

Reflection of Plane Waves from Surface of a Generalized Thermo-Viscoelastic Porous Solid Half-Space with Impedance Boundary Conditions

Baljeet SINGH

*Department of Mathematics
Post Graduate Government College
Sector-11, Chandigarh - 160 011, India
bsinghgc11@gmail.com*

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A phenomenon of reflection of plane waves from a thermally insulated surface of a solid half-space is studied in context of Lord-Shulman theory of generalized thermo-viscoelasticity with voids. The governing equations of generalized thermo-viscoelastic medium with voids are specialized in x - z plane. The plane wave solution of these equations shows the existence of three coupled longitudinal waves and a shear vertical wave in a generalized thermo-viscoelastic medium with voids. For incident plane wave (longitudinal or shear), three coupled longitudinal waves and a shear vertical wave reflect back in the medium. The mechanical boundary conditions at free surface of solid half-space are considered as impedance boundary conditions, in which the shear force tractions are assumed to vary linearly with the tangential displacement components multiplied by the frequency. The impedance corresponds to the constant of proportionality. The appropriate potentials of incident and reflected waves in the half-space will satisfy the required impedance boundary conditions. A non-homogeneous system of four equations in the amplitude ratios of reflected waves is obtained. These amplitude ratios are functions of material parameters, impedance parameter, angle of incidence, thermal relaxation and speeds of plane waves. Using relevant material parameters for medium, the amplitude ratios are computed numerically and plotted against certain ranges of impedance parameter and the angle of incidence.

Keywords: Generalized thermo-viscoelasticity, voids, thermal relaxation, plane waves, reflection, amplitude ratios.

1. Introduction

Cowin and Nunziato [1] developed the theory of elastic material with voids. Iesan [2, 3] developed the theory of thermoelastic material with voids. Various dynamical problems and plane strain problems in theory of elasticity and thermoelasticity with voids have been appeared in literature. For example, Iesan [4], Ciarletta and

Scalia [5], Chirita and Scalia [6], Chirita et al. [7], Iesan and Nappa [8], Chirita and D'Apice [9, 10] and Ciarletta et al. [11] have studied various outstanding dynamical problems in theory of thermoelasticity with voids. Various problems on plane wave propagation in elasticity and thermoelasticity with voids were also studied. For example, Puri and Cowin [12], Chandrasekharaiah [13, 14], Singh [15], Ciarletta and Straughan [16], Ciarletta, et al. [17] and Bucur et al. [18].

Iesan [19, 20] developed theories of thermoviscoelastic materials with voids by incorporating the memory effects. Some problems on waves and vibrations in thermoviscoelastic material with voids were studied by Sharma and Kumar [21], Svanadze [22], Tomar et al. [23], Chirita [24], Chirita and Danescu [25], D'Apice and Chirita [26] and Bucur [27]. Exploring various problems on wave propagation in thermoviscoelastic materials with voids is useful in civil engineering, seismology, nano-technology and bio-materials [28]. In the present paper, we consider a generalized thermoviscoelastic solid half-space with voids, whose surface is subjected to impedance boundary conditions as in Godoy [29], where the tangential components of stress tensor depends linearly on tangential displacement components times frequency, respectively. A problem on reflection of plane (longitudinal or shear) wave in a generalized thermoviscoelastic medium with voids under these impedance boundary conditions is considered. The reflection coefficients (or amplitude ratios) of various reflected waves are analysed numerically to show the dependence on angle of incidence, viscous, thermal and voids parameters and impedance parameters.

2. Basic equations

Following Iesan [19] and Lord and Shulman [30], the system of field equations for isotropic and homogeneous generalized thermoviscoelastic porous solid in absence of body forces and heat sources are:

(a) the equations of motion:

$$t_{sr,s} = \rho \ddot{u}_r \quad (1)$$

$$H_{r,r} + g = \rho K^* \ddot{\phi} \quad (2)$$

(b) the energy equation:

$$\rho T_0 \dot{\eta} = Q_{r,r} \quad (3)$$

(c) the constitutive equations:

$$t_{rs} = \lambda_0 e_{mm} \delta_{rs} + 2\mu_0 e_{rs} + b_0 \phi \delta_{rs} - \beta T \delta_{rs} \quad (4)$$

$$H_r = \alpha_0 \phi_{,r} + \tau^* T_{,r} \quad (5)$$

$$g = -b_0 e_{mm} - \xi_0 \phi + mT \quad (6)$$

$$\rho \eta = \beta e_{mm} + aT + m\phi \quad (7)$$

$$Q_r + \tau_0 \dot{Q}_r = \kappa T_{,r} + \zeta \dot{\phi}_{,r} \tag{8}$$

$$e_{rs} = \frac{1}{2}(u_{r,s} + u_{s,r}) \tag{9}$$

Using equations (4) to (9) in equations (1) to (3), we can obtain following equations:

$$\mu_0 u_{s,rr} + (\lambda_0 + \mu_0) u_{m,ms} + b_0 \phi_{,s} - \beta T_{,s} = \rho \ddot{u}_s \tag{10}$$

$$\alpha_0 \phi_{,rr} - \gamma_0 u_{r,r} - \xi_0 \phi + \tau^* T_{,rr} + mT = \rho K^* \ddot{\phi} \tag{11}$$

$$\kappa T_{,rr} + \zeta \dot{\phi}_{,rr} = \beta T_0 (\dot{u}_{r,r} + \tau_0 \ddot{u}_{r,r}) + mT_0 (\dot{\phi} + \tau_0 \ddot{\phi}) + C_e (\dot{T} + \tau_0 \ddot{T}) \tag{12}$$

where the following notations are used:

$$C_e = aT_0, \lambda_0 = \lambda + \lambda^* \frac{\partial}{\partial t}, \mu_0 = \mu + \mu^* \frac{\partial}{\partial t}, b_0 = b + b^* \frac{\partial}{\partial t}, \alpha_0 = \alpha + \alpha^* \frac{\partial}{\partial t},$$

$$\gamma_0 = b + \gamma^* \frac{\partial}{\partial t}, \xi_0 = \xi + \xi^* \frac{\partial}{\partial t}.$$

and t_{rs} are the components of the stress tensor, H_r are the components of the equilibrated stress vector, g is the intrinsic equilibrated body force, η is the entropy per unit mass, Q_r are the components of the heat flux vector, e_{rs} are the components of the strain tensor, ρ is the mass density of the medium, K^* is the equilibrated inertia, u_r are the components of the displacement vector, ϕ is the void volume fraction, θ is the change in temperature from the constant reference temperature T_0 and δ_{rs} are the components of the Kronecker delta, λ and μ are well known Lamé's constant parameters, b , α , ξ and ξ^* are the constant parameters corresponding to voids present in the medium, β , τ^* , m , κ , ζ and a are the constant thermal parameters and λ^* , μ^* , b^* , α^* and γ^* are the constant viscoelastic parameters, τ_0 is thermal relaxation time.

Equations (10) to (12) are specialized in x-z plane as:

$$\mu_0(u_{1,11} + u_{1,33}) + (\lambda_0 + \mu_0)(u_{1,11} + u_{3,31}) + b_0 \phi_{,1} - \beta T_{,1} = \rho \ddot{u}_1 \tag{13}$$

$$\begin{aligned} &\mu_0(u_{3,11} + u_{3,33}) + (\lambda_0 + \mu_0)(u_{1,13} + u_{3,33}) + b_0 \phi_{,3} - \beta T_{,3} \\ &= \rho \ddot{u}_3 \alpha_0 (\phi_{,11} + \phi_{,33}) - \gamma_0 (u_{1,1} + u_{3,3}) - \xi_0 \phi + \tau^* (T_{,11} + T_{,33}) + mT \tag{14} \\ &= \rho K^* \ddot{\phi} \end{aligned}$$

$$\begin{aligned} &\kappa (T_{,11} + T_{,33}) + \zeta (\dot{\phi}_{,11} + \dot{\phi}_{,33}) - \beta T_0 \{(\dot{u}_{1,1} + \dot{u}_{3,3}) + \tau_0 (\ddot{u}_{1,1} + \ddot{u}_{3,3})\} \\ &- mT_0 (\dot{\phi} + \tau_0 \ddot{\phi}) - C_e (\dot{T} + \tau_0 \ddot{T}) = 0 \tag{15} \end{aligned}$$

Using the following Helmholtz representations of displacement components in terms of potentials:

$$u_1 = \frac{\partial q}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial q}{\partial z} + \frac{\partial \psi}{\partial x} \tag{16}$$

the equations (13) to (16) result into the following equations:

$$\mu_0\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2}\right) = \rho\ddot{\psi} \tag{17}$$

$$(\lambda_0 + 2\mu_0)\left(\frac{\partial^2q}{\partial x^2} + \frac{\partial^2q}{\partial z^2}\right) + b_0\phi - \beta T = \rho\ddot{q} \tag{18}$$

$$\alpha_0\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2}\right) - \gamma_0\left(\frac{\partial^2q}{\partial x^2} + \frac{\partial^2q}{\partial z^2}\right) - \xi_0\phi + \tau^*\left(\frac{\partial^2T}{\partial x^2} + \frac{\partial^2T}{\partial z^2}\right) + mT = \rho K^* \ddot{\phi} \tag{19}$$

$$\begin{aligned} &\kappa\left(\frac{\partial^2T}{\partial x^2} + \frac{\partial^2T}{\partial z^2}\right) + \zeta\left(\frac{\partial^2\dot{\phi}}{\partial x^2} + \frac{\partial^2\dot{\phi}}{\partial z^2}\right) - \beta T_0\left\{\left(\frac{\partial^2\dot{q}}{\partial x^2} + \frac{\partial^2\dot{q}}{\partial z^2}\right) + \tau_0\left(\frac{\partial^2\ddot{q}}{\partial x^2} + \frac{\partial^2\ddot{q}}{\partial z^2}\right)\right\} \\ &- mT_0(\dot{\phi} + \tau_0\ddot{\phi}) - C_e(\dot{T} + \tau_0\ddot{T}) = 0 \end{aligned} \tag{20}$$

We seek the plane wave solutions of equations (18) to (21) of the following form:

$$\{q, \phi, T, \psi\} = \{A, B, C, D\}exp[ik(\sin\theta x + \cos\theta z - Vt)] \tag{21}$$

where A, B, C, D and E are arbitrary constants. k is the wavenumber, V is the complex phase speed and θ is the angle of propagation. With the help of (22), the non-trivial plane wave solution of equation (18) leads to:

$$V^2 = \frac{\mu_0}{\rho} \tag{22}$$

which is the speed of shear vertical (SV) wave.

With the help of (22) the plane wave solutions of equations (19) to (21) lead to following cubic velocity equation:

$$\begin{aligned} &\left(\frac{1 - \bar{\xi}_0}{\rho} - \epsilon_2\bar{m}\right)\Gamma^3 - [\bar{\kappa}(1 - \bar{\xi}_0) + \frac{c_2}{\rho} + \epsilon_3c_3 + \epsilon_3\bar{m} + \frac{c_1(1 - \bar{\xi}_0)}{\rho} - \epsilon_2c_1\bar{m} + \frac{b_0\bar{\gamma}_0}{\rho} \\ &+ \epsilon_1b_0\bar{m} + \beta\gamma_0\epsilon_2 + \beta\epsilon_1(1 - \bar{\xi}_0)]\Gamma^2 + [\bar{\kappa}c_2 - \epsilon_3c_3 + \bar{\kappa}c_1(1 - \bar{\xi}_0) + \frac{c_1c_2}{\rho} \\ &- c_1c_3\epsilon_3 + c_1\bar{m}\epsilon_3 + b_0\bar{\gamma}_0\bar{\kappa} - b_0c_3\epsilon_1 - \beta\epsilon_3\bar{\gamma}_0 + \beta c_2\epsilon_1]\Gamma - (c_1c_2\bar{\kappa} - c_1c_3\epsilon_3) = 0 \end{aligned} \tag{23}$$

where:

$$\begin{aligned} \Gamma &= \rho v^2, \quad c_1 = \lambda_0 + 2\mu_0, \quad c_2 = \frac{\alpha_0}{K^*}, \quad c_3 = -\frac{\tau^*}{K^*}, \quad \epsilon_1 = \frac{\beta T_0}{\rho C_e}, \\ \epsilon_2 &= \frac{mT_0}{\rho C_e}, \quad \epsilon_3 = \frac{\zeta^*\omega^2}{C_e}, \quad \bar{\kappa} = \frac{\kappa}{C_e(\tau_0 + \frac{i}{\omega})}, \quad \bar{\gamma}_0 = \frac{\gamma_0}{\rho K^*\omega^2}, \\ \bar{m} &= \frac{m}{\rho K^*\omega^2}, \quad \bar{\xi}_0 = \frac{\xi_0}{\rho K^*\omega^2}, \quad \zeta^* = \frac{i\zeta}{\omega(\tau_0 + \frac{i}{\omega})}. \end{aligned}$$

The real parts of the roots of cubic velocity equation (24) correspond to the speeds of three coupled longitudinal ($P1, P2$ and $P3$) waves.

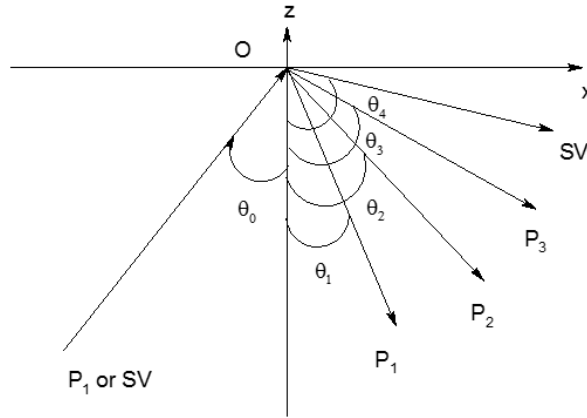


Figure 1 Reflection of plane waves at a stress-free surface of a poro-thermo-viscoelastic solid half-space

3. Reflection from a plane surface

We consider a half-space of a generalized thermoviscoelastic medium with voids. The plane surface of the half-space is taken along the x -axis. The negative z -axis is taken as normal into the half-space as shown in Figure 1. Following Godoy et al. [29], we assume that the surface of half-space is subjected to impedance boundary conditions, where the tangential tractions are proportional to tangential displacement components time frequency, respectively. Therefore, in the present problem, the impedance boundary conditions at $z = 0$ are expressed as:

$$t_{33} = 0, \quad t_{31} + \omega Z u_1 = 0, \quad H_3 = 0, \quad Q_3 = 0, \quad (24)$$

where: $t_{33} = \lambda_0(e_{11} + e_{33}) + 2\mu_0 e_{33} + b_0\phi - \beta T$, $t_{31} = 2\mu_0 e_{31}$, $Q_3 = \kappa \frac{\partial T}{\partial z} + \zeta \frac{\partial \dot{\phi}}{\partial z}$, $H_3 = \alpha_0 \frac{\partial \phi}{\partial z} + \tau^* \frac{\partial T}{\partial z}$ and ω is frequency of wave and Z is impedance parameters of dimension stress/velocity, which is assumed strictly real. For $Z = 0$, the impedance boundary conditions reduce to traction-free boundary conditions and $|Z| \rightarrow +\infty$ corresponds to vanishing of tangential component of displacement vector. For an incident P_1 or SV wave at plane surface $z = 0$, the reflected P_1, P_2, P_3 and SV waves propagate in the half-space ($z < 0$). The appropriate potentials for

incident and reflected waves in the half-space are:

$$q = A_0 \exp\{ik_1(x \sin \theta_0 + z \cos \theta_0 - v_1 t)\} + \sum_{j=1}^3 A_j \exp\{ik_j(x \sin \theta_j - z \cos \theta_j - v_j t)\} \tag{25}$$

$$\phi = p_1 A_0 \exp\{ik_1(x \sin \theta_0 + z \cos \theta_0 - v_1 t)\} + \sum_{j=1}^3 p_j A_j \exp\{ik_j(x \sin \theta_j - z \cos \theta_j - v_j t)\} \tag{26}$$

$$T = q_1 A_0 \exp\{ik_1(x \sin \theta_0 + z \cos \theta_0 - v_1 t)\} + \sum_{j=1}^3 q_j A_j \exp\{ik_j(x \sin \theta_j - z \cos \theta_j - v_j t)\} \tag{27}$$

$$\psi = B_0 \exp\{ik_4(x \sin \theta_0 + z \cos \theta_0 - v_4 t)\} + B_1 \exp\{ik_4(x \sin \theta_4 - z \cos \theta_4 - v_4 t)\} \tag{28}$$

where $v_i = Re(V_i)$, ($i = 1, 2, \dots, 4$) and the expression for $\frac{p_j}{k_j^2}, \frac{q_j}{k_j^2}$, ($j = 1, 2, 3$) are given as:

$$\frac{p_j}{k_j^2} = \frac{(\tau^* - \frac{m}{k_j^2})(\lambda_0 + 2\mu_0 - \rho v_j^2) + \frac{\beta \gamma_0}{k_2^2}}{b_0(\tau^* - \frac{m}{k_j^2}) + \beta(\alpha_0 + \frac{\xi_0}{k_j^2} - \rho K^* v_j^2)}$$

$$\frac{q_j}{k_j^2} = \frac{-(\alpha_0 + \frac{\xi_0}{k_j^2} - \rho K^* v_j^2)(\lambda_0 + 2\mu_0 - \rho v_j^2) + \frac{b_0 \gamma_0}{k_2^2}}{b_0(\tau^* - \frac{m}{k_j^2}) + \beta(\alpha_0 + \frac{\xi_0}{k_j^2} - \rho K^* v_j^2)}$$

The potentials given in equations (26) to (29) satisfy boundary conditions (25) if following relations (Snell’s law for present problem) hold:

$$\frac{\sin \theta_0}{v_1 \text{ or } v_4} = \frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2} = \frac{\sin \theta_3}{v_3} = \frac{\sin \theta_4}{v_4} \tag{29}$$

$$k_1 v_1 = k_2 v_2 = k_3 v_3 = k_4 v_4 \tag{30}$$

and:

(a) incident P wave:

$$\sum_{j=1}^4 a_{ij} Z_j = b_i, \quad (i = 1, 2, \dots, 4) \tag{31}$$

where: $Z_j = \frac{A_j}{A_0}$, ($j = 1, 2, 3$) and $Z_4 = \frac{B_1}{A_0}$ are reflection coefficients of reflected P_1, P_2, P_3 and SV waves, and: $b_1 = -1, b_2 = -1, b_3 = 1, b_4 = 1$,

$$a_{1j} = \frac{(\frac{v_1}{v_j})^2 [(\lambda - i\omega\lambda^*) + 2(\mu - i\omega\mu^*)(1 - (\frac{v_j}{v_1})^2 \sin^2 \theta_0) - (b - i\omega b^*)\frac{p_j}{k_j^2} + \beta\frac{q_j}{k_j^2}]}{(\lambda - i\omega\lambda^*) + 2(\mu - i\omega\mu^*) \cos^2 \theta_0 - (b - i\omega b^*)\frac{p_1}{k_1^2} + \beta\frac{q_1}{k_1^2}}$$

$$\begin{aligned}
 a_{14} &= -\frac{2(\mu - i\omega\mu^*)\frac{v_1}{v_4} \sin \theta_0 \sqrt{1 - (\frac{v_4}{v_1})^2 \sin^2 \theta_0}}{(\lambda - i\omega\lambda^*) + 2(\mu - i\omega\mu^*) \cos^2 \theta_0 - (b - i\omega b^*)\frac{p_1}{k_1^2} + \beta\frac{q_1}{k_1^2}} \\
 a_{2j} &= (\frac{v_1}{v_j}) \left[\frac{2\sqrt{1 - (\frac{v_j}{v_1})^2 \sin^2 \theta_0} + iZ_j}{-2 \cos \theta_0 + iZ_1} \right], \quad Z_j = \frac{v_j Z}{\mu - i\omega\mu^*}, \quad (j = 1, 2, 3) \\
 a_{24} &= (\frac{v_1}{v_4})^2 \left[\frac{1 - 2(\frac{v_4}{v_1})^2 \sin^2 \theta_0 + iZ_4 \sqrt{1 - (\frac{v_4}{v_1})^2 \sin^2 \theta_0}}{\sin \theta_0 (-2 \cos \theta_0 + iZ_1)} \right], \quad Z_4 = \frac{v_4 Z}{\mu - i\omega\mu^*} \\
 a_{3j} &= \frac{(\frac{v_1}{v_j})(\omega p_j \zeta + i q_j \kappa) \sqrt{1 - (\frac{v_j}{v_1})^2 \sin^2 \theta_0}}{(\omega p_1 \zeta + i q_1 \kappa) \cos \theta_0}, \quad (j = 1, 2, 3), \quad a_{34} = 0, \\
 a_{4j} &= \frac{(\frac{v_1}{v_j})[(\alpha - i\omega\alpha^*)p_j + \tau^* q_j] \sqrt{1 - (\frac{v_j}{v_1})^2 \sin^2 \theta_0}}{[(\alpha - i\omega\alpha^*)p_1 + \tau^* q_1] \cos \theta_0}, \quad (j = 1, 2, 3), \quad a_{44} = 0
 \end{aligned}$$

(b) incident *SV* wave:

$$\sum_{j=1}^4 c_{ij} Y_j = d_i, \quad (i = 1, 2, \dots, 4) \tag{32}$$

where $Y_j = \frac{A_j}{B_0}$ ($j = 1, 2, 3$) and $Y_4 = \frac{B_1}{B_0}$ are reflection coefficients of reflected P_1, P_2, P_3 and *SV* waves, and $d_1 = -1, d_2 = -1, d_3 = 0, d_4 = 0,$

$$\begin{aligned}
 c_{1j} &= \frac{(\frac{v_4}{v_j})^2 [(\lambda - i\omega\lambda^*) + 2(\mu - i\omega\mu^*) [1 - (\frac{v_j}{v_4})^2 \sin^2 \theta_0] - (b - i\omega b^*)\frac{p_j}{k_j^2} + \beta\frac{q_j}{k_j^2}]}{(\mu - i\omega\mu^*) \sin 2\theta_0}, \\
 c_{14} &= -1
 \end{aligned}$$

$$\begin{aligned}
 c_{2j} &= \sin \theta_0 (\frac{v_4}{v_j}) \left[\frac{2\sqrt{1 - (\frac{v_j}{v_4})^2 \sin^2 \theta_0} + iZ_j}{1 - 2 \sin^2 \theta_0 - iZ_4 \cos \theta_0} \right], \quad (j = 1, 2, 3), \\
 c_{24} &= \frac{1 - 2 \sin^2 \theta_0 + iZ_4 \cos \theta_0}{1 - 2 \sin^2 \theta_0 - iZ_4 \cos \theta_0}, \\
 c_{3j} &= (\frac{v_4}{v_j})(\omega \zeta p_j + i \kappa q_j) \sqrt{1 - (\frac{v_j}{v_4})^2 \sin^2 \theta_0}, \quad (j = 1, 2, 3), \quad c_{34} = 0, \\
 c_{4j} &= (\frac{v_4}{v_j}) [(\alpha - i\omega\alpha^*)p_j + \tau^* q_j] \sqrt{1 - (\frac{v_j}{v_4})^2 \sin^2 \theta_0}, \quad (j = 1, 2, 3), \quad c_{44} = 0.
 \end{aligned}$$

4. Numerical results and discussion

To get an idea about the dependence of amplitude ratios of various reflected waves on angle of incidence, impedance parameter and other material parameters, the copper is treated as a thermoviscoelastic material with voids. The following physical constants of copper material are considered:

$$\begin{aligned}
 \lambda &= 7.76 \times 10^{11} \text{ dyn/cm}^2, & \mu &= 3.86 \times 10^{11} \text{ dyn/cm}^2, & \rho &= 8.954 \text{ gm/cm}^3, \\
 c &= 3.4303 \times 10^4 \text{ dyn/cm}^2 \text{ } ^\circ\text{C}, & b &= 2 \times 10^3 \text{ dyn/cm}^2, & \alpha &= 1.688 \text{ dyn}, \\
 \beta &= 0.4 \times 10^{-1} \text{ dyn/cm}^2 \text{ } ^\circ\text{C}, & \xi &= 1.475 \text{ dyn/cm}^2, & m &= 0.2 \times 10^7 \text{ dyn/cm}^2 \text{ } ^\circ\text{C},
 \end{aligned}$$

$\kappa = 0.386 \times 10^8 \text{ dyn/s } ^\circ\text{C}$, $T_0 = 293 \text{ K}$, $K^* = 1.75 \times 10^{-11} \text{ cm}^2$,
 and we set:
 $\lambda^* = 0.1 \text{ dyn s/cm}^2$, $\mu^* = 0.2 \text{ dyn s/cm}^2$, $b^* = 0.1 \times 10^{-3} \text{ dyn s/cm}^2$,
 $\xi^* = 0.3 \text{ dyn s/cm}^2$, $\alpha^* = 0.1 \text{ dyn s}$, $\gamma^* = 0.5 \times 10^{-7} \text{ dyn s/cm}^2$,
 $\tau^* = 0.3 \times 10^{-7} \text{ dyn/ } ^\circ\text{C}$, $\zeta = 1.5 \times 10^{-11} \text{ dyn}$.

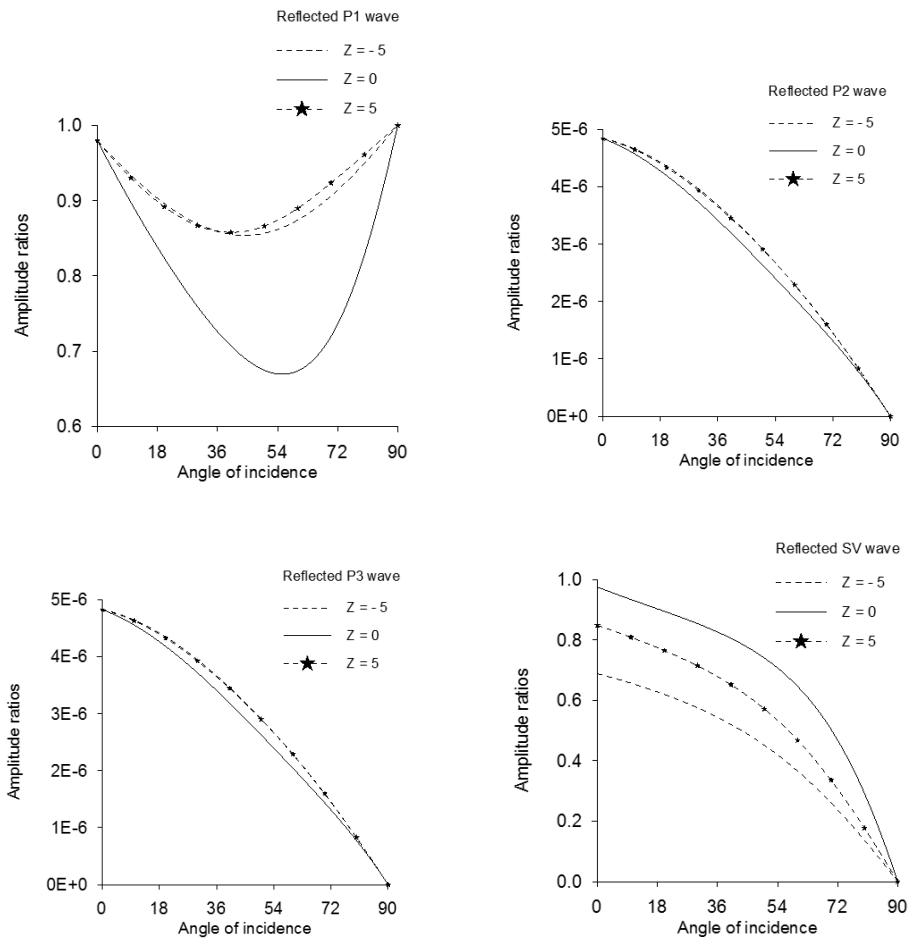


Figure 2 Variations of the amplitude ratios of reflected P1, P2, P3 and SV waves against the angle of incidence θ_0 of incident P1 wave when $Z = -5, 0$ and 5

For above values of material parameters, the non-homogeneous systems (32) and (33) of linear equations in amplitude ratios of reflected waves are solved by using Fortran program of Gauss elimination method. For incident P1 wave, the amplitude ratios of reflected waves are plotted against the range $0^\circ \leq \theta_0 \leq 90^\circ$ of angle of incidence in Figure 2 by solid lines, when impedance parameter $Z = 0$. The amplitude ratios of reflected P1 wave is 0.98 at $\theta_0 = 0^\circ$ (normal incidence). It decreases

to a value 0.6695 at $\theta_0 = 55^\circ$ and then increases to a value one at $\theta_0 = 90^\circ$ (grazing incidence). The amplitude ratios of reflected $P2$ and $P3$ waves are very smaller in comparison to that of $P1$ wave. The maximum values of the amplitude ratios of reflected $P2$ and $P3$ waves are $0.4841\text{e-}05$ and $0.4825\text{e-}05$ at normal incidence. These reduce to zero at grazing incidence. The amplitude ratios of reflected SV is 0.9742 at normal incidence and it also reduces to zero at grazing incidence. Similar variations for impedance parameters $Z = -5$ and $Z = 5$ are also shown in Figure 2 by dashed line and dashed line with star as center symbols, respectively. The comparison of these dashed lines with solid line shows the effect of impedance parameter at each angle of incidence of $P1$ wave.

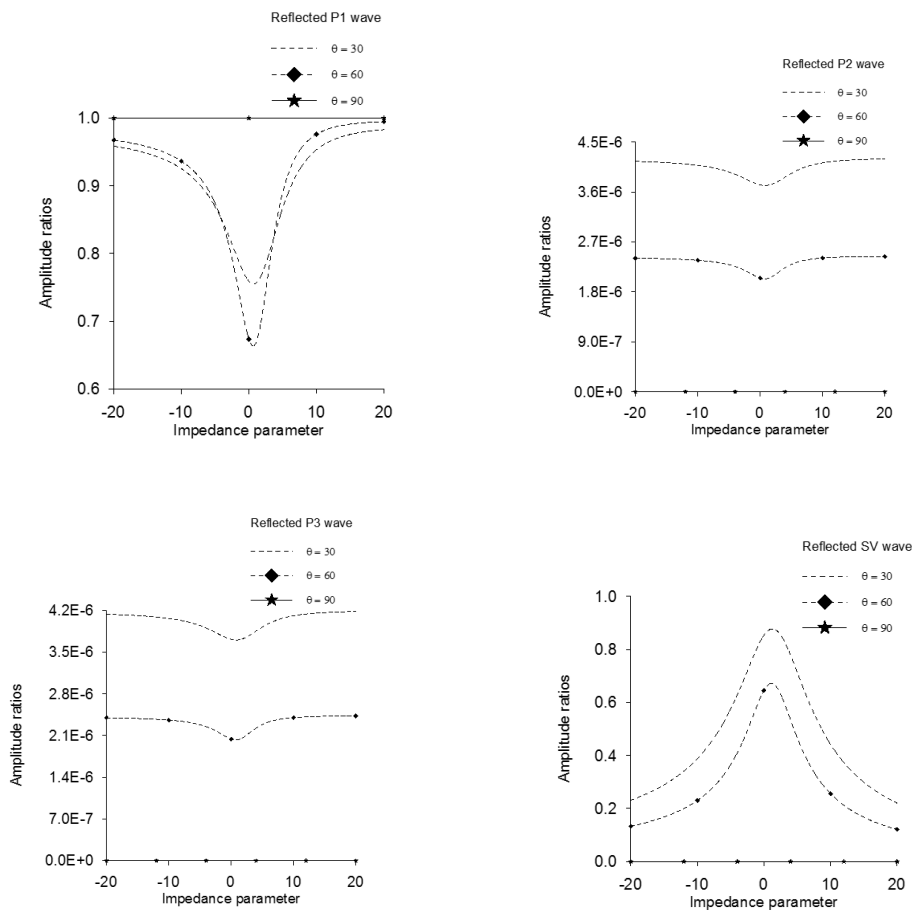


Figure 3 Variations of the amplitude ratios of reflected $P1$, $P2$, $P3$ and SV waves against the impedance parameter Z for incident $P1$ wave when $\theta = 30^\circ$, 60° and 90°

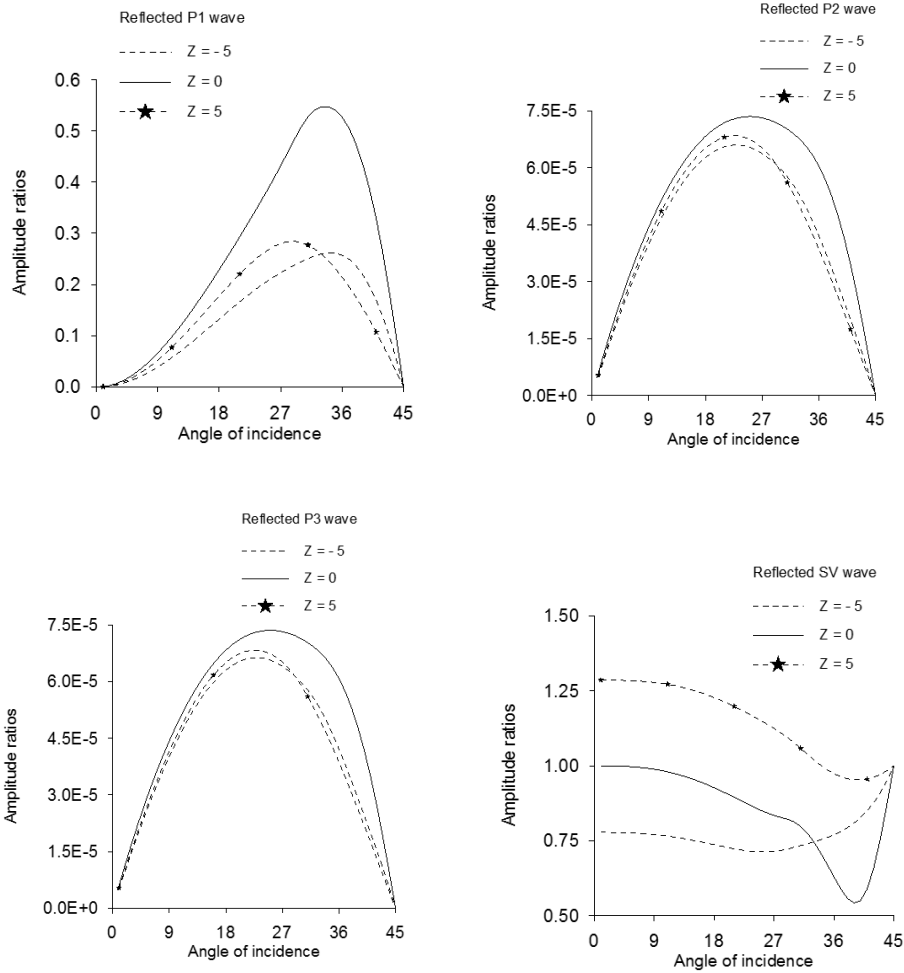


Figure 4 Variations of the amplitude ratios of reflected P1, P2, P3 and SV waves against the angle of incidence 0 of incident SV wave when $Z = -5, 0$ and 5

For incident *P1* wave, the amplitude ratios of reflected waves are plotted against the range $-20 \leq Z \leq 20$ of impedance parameter in Figure 3 by dashed line, dashed line with squares and solid line with stars for $\theta_0 = 30^\circ, 60^\circ$ and 90° , respectively. The comparison of these three variations shows the effect of three different angle of incidences in a particular range of impedance parameter. It is observed that there is no impact of impedance at grazing incidence.

For incident *SV1* wave, the amplitude ratios of reflected waves are plotted against the range $1^\circ \leq \theta_0 \leq 45^\circ$ of angle of incidence in Figure 4 by solid lines, when impedance parameter $Z = 0$. Beyond $\theta_0 > 45^\circ$, a phase change occurs. The amplitude ratios of reflected *P1* wave is zero at $\theta_0 = 1^\circ$ (near normal incidence). It increases to its maximum value 0.5472 at $\theta_0 = 34^\circ$ and then decreases sharply to its minimum value zero at $\theta_0 = 90^\circ$ (grazing incidence). In this case also, the ampli-

tude ratios of reflected $P2$ and $P3$ waves are observed very smaller in comparison to that of $P1$ wave. The maximum values of the amplitude ratios of reflected $P2$ and $P3$ waves are $0.7350e - 04$ and $0.7355e - 04$ at $\theta_0 = 25^\circ$. These amplitude ratios of reflected $P2$ and $P3$ waves reduce to zero at 1° and 45° . The amplitude ratios of reflected SV is one at $\theta_0 = 1^\circ$ and it reduces to 0.5416 at $\theta_0 = 39^\circ$ and increases sharply to one at 45° . Similar variations for impedance parameters $Z = -5$ and $Z = 5$ are also shown in Figure 4 by dashed line and dashed line with star as center symbols, respectively. The comparison of these dashed lines with solid line shows the effect of impedance parameter at each angle of incidence of SV wave.

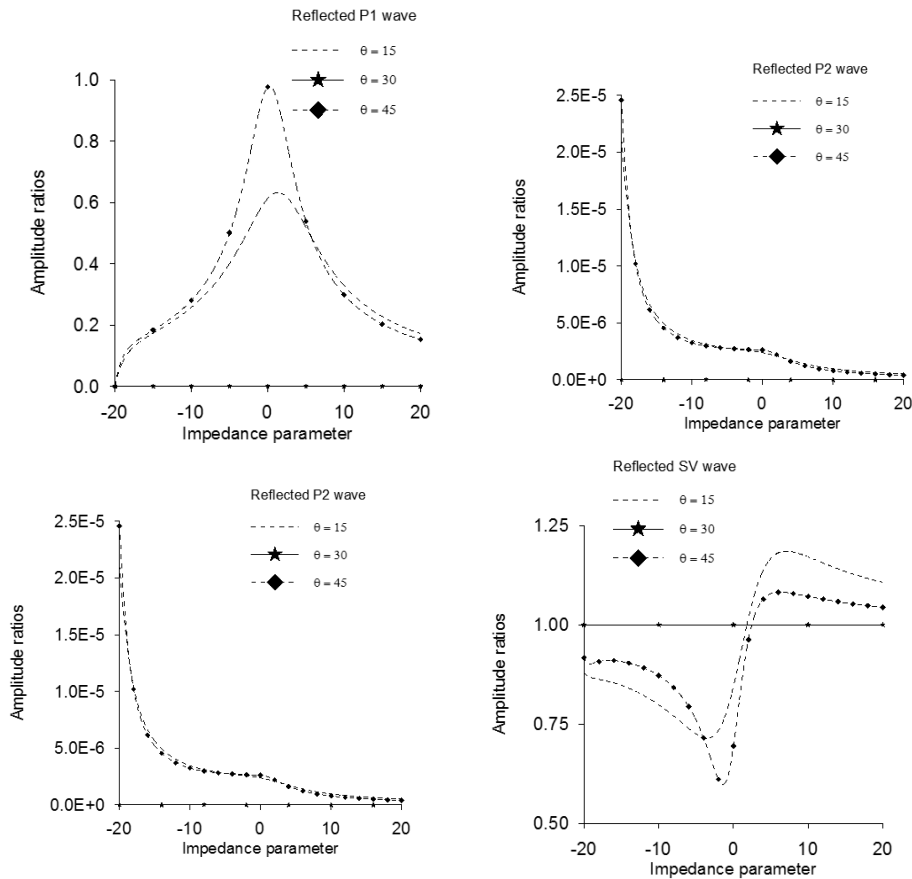


Figure 5 Variations of the amplitude ratios of reflected P1, P2, P3 and SV waves against the impedance parameter Z for incident SV wave when $\theta = 15^\circ, 30^\circ$ and 45°

For incident SV wave, the amplitude ratios of reflected waves are plotted against the range $-20 \leq Z \leq 20$ of impedance parameter in Figure 5 by dashed line, dashed line with stars and solid line with squares for $\theta = 15^\circ, 30^\circ$ and 45° , respectively.

The comparison of these three variations shows the effect of three different angle of incidences in a particular range of impedance parameter. It is observed that there is no impact of impedance at $\theta_0 = 45^\circ$.

5. Conclusions

Plane waves in a thermoviscoelastic medium with voids is studied in context of Lord and Shulman theory of generalized thermoelasticity. The solution of specialized governing equations of medium shows the existence of three coupled longitudinal waves (P_1 , P_2 and P_3) and a shear vertical (SV) wave. The relations between the amplitude ratios of various reflected waves are obtained for incidence of both P_1 and SV waves. For a particular material representing the medium, the amplitude ratios of the reflected waves are computed and plotted against the angle of incidence and impedance parameter. The numerical discussion of these plots provide some vital observations:

(i) The introduction of impedance parameter in tangential stress component changes significantly the amplitude ratios of reflected waves for incidence of both P_1 and SV waves.

(ii) For incident P_1 wave, the impedance parameter significantly changes the amplitude ratios of reflected waves at each angle of incidence except grazing incidences. From figure 2, it is also observed that the presence of impedance parameter changes significantly the amplitude ratios of reflected shear vertical wave at normal incidence and the amplitude ratios of reflected longitudinal waves remain unaffected at normal incidence.

(iii) For incident SV wave, the impedance parameter significantly changes the amplitude ratios of reflected waves at each angle of incidence except at $\theta_0 = 45^\circ$. Again from figure 4, it is also observed that the presence of impedance parameter changes significantly the amplitude ratios of reflected shear vertical wave at normal incidence and the amplitude ratios of reflected longitudinal waves remain unaffected at normal incidence.

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