Mechanics and Mechanical Engineering
Vol. 22, No. 4 (2018) 875–883

https://doi.org/10.2478/mme-2018-0068

Friction Compensation in the Inverted Pendulum Controller by Means of a Neural Network

Marek Balcerzak
Division of Dynamics, Lodz University of Technology
ul. Stefanowskiego 1/15, 90-924 Lodz, Poland
marek.balcerzak@dokt.p.lodz.pl

Received (1 December 2017)
Revised (6 May 2018)
Accepted (25 June 2018)

This paper presents an experimental confirmation of the novel method of friction modelling and compensation. The method has been applied to an inverted pendulum control system. The practical procedure of data acquisition and processing has been described. Training of the neural network friction model has been covered. Application of the obtained model has been presented. The main asset of the presented model is its correctness in a wide range of relative velocities. Moreover, the model is relatively easy to build.

Keywords: friction model, neural network, inverted pendulum, friction compensation.

1. Introduction

This paper presents a novel approach to friction modelling and compensation. Friction modelling is an important issue in analysis and simulation of mechanical systems. High-quality friction models leads to minimization of a discrepancy between simulations and real behavior of mechanical systems. Different friction models have been proposed [1]. They differ by complexity and range of applications.

However, a problem arises when friction characteristics differ depending on the range of velocities. For example, if small velocities are analyzed, the Coulomb friction may play a crucial role. For bigger velocities, the linear (viscous friction) or quadratic (air drag) dependences between friction function and relative speed may appear. Different behaviors are also possible. This is particularly important in control systems [2] that have to operate in wide range of velocities.

This paper presents a novel approach to friction modelling and compensation. The obtained friction model has been applied to the inverted pendulum control system. The data acquisition procedure has been described. Important issues concerning data processing have been depicted: low-pass filtering [3], differentiation by B-splines [4] fit with continuous derivatives, building the experimentally-obtained friction characteristic. A neural network was trained to “remember” the characteristic [5]. In such a manner, a neural friction model has been obtained. A great asset of this model is its correctness in a wide range of relative velocities.

2. Building the Friction Model

In order to create a successful friction compensator, an accurate friction model has to be created. In this paper, an artificial neural network (ANN) [5] has been used for modelling. The purpose of the ANN is to “learn” the friction characteristics. It has been assumed that the pendulum is a uniform, rigid bar of mass M and length l and the friction torque depends on the angular velocity of the pendulum only. The data used to train the neural network has been obtained from free vibrations. A scheme of free vibrations measurement is presented in the Fig. 1.

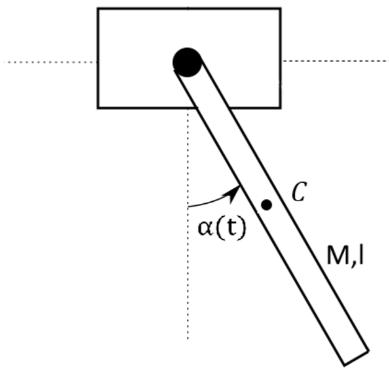

Figure 1 Measurement of pendulum’s bar free vibrations

Free vibrations of the pendulum are described by the differential equation (1):

$$\ddot{\alpha}(t) = -\frac{3g}{2l}\sin(\alpha(t)) - \frac{3\tau(\dot{\alpha}(t))}{Ml^2}\,, \tag{1}$$

where τ is the friction torque. The equation (1) implies that in order to obtain the friction characteristics it is necessary to register the pendulum angle, angular velocity and angular acceleration simultaneously. The data has been obtained by registration of free vibrations with various initial amplitudes. In order to reduce the noise that appear due to discretization of the signal from the incremental encoder, the registered data has been processed by means of the low-pass Butterworth filter [3]. After filtering, the saved motions $\alpha(t)$ have been interpolated by means of B-splines [4]. Continuity of the first and the second derivative has been maintained in the interpolation process. Therefore, derivatives of $\alpha(t)$ can be obtained directly from interpolated curves. The process of data acquisition is presented in Figs. 2–4. Fig. 2 presents an exemplary registered motion (the dotted line) and the corresponding data after passing the low-pass filter (the solid line). It can be noticed that no significant distortions appear due to filtering. Fig. 3 presents derivatives of the motion obtained from B-splines. Fig. 4 shows the data from the Fig. 3 after transformation to the $(\dot{\alpha} - \ddot{\alpha})$ plane.

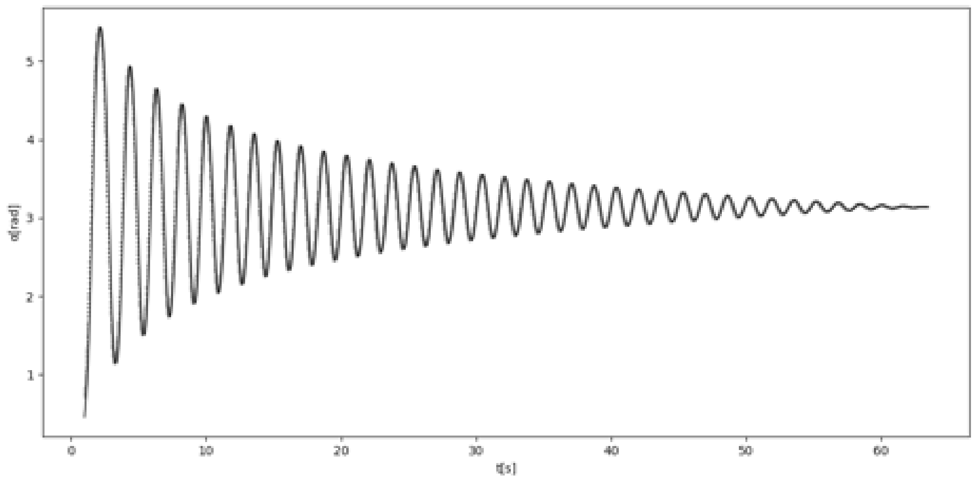

Figure 2 The recorded motion (grey dotted line) and the data after filtering (black solid line)

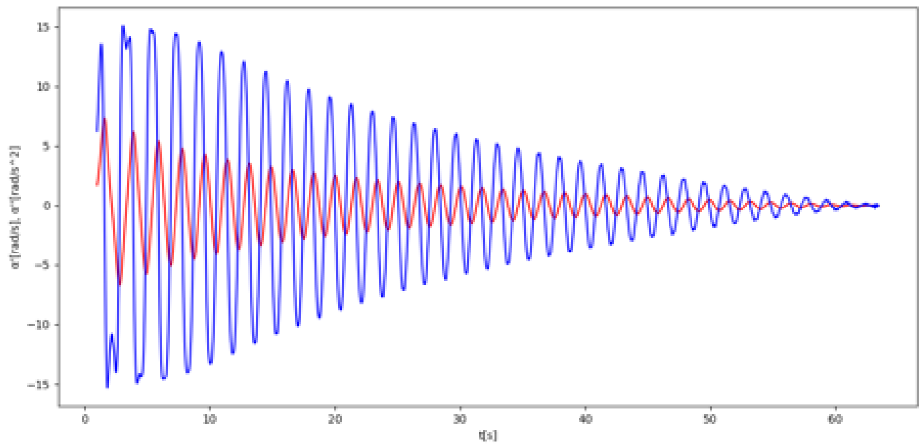

Figure 3 Derivatives of the recorded function $\alpha(t)$ – the first derivative in red, the second derivative in blue

Free vibrations of the pendulum with different initial amplitudes have been recorded and processed in the same manner. From all the obtained data, a set of points $\big(\dot{\alpha}, 3\tau(\dot{\alpha}(t))\big/Ml^2\big)$ satisfying the equation (1) has been received. This set of points, which represents an experimentally obtained friction characteristic, is presented in the Fig. 5.

The task of the neural network is to provide a curve that represents the set presented in the Fig. 5 as closely as possible. Due to specification of particular application, a neural network with one hidden layer that contains neurons with sigmoidal activation function and one output layer that includes a neuron with linear activation function, has been chosen. Then, training of neural networks has

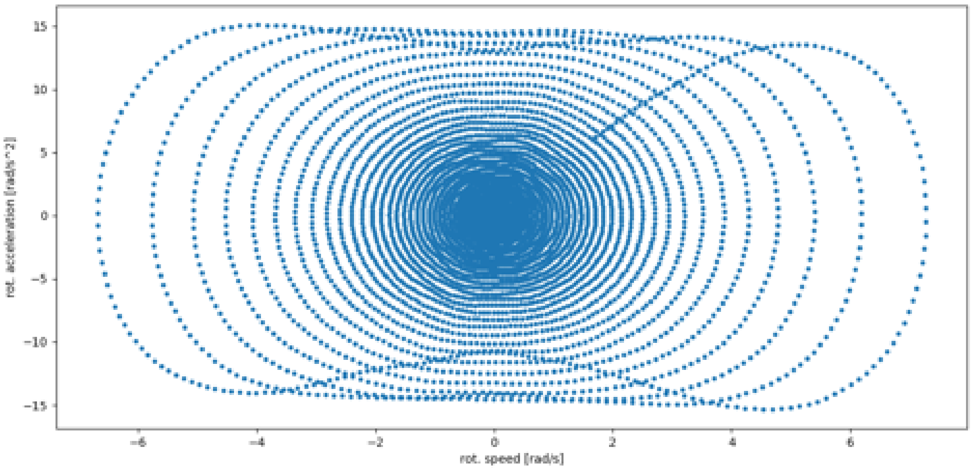

Figure 4 Angular velocity and angular acceleration data after transformation to the ($\dot{\alpha}$- $\ddot{\alpha}$) plane

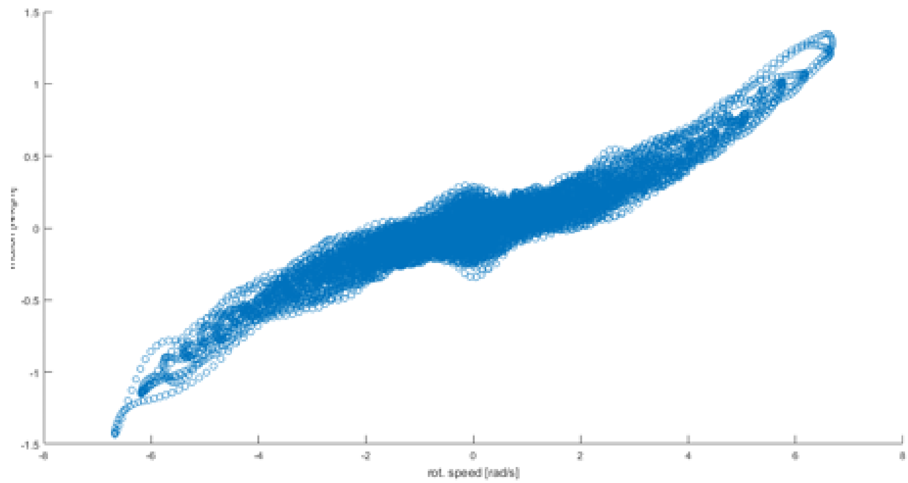

Figure 5 The experimentally-obtained friction characteristic

been performed. ANNs with 3, 5, 7, 9 and 10 neurons in the hidden layer have been tested. During learning, the Levenberg-Marquardt algorithm [5] has been used. Values of the mean squared fit error for different numbers of neurons in the hidden layer are presented in the Fig. 6.

The graph from the Fig. 6 shows that although the highest quality of fit is obtained for 10 neurons in the hidden layer, only 3 neurons in the hidden layer assure only slightly worse accuracy of the fit. Comparison of fitted curves obtained from networks with 3 neurons and with 10 neurons is presented in the Fig. 7. It can be noticed that the curves almost overlap.

In such a way, a neural-network-based friction model has been obtained. Unlike

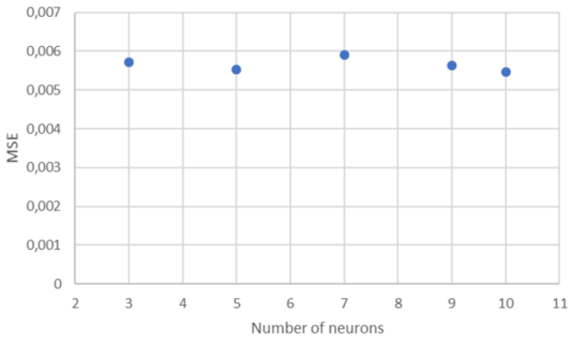

Figure 6 The mean squared fit error depending on the number of neurons

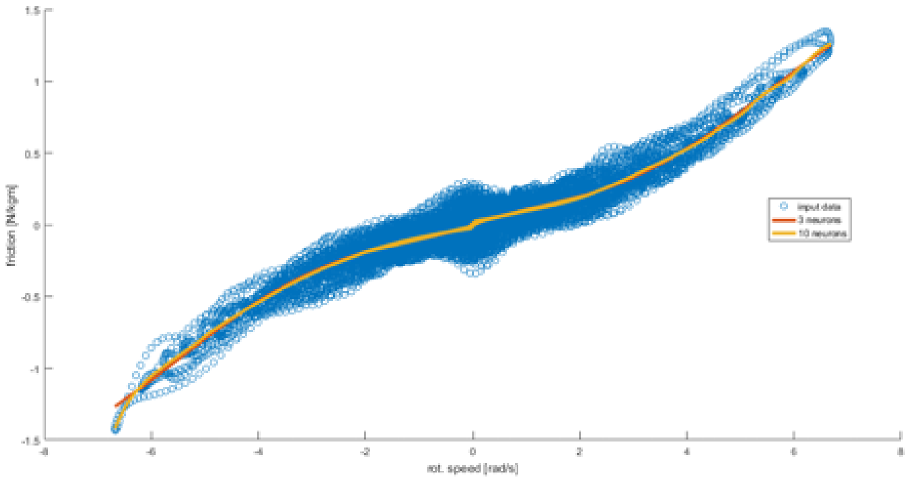

Figure 7 The mean squared fit error depending on the number of neurons

simple friction models [1], the presented model is valid for a large range of relative velocities. However, its complexity is significant.

3. The Friction Compensator

The obtained friction model can be applied as the friction compensator in an inverted pendulum control system. The inverted pendulum is a kind of pendulum in which the axis of rotation is fixed to a cart (Fig. 8). The cart is able to move along the horizontal axis x in a controlled way. The fundamental problem of the inverted pendulum is to find such a control of the cart that keeps the pendulum's bar in the vicinity of the upright vertical position $\alpha(t) = 0$ even if external disturbances appear.

It has been assumed that the pendulum's drive is velocity-controlled. It means that the control signal $u(t)$ supplied to the drive is equal to the desired velocity of the cart. If the drive is stiff enough, then the motion of the pendulum's bar does not

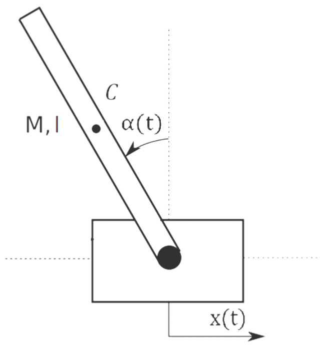

Figure 8 Sketch of the considered control object – the inverted pendulum

influence position of the cart $x(t)$. Providing that the drive can be approximated by a linear differential equation of the first order, the dependence between acceleration of the cart and the control signal is as follows:

$$\ddot{x}(t) = a[u(t) - \dot{x}(t)]\,, \tag{2}$$

where $u(t)$ is the control signal and a is a drive constant, which can be determined in the identification process.

The equation of motion of the inverted pendulum can be easily derived using Lagrange approach [6]. Assume that the pendulum's bar is uniform, its mass center C is in the middle of its length and it is loaded by a friction torque τ. Then, the following equation of motion (3) is obtained:

$$\ddot{\alpha}(t) = \frac{3g}{2l}\sin(\alpha(t)) + \frac{3\ddot{x}(t)}{2l}\cos(\alpha(t)) - \frac{3\tau(\dot{\alpha}(t))}{Ml^2}\,, \tag{3}$$

where l is the length of the bar. Equations (2) and (3) constitute a complete mathematical description of the inverted pendulum. After inserting the equation (2) into (3), acceleration of the cart is removed from (3) and acceleration of the bar depends directly on the control signal (4):

$$\ddot{\alpha}(t) = \frac{3g}{2l}\sin(\alpha(t)) + \frac{3a}{2l}[u(t) - \dot{x}(t)]\cos(\alpha(t)) - \frac{3\tau(\dot{\alpha}(t))}{Ml^2}\,. \tag{4}$$

Assume that initially the pendulum is governed by a linear LQR controller [2]. Then, the control signal is calculated according to the formula (5):

$$u(t) = [k_1, k_2, k_3, k_4]^T \cdot \mathbf{x}(t) = \mathbf{k} \cdot \mathbf{x}(t)\,, \tag{5}$$

where $\mathbf{k} = [k_1, k_2, k_3, k_4]^T$ is the vector of controller parameters and $k_1, \ldots, k_4$ are constants to be determined from the control object linearized model using the Riccati equation [2]. Let the compensated control $u^*(t)$ be defined according to the equation (6):

$$u^*(t) = \frac{3a\,[\dot{x}\,[\cos(\alpha) - 1] + u(t)] + 3g\,[\alpha - \sin(\alpha)] + 2\tau(\dot{\alpha})/Ml}{3a\cos(\alpha)}\,. \tag{6}$$

Obviously, the compensated control (6) can be calculated only if the friction characteristic $\tau()$ is known – here the neural model from the previous chapter is applied. Inserting the control $u^*(t)$ in the place of $u(t)$ in the equation (4) yields the linear model of an inverted pendulum (7):

$$\ddot{\alpha}(t) = \frac{3g}{2l}\alpha(t) + \frac{3a}{2l}[u(t) - \dot{x}(t)] \,. \tag{7}$$

Therefore, the control $u^*(t)$ makes the system behave as if the control object was linear.

4. Numerical Test

The control system defined by the equations (2), (4) has been simulated with both: standard control (5) and the compensated control (6). The following values of parameters have been used: $a = 19.72688$, $g = 9.81$, $l = 1.0$. The parameter M was redundant due to the fact that the friction torque value obtained in modelling process was already divided by the moment of inertia of the bar. The following initial conditions were applied: $\alpha(0) = 0.3$, $\dot{\alpha}(0) = 1.0$. The parameters of the LQR controller have been obtained for state weights $\mathbf{Q} = [2.0, 1.0, 1.0, 1.0]$ and the input weight $\mathbf{R} = [1.0]$. The comparison between stabilization of the pendulum with the compensator and without compensator is depicted in the Fig. 9. Figure 10 presents a zoom of the Fig. 9.

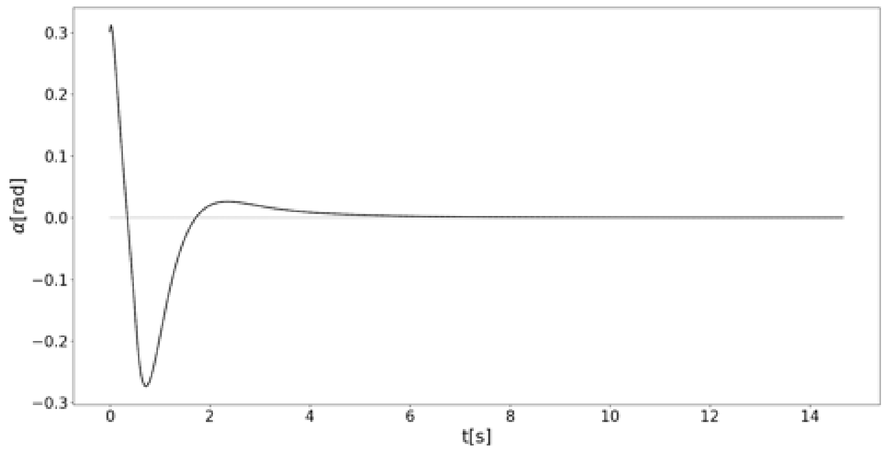

Figure 9 The comparison between stabilization of the pendulum with the compensator (dashed line) and without compensator (solid line) – numerical simulation

It can be noticed that application of the friction compensator results in slightly smaller overshoot in the system. However, the difference is minor. The test has been repeated on the real inverted pendulum laboratory stand. The recorded graph of $\alpha(t)$ is presented in the Fig. 11.

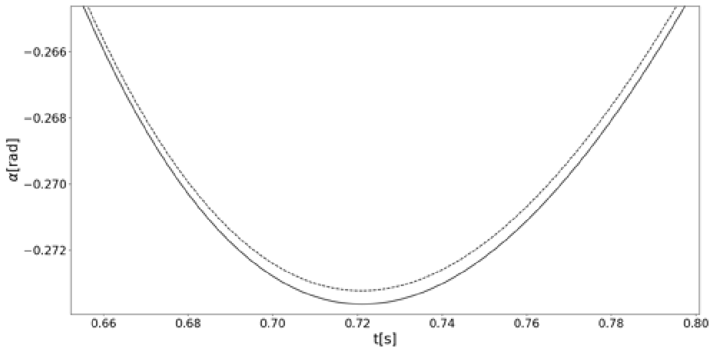

Figure 10 Zoom of Fig. 9

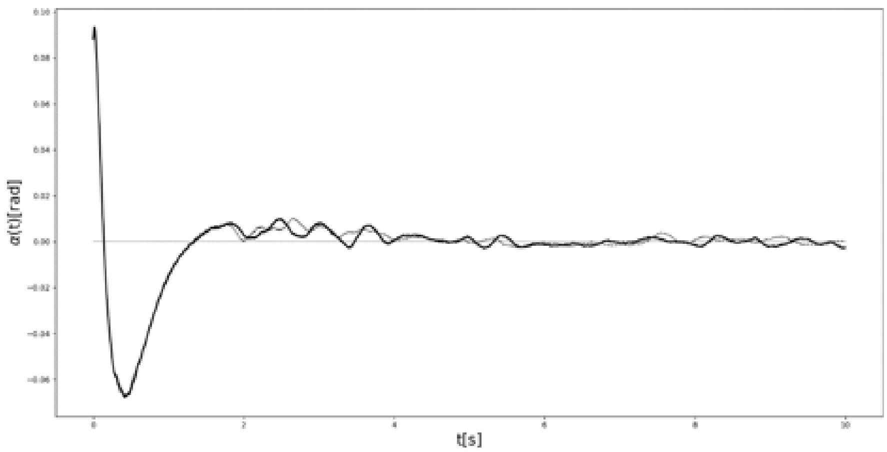

Figure 11 The comparison between stabilization of the pendulum with the compensator (dashed line) and without compensator (solid line) – test on the laboratory stand

5. Conclusions

The paper presents a novel approach to friction modelling and compensation with application to the inverted pendulum control system. The data acquisition procedure has been described. Important issues concerning data processing have been depicted: low-pass filtering, differentiation by B-splines fit with continuous derivatives, building the experimentally-obtained friction characteristic. A neural network was trained to "remember" the characteristic. In such a manner, a neural friction model has been obtained. A great asset of this model is its correctness in a wide range of relative velocities.

The friction model has been applied to compensate friction of an inverted pendulum system. The control system with compensator works properly. However, due to relatively small values of friction in the system, improvement in operation is minor. Nevertheless, such approach may be very useful for systems in which the friction plays crucial role.

6. Acknowledgements

This study has been supported by Polish Ministry of Science and Higher Education under the program "Diamond Grant", project no. D/2013 019743.

This study has been supported by Polish National Centre of Science (NCN) under the program MAESTRO: "Multi-scale modelling of hysteretic and synchronous effects in dry friction process", project no. 2012/06/A/ST8/00356.

References

[1] **Marques, F., Flores, P., Claro, J. P., & Lankarani, H.M.**: A survey and comparison of several friction force models for dynamic analysis of multibody mechanical systems, *Nonlinear Dynamics*, 86(3), **2016**, 1407-1443.

[2] **Ogata, K.**: *Modern control engineering,* Prentice Hall, **2002**.

[3] **Zhongshen, L.**: The design of butterworth lowpass filter based on MATLAB [J], *Heilongjiang Electronic Technology,* **3**, **2003**, 017.

[4] **Salomon, D.**: *Curves and Surfaces for Computer Graphics*, Springer, **2006**.

[5] **Hagan, M. T., & Menhaj, M. B.**: Training feed-forward networks with the Marquardt algorithm, *IEEE transactions on Neural Networks*, **5**(6), **1994**, 989-993.

[6] **Taylor, J.R.**: *Classical mechanics*, University Science Books, **2005**.