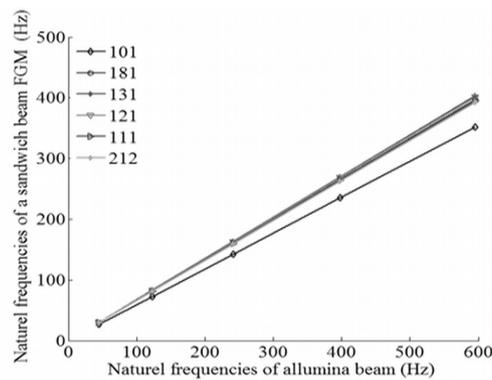


Figure 8 Natural frequencies of a clamped - clamped In sandwich beam compared to the natural frequencies of an alumina beam for $p = 0.1$

Figures 7 and 8 show that the natural frequencies of a sandwich beam FGM (for all the boundary conditions) are proportional to the natural frequency of the base materials (ceramics, metal). The plot of the beam with combination "0-1-0" has the smallest frequencies value while that corresponding to "1-3-1" has the largest value.

5. Conclusion

In this paper, the free vibrations of sandwich beams FGM assuming that the material properties vary depending on the thickness with a power distribution (P-FGM) is examined. The main purpose of this work is to study the thickness influence of the sandwich FGM beams skin on the frequencies for different volume fraction. The exact solution has been presented with various boundary conditions. The variation in the volume fractions of the FGM materials on the natural frequencies is also studied and compared with the natural frequencies of a two-material and those of the base materials.

References

- [1] **Kessas, S.:** *Etude de l'effet de cisaillement transverse sur le comportement des poutres composites*, Thèse de Magister en génie civil, Université Mentouri de Constantine, Algerie, **2010**.
- [2] **Gorman, D.J.:** *Free vibration analysis of beams and shafts*, John Wiley&Sons, **1975**.
- [3] **Plantema, F.J.:** *Sandwich Construction, the Bending and Buckling of Sandwich Beam, Plates and Shells*, John Wiley and Sons, New York, **1966**.
- [4] **Allen, H.G.:** *Analysis and Design of Structural Sandwich Panels*, Pergamon Press, Oxford, **1969**.
- [5] **Whitsny, J.M.:** *Structural Analysis of Laminated Anisotropic Plates*, Technomic, Lancaster, **1987**.
- [6] **Zenkert, D.:** *An Introduction to Sandwich Construction*, Chameleon Press, London, **1995**.
- [7] **Vinston, J.R.:** *The behavior of sandwich structures of isotropic and composite materials*, Technomic, Lancaster, **1999**.
- [8] **Koizumi, M.:** The concept of FGM functionally gradient materials, *Ceramic Transactions*, **34**(1), 3–10, **1993**.
- [9] **Suresh, S., Mortensen, A.:** *Fundamentals of functionally graded materials*, IOM Communications, London, **1998**.
- [10] **Tanigawa, Y.:** Some basic thermo-elastic problems for nonhomogeneous structural materials, *Applied Mechanics Reviews*, **48**, 287–300, **1995**.
- [11] **Bao, G. and Wang, L.:** Multiple cracking in functionally graded ceramic/metal coatings, *International Journal of Solids and Structures*, **32**(19), 2853–2871, **1995**.
- [12] **Marur, P.R.:** textitFracture behaviour of functionally graded materials, Ph. D. dissertation, Auburn University, Alabama, **1999**.
- [13] **Williamsson, R. and Drake, J.T.:** Finite element analyses of thermal residual stresses at graded ceramic-metal interfaces—part I: model description and geometrical effects, *Journal of Applied Physics*, **74**(2), 1310–1320, **1995**.
- [14] **Elmeiche, A., Megueni, A., Lousdad, A.:** Free vibration analysis of functionally graded Nanobeams based on different order beam theories using Ritz method, *Periodica Polytechnica Mechanical Engineering*, Online First **2016**, DOI: 10.3311/PPme.8707.
- [15] **Venkataraman, S. and Sankar, B.V.:** Analysis of sandwich beams with functionally graded core, *Proceedings of the 42nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Seattle, AIAA*, 1281, 16–19, **2001**.

- [16] **Anderson, T.A.:** A 3-D elasticity solution for a sandwich composite with functionally graded core subjected to transverse loading by a rigid sphere, *Composite Structures*, **60**(3), 265–274, **2003**.
- [17] **Pan, E. and Han, F.:** Exact solution for functionally graded and layered magneto-electro-elastic plates, *International Journal of Engineering Science*, **43**(3-4), 321–339, **2005**.
- [18] **Shen, H.S.:** Postbuckling of FGM plates with piezoelectric actuators under thermo-electromechanical loadings, *International Journal of Solids and Structures*, **42**(23), 6101–6121, **2005**.
- [19] **Bendine, K., Boukhoulda, B.F., Nouari, M. and Satla, Z.:** Structural modeling and active vibration control of smart FGM plate through ANSYS, *International Journal of Computational Methods*, 1750042, **2016**.
- [20] **Bendine, K. and Wankhade, R.L.:** Vibration control of FGM piezoelectric plate based on LQR genetic search, *Open Journal of Civil Engineering*, **6**(1), **2016**.
- [21] **He, X.Q., Ng, T.Y., Sivashanker, S. and Liew, K.M.:** Active control of FGM plates with integrated piezoelectric sensors and actuators, *International Journal of Solids and Structures*, **38**, 1641–1655, **2001**.
- [22] **Liew, K.M., He, X.Q., Ng, T.Y. and Sivashanker, S.:** Active Control of FGM Plates Subjected to a temperature gradient: Modeling via finite element method based on FSDT, *International Journal for Numerical Methods in Engineering*, **521**, 253–271, **2001**.
- [23] **Delale, F., Erdogan, F.:** The crack problem for a non homogeneous plane, *ASME Journal of Applied Mechanics*, **50**, 609–614, **1983**.
- [24] **Simsek, M.:** Fundamental frequency analysis of functionally graded beams by using different higher-order beam theories, *International Journal of Nuclear Engineering and Design*, **240**, 697–705, **2010**.
- [25] **Nguyen, T.K., Nguyen, T.T.P. and Vo, T.:** Vibration and buckling analysis of functionally graded sandwich beams by a new higher-order shear deformation theory, *International Journal of Composites, Part B*, **76**, 273–285, **2015**.

