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Mathematical Modeling and Analysis of Thermostressed State of Bimetallic Plate Under Electromagnetic Action in the Mode with Pulse Modulated Signal

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The thermostressed state of a bimetallic plate under the electromagnetic action in the mode with pulse modulated signal (MPMS) at the resonant frequencies of the electromagnetic field is studied. There are established the critical values of the parameters of the electromagnetic action at which the bimetallic plate loses its load-carrying ability as a constituent element.

Keywords: thermostressed state, electromagnetic pulsed action, bimetallic plate, load-carrying ability.

1. Introduction

Bimetallic plates are widely used as structural elements in appliances and devices of modern science and engineering [1], as well as electromagnetic adapters and screens. During their operation and utilization, these devices are exposed to the influence of fields of different physical nature, including pulsed electromagnetic fields (EMF), see [2–4]. Under the action of pulsed EMF, in the bimetallic plates the temperature fields and stresses arise. When the value of the stress intensities $\sigma_i^{(n)}$ in the constituent n $(n = 1, 2)$ layers reaches the limit of elasticity $\sigma_d^{(n)}$ of the materials of the component layers, the bimetallic plate loses its efficiency as a structural element. The aim of this work is to study the influence of pulsed EMF in the mode with pulse modulated signal (MPMS), when the frequency ω of carrying

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electromagnetic oscillations takes the value of the resonant frequencies ω_{rj} $(j = 1, 2)$ of pulsed EMF, which are equal to a half of the value of the natural frequencies ω_{vj} of mechanical vibrations of the bimetallic plate, i.e. $\omega_{rj} \approx 1/2\omega_{vj}$ [5,6].

The thermomechanical behavior of the bimetallic plate under the electromagnetic action in the MPMS at the frequencies of electromagnetic field $\omega \neq \omega_{rj}$ is investigated in [7,8], where it is established that for such frequencies the efficiency of bimetallic plates is saved.

2. Formulation of the Problem

We consider a bimetallic plate of the constant thickness, which is infinite by the coordinates x, y of a rectangular Cartesian coordinate system, whose coordinate plane XOY coincides with the plane $z = 0$ of the connection of the component layers of the thicknesses h_1 and h_2 respectively. The bimetallic plate is subjected to the influence of the external nonstationary EMF given by the values $H_y^{\pm}(t)$ of the tangent to bases of the layer $z = -h_1$, $z = h_2$ component $H_y^{(n)}(z,t)$ of the magnetic field strength vector in the n-th $(n = 1, 2)$ composite layer of the plate. The surfaces of the bimetallic plate are in conditions of convective heat exchange with the environment and free of power load.

On the contact surface of the composite layers the conditions of ideal electromagnetic, thermal and mechanical contacts are fulfilled [1]. Under these conditions, the influence of nonstationary EMF is manifested by two physical factors – Joule heat $Q^{(n)} = rot^2 \vec{H}^{(n)} \big/ \sigma_n$ and the ponderomotive force $\vec{F}^{(n)} = \mu_n \, rot \, \vec{H}^{(n)} \times \vec{H}^{(n)}$, where σ_n, μ_n are the heat-conductivity factor and the magnetic permeability of the n-th layer. These two physical factors induce the nonstationary temperature $T^{(n)}$ and the stress tensor $\hat{\sigma}^{(n)}$, which determine thermostressed state of the bimetallic plate.

We consider an uniaxial deformation, in which the components of the dynamic stress tensor $\sigma_{jj}^{(n)}$ $(j = x, y, z)$ that cause the stress intensities

$$\sigma_i^{(n)} = \sqrt{(3I_2(\hat{\sigma}^{(n)}) - I_1^2(\hat{\sigma}^{(n)}))/2} \tag{1}$$

in the n-th composite layer are the nonzero values. Here $I_j\left(\hat{\sigma}^{(n)}\right)$ $(j = 1,\ 2)$ are the j-th invariant of the stress tensor $\hat{\sigma}^{(n)}$ of the total stresses

$$\hat{\sigma}^{(n)} = \hat{\sigma}^{(n)Q} + \hat{\sigma}^{(n)F},$$

where $\hat{\sigma}^{(n)Q}$ and $\hat{\sigma}^{(n)F}$ are the stresses caused by Joule heat $Q^{(n)}$ and the ponderomotive forces

$$\vec{F}^{(n)} = \left\{\ 0;\ 0;\ F_z^{(n)}\ \right\}$$

in the n-th layer. Provided max $\sigma_i^{(n)} \geq \sigma_d^{(n)}$ and $\max \sigma_i^{(n)\,*} \geq \sigma_M$, the bimetallic plate loses its load-carrying ability and properties as a contact connector. Here $\sigma_d^{(n)}$ is the limit of elasticity of the n-th layer, σ_M is the contact joint strength limit [6].

3. Results and Discussions

The electromagnetic action in MPMS is mathematically described by the value of the function $H_y^{\pm}(t)$ as, [6]

$$H_y^{\pm}(t) = kH_0 \left(e^{-\beta_1 t} - e^{-\beta_2 t}\right) \cos \omega t\,.$$

Here k is a normalizing factor, β_1 and β_2 are characteristic parameters of time of rising edge t_{iner} and falling edge t_{dekr} of the modulated pulse of the duration t_i respectively, H_0 is the amplitude of sinusoidal carrying electromagnetic oscillations of the frequency ω [6, 8].

Substituting the expression of the function $H_y^{\pm}(t)$ into the general solutions for the problem of thermomechanics for the bimetallic plate under the uniform nonstationary electromagnetic action presented in [7, 8], we obtain the solution of a such problem under the action in the MPMS.

Numerical analysis is carried out for the bimetallic plate whose constituent layers have the same thickness $h_1 = h_2 = 1$ mm and are made of nonferromagnetic materials stainless 189 steel and copper. The durations of modulated pulse were $t_i = 10^{-4}s$, $10^{-3}s$, $10^{-2}s$. The parameters β_1 and β_2 were chosen so that the ratio of the time t_{iner} to the time of pulse t_{dekr} was equal to $t_{iner}/t_{dekr} \approx 0.1$. In this case $\beta_1 = -\ln\,\varepsilon/t_i$, $\beta_2 = 2\beta_1$, $\varepsilon = 10^{-3}$, $k = 4$, $\omega_{r1} = 4,443 \cdot 10^6$ $1/s$, $\omega_{r2} = 9,138 \cdot 10^6$.

The investigation of thermomechanical behavior, load-carrying ability and properties of the contact joint of the layers of the bimetallic plate under the action in MPMS are carried out for the first resonant frequency ω_{r1}.

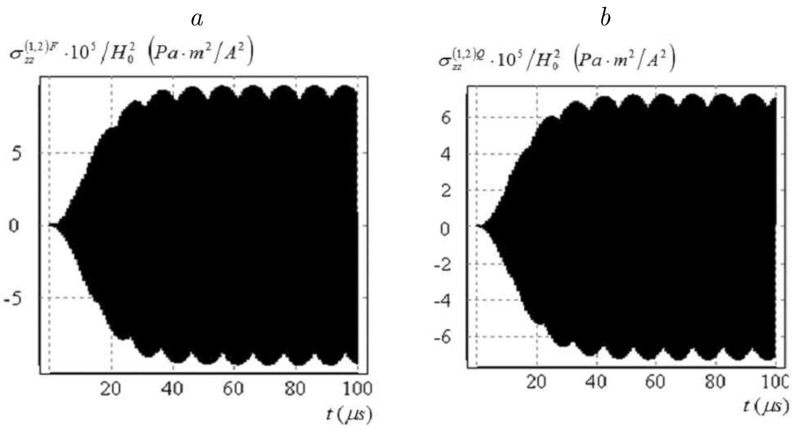

Figure 1 Variation in time of $\sigma_{zz}^{(n)}$ stresses (a, b) on the contact surface

In Figs. 1 and 2, the time variations of the terms $\sigma_{zz}^{(n)F}$ and $\sigma_{zz}^{(n)Q}$ of the stress tensor component $\sigma_{zz}^{(n)}$ on the contact surface of the layers are shown for the duration of the action in MPMS $t_i = 100$ μs. These values in both layers on the contact surface are equal, confirming the condition of ideal mechanical contact. The components $\sigma_{zz}^{(n)F}$ are

$$\sigma_{xx}^{(n)F} = \gamma_n \sigma_{zz}^{(n)F} \Big/ (1 - \gamma_n),$$

where γ_n is a Poisson's ratio of the material of the n-th layer. Hereinafter, all values are related to the square of the amplitude of the carrying signal H_0^2.

We have established that the components $\sigma_{zz}^{(n)F}$ caused by the action of the ponderomotive force and the components $\sigma_{zz}^{(n)Q}$ caused by Joule heat for the first resonance frequency ω_{r1} are values of the same order. These elements attain the mode of stable oscillations over the time $t \approx t_i/3$.

a *b*

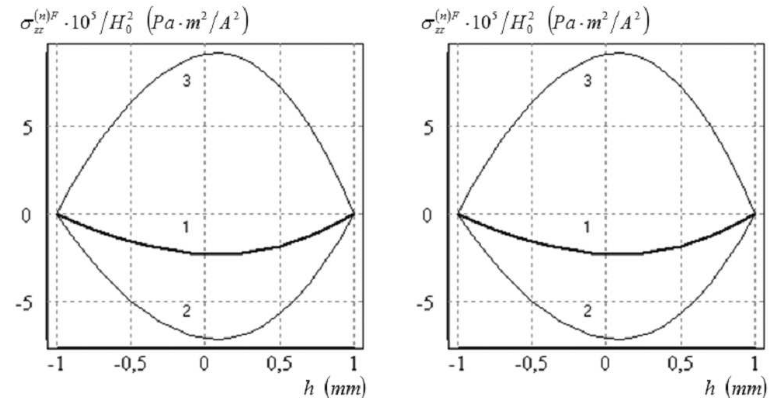

Figure 2 Distribution of $\sigma_{zz}^{(n)}$ stresses (a, b) in the thickness of bimetallic plate at different moments

Figure 2 presents the distribution of the values of the elements $\sigma_{zz}^{(n)F}$, $\sigma_{zz}^{(n)Q}$ of the components $\sigma_{zz}^{(n)}$ of the dynamic stresses tensor $\sigma_{xx}^{(n)}$ in the bimetallic plate thickness for the duration of action in MPMS $t_i = 100\ \mu s$ at different time moments. Curves 1–3 correspond to the time moments $t = 10\ \mu s$, $t = 25\ \mu s$, and $t = 50\ \mu s$. We can see that all stresses components reach their maximum values at the time $t = 50\ \mu s$, which is equal to $t \approx t_i/2$.

a *b*

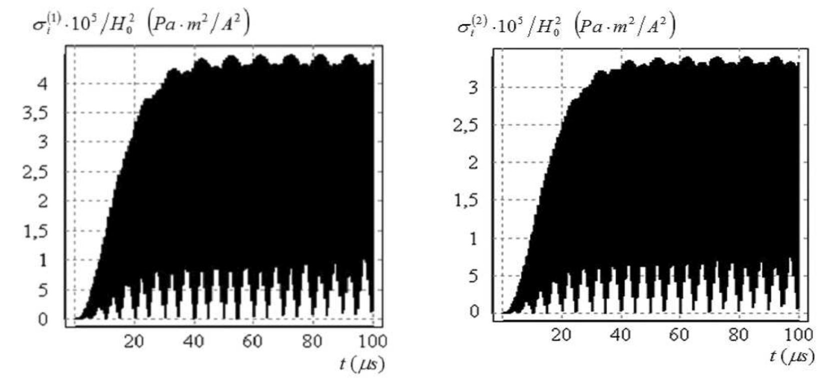

Figure 3 Changes in time of $\sigma_i^{(n)}$ stresses intensity on the contact surface of steel (a) and copper (b) layer

In Fig. 3, the change in time of the intensities $\sigma_i^{(n)}$ of the total stresses $\hat{\sigma}_{jj}^{(n)} = \hat{\sigma}_{jj}^{(n)F} + \hat{\sigma}_{jj}^{(n)Q}$ are illustrated in the first steel layer (Fig. 3 a) and in the second copper layer (Fig. 3 b) on the contact surface $z = 0$ for the duration of action in MPMS $t_i = 100\ \mu s$.

In Fig. 4, it is shown the distribution of stresses intensity $\sigma_i^{(n)}$ in the bimetallic plate thickness for the duration of action in MPMS $t_i = 100\ \mu s$ for the moments $t = 10\ \mu s$ and $t = 25\ \mu s$ (curves 1, 2).

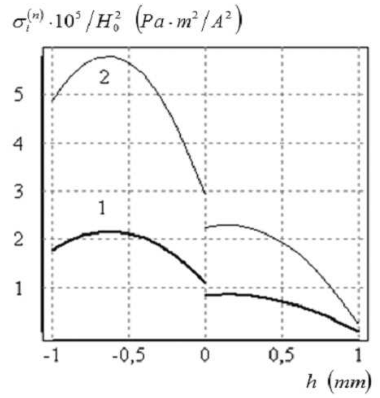

Figure 4 The plate thickness distribution of $\sigma_i^{(n)}$ stresses intensity at different moments for duration of action in MPMS $t_i = 100\ \mu s$

The value H_0 dependence of the maximal values of the stresses intensities $\sigma_{i\ \max}^{(n)}$ on the contact surface of the layers for the duration of modulated pulse $t_i = 10^{-4}s$; $10^{-3}s$; $4 \cdot 10^{-3}s$ (curves 1-3) are shown in Fig. 5.

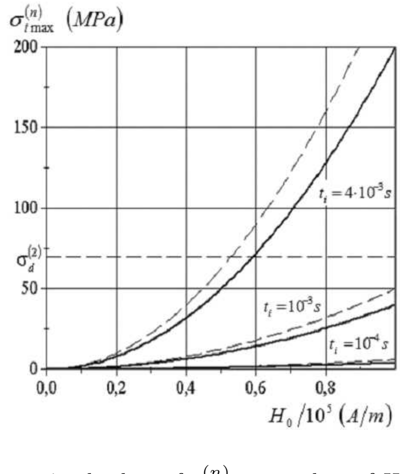

Figure 5 Dependence of the maximal values of $\sigma_{i\ \max}^{(n)}$ on values of H_0 for different pulse durations

The solid curves correspond to the steel layer and the dashed ones – to the copper layer. It is shown that the strength limit σ_M of the contact joint of the given bimetallic plate for the frequency ω_{r1} can be attained for the duration of the modulated pulse $t_i > 4 \cdot 10^{-3}s$ and for the amplitude of sinusoidal carrying electromagnetic oscillations $H_0 \approx 0.5 \cdot 10^5/m$, and the limit of elasticity for copper layer $\sigma_d^{(2)}$ is attained for the same duration t_i and $H_0 \approx 0.55 \cdot 10^5/m$.

4. Conclusions

We have obtained that at the resonant frequencies of the electromagnetic action in the MPMS, the components of the stresses caused by Joule heat and ponderomotive force take the equal values. The stresses normal to the base of the plate are greater than the shearing stresses. Based on the dependence of the stress intensities on the maximum amplitude value of the electric field intensity for different durations of modulated pulse there are established the critical values of the parameters of the given electromagnetic action, for which the bimetallic plate loses its carrying ability.

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