

Structural Optimization Coupled with Materials Selection for Stiffness Improvement

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An application of a Finite Element Model updating is presented in this paper. Two Finite Element models were considered: a reinforced plate and a thin-walled beam. The two parts were numerically calculated in ANSYS Mechanical APDL and MATLAB programs. ANSYS performs Finite Element calculations, and a MATLAB programming code was used to control the optimization procedure. Geometric variables were chosen, to evaluate the value of the defined objective function. The material was picked using available selection charts, to find the most adequate one for the study. It has been concluded that the transversal displacement of the models modified by the optimization process decreased sharply in relation to the original state.

Keywords: Finite Element Method, Finite Element Model Updating, optimization, MATLAB, ANSYS, structural static analysis, materials selection, stiffness.

1. Introduction

Acceleration levels noted for mobile parts in industrial machines gained in the last decade. That happened mainly due to higher manufacturing rate demanded by increasing consumption level. The machines already designed were set for a lower acceptable acceleration values. This might result in their stiffness being too low for correct operation at higher accelerations. Presently, the acceleration level can reach an order of magnitude larger than gravity. Therefore, it is needed to improve the stiffness in order to achieve correct machine operation. Undesired vibrations might be a problem that could eventually lead to failure. Our results can mainly be applied to industrial devices comprising mobile parts, such as automatized plotters

and laser cutting machines. To improve the stiffness both, the material and the geometry of the model should be studied [1].

In the last years, there has also been an increase in the use of computers in engineering and applied sciences. This is due to the extreme improvement of the personal computers capabilities, which made possible to solve complex engineering problems from several hours to few days. The FEM programs, like *ANSYS*, are extremely powerful, especially when they are allied with optimization procedures. The FEM has some limitations in the results accuracy when modelling complex structures [2]; however, there have been many developments in this field. For instance, Bin has modelled an aerial working vehicle with good correlation between results [3]. Several optimization procedures have been proposed presently with application to structural static analysis [4–8]. Kalanta *et al.* shown that 2-D optimization problems, namely in terms of mathematical models and solution algorithms, can be used for solutions of 3-D optimization problems [9].

Silva and Meireles developed a Finite Element Model Updating methodology for static analysis, where the main aim was to optimize the mechanical behaviour of steel objects subjected to bending and torsion uncoupled loadings [10,11]. Some authors applied optimization methods to beams of various cross-sections manufactured by cold forming in order to optimize relevant geometric variables of the structures [12]. A modern global optimization path effectively working in optimisation of cross-sections has been presented by Liu *et al.* in [13]. It is found that the developed optimization process can effectively learn from the optimisation processes and apply the knowledge on related design optimisation problems. This is an efficient learning mechanism that is not present in most optimization schemes.

Magnucka-Blandzi and Magnucki wrote a paper about cold-formed thin-walled channel beams having either open or closed flanges [14]. Leng *et al.* demonstrated the application of formal optimization tools with the aim of maximizing the compressive strength of an open cold-formed steel cross section. In this work, the cross-section shape is not limited by pre-determined elements (flanges, webs, stiffeners, etc.), as is commonly required to meet the necessity of conventional code-based procedures for design that employ simplified closed-form stability analysis [15]. The design optimization of oval hollow-box beams made of stainless steel was discussed by Theofanous *et al.* in which authors studied the structural response of an oval hollow section manufactured from stainless steel and working at the compressed load [16].

Designs which include several beams were studied by Lagaros *et al.* who attempted an optimum design of 3-D steel structures with perforated I-sections [17]. Tsavdaridis and D'Mello studied the optimization of new shapes in from of elliptically-based web opening [18]. The work developed by the authors improves the structural behavior of perforated beams while aiming an economic design in terms of manufacture and usage. McKinstray *et al.*, studied the optimal design of fabricated steel beams for applications on long-span portal frames [19]. The design optimization includes a number of several relevant factors, such as ultimate and serviceability limit states, and displacement limits, as recommended by the Steel Construction Institute (SCI). The authors used a genetic algorithm (GA) in order to optimize geometric variables of the plates, which were used for columns, rafters and haunches. Tran and Li proposed a global optimization procedure for the design of

cross-section of channel beams under uniformly distributed transverse loading [20]. The optimization presented by the authors is carried out by application of the trust region method (TRM) utilizing factors, such as failure modes of yielding strength, displacement limitation, local and distortional and combined buckling.

2. Related Theory

2.1. Sequential Quadratic Programming (SQP)

SQP is a range of robust methods in the field of nonlinear programming. For example, Schittkowski [21], proposed and verified a version performing better than many other attempts in terms of efficiency and accuracy of effective results. The development was verified in big number of different tests. Having as a basis the work of Biggs [22], Han [23], and Powell [24, 25], this method permits the close mimic of the Newton's method in constrained optimization in the same way as it is done for unconstrained optimization. It works by utilizing of the Hessian of the Lagrangian functions and a quasi-Newton updating method at each iteration, which is then applied in generation of a Quadratic Problem (QP) subproblem. The solution of this subproblem is then used to form a search direction for a line search procedure. An overview of SQP can be found in Fletcher [26], Gill *et al.* [27], Powell [28], and Schittkowski [29]. The general method is described next. The SQP method formulates the QP subproblem using a quadratic approximation of the Lagrangian function as:

$$L(x, \lambda) = f(x) + \sum_{i=1}^m \lambda_i g_i(x). \quad (1)$$

A simplification of Eq.(1) is done using the assumption that bound constraints have been described in form of inequality conditions. The QP sub-problem is defined through linearisation of the nonlinear constraints.

The QP can be defined as a set of the following equations:

$$\begin{aligned} \min \frac{1}{2} d^T \mathbf{H}_k d + \nabla f(x_k)^T d \\ \nabla g_i(x_k)^T d + g_i(x_k) = 0, i = 1 \dots m_e \\ \nabla g_i(x_k)^T d + g_i(x_k) \leq 0, i = m_e + 1 \dots m. \end{aligned} \quad (2)$$

After solving of the above, a new iterate takes the form:

$$x_{k+1} = x_k + \alpha_k d_k. \quad (3)$$

Parameter α_k is known as step length. Its determination happens by means of an appropriate line search procedure, in order for an enough decrease in a merit function. The matrix H_k is a positive definite approximation of the Hessian matrix of the Lagrangian function, as shown in Eq.(1). A constrained task can usually be solved in smaller number of iterations that an unconstrained problem in nonlinear optimization using the SQP. The main reason for this fact, is that, the limits that are im-posed in the constrained optimization problem is a useful information that allows the optimizer to find feasibility with more easiness, by directing the search and set-ting the step length more efficiently [30].

2.2. Active Set Algorithm

The optimization function used in the MATLAB programming code was `fmincon`. While running, it attempts to find the minimum of a constrained nonlinear multi-variable using a nonlinear programming solver. It searches for a minimal value in a problem described by:

$$\min_x f(x) \text{ with } \begin{cases} c(x) \leq 0 \\ c_{eq}(x) = 0 \\ \mathbf{A}x \leq b \\ \mathbf{A}_{eq}x = b_{eq} \\ l_b \leq x \leq u_b, \end{cases} \quad (4)$$

where b and b_{eq} are vectors, \mathbf{A} and \mathbf{A}_{eq} matrices, $c(x)$ and $c_{eq}(x)$ are functions which return vectors, and $f(x)$ is a function returning a scalar value. All the $f(x)$, $c(x)$, and $c_{eq}(x)$ functions can be nonlinear and x , l_b , and u_b can be passed as vectors or matrices [31].

In a constrained optimization task, such as in this, the aim is usually to modify the problem, making it a sub-problem which requires less difficulty and can be solved in an iterative process. Early methods used for approximation of the constrained problem as a basic unconstrained one usually using a penalty function for constraints which are close or beyond the constraint boundary. This ensured solving the constrained problem by use several sequential parameterized unconstrained optimizations. These optimizations caused the sequence limit to converge to the constrained problem. The early methods are nowadays recognized of low efficiency, and therefore became obsolete. They have been replaced by newer attempts that are applying solving of the Karush-Kuhn-Tucker (KKT) equations. They need some conditions to achieve optimality on a constrained optimization problem. The KKT equations are both required and enough for a global solution point in the case of problems which belong to the convex programming problem class. To be considered as such, $f(x)$ and $G_i(x)$, $i = 1, m$, must be convex functions. These equations can be expressed as:

$$\begin{aligned} \nabla f(x^*) + \sum_{i=1}^m \lambda_i \nabla G_i(x^*) &= 0, \\ \lambda_i \nabla G_i(x^*) &= 0, \quad i = 1 \dots m_e \\ \lambda_i &\leq 0, \quad i = m_e + 1 \dots m. \end{aligned} \quad (5)$$

in addition to the original constraints:

$$\begin{aligned} g(x) &= 0 \\ h(x) &\leq 0 \\ x_l &\leq x \leq x_u. \end{aligned} \quad (6)$$

here the optimization parameters are held in the x vector, $q(x)$, $g(x)$ and $h(x)$ are functions. The first of the Eqs. (6) describes the canceling of the gradients between the objective function and the active constraints at the solution point. This requires the Lagrange multipliers (λ_i , $i = 1, m$) which are needed for the balance of the

deviations in the same magnitude of both, the objective function and constraint gradients, respectively. Because only the active constraints are included in the cancelling, the inactive ones must not be included in the process, and, therefore, the Lagrange multipliers equal to 0. This fact is implicitly included in the last two KKT equations. The solution of the KKT equations serve as a start point for various nonlinear programming algorithms, computing a direct calculation of the Lagrange multipliers. For instance, constrained quasi-Newton methods lead to superlinear convergence by doing the accumulation of the second-order information related to the KKT equations utilizing a quasi-Newton updating procedure. Such procedures are recognized as Sequential Quadratic Programming (SQP) methods. At each major iteration the Quadratic Programming (QP) sub-problem must be solved. This solving method is also known as Iterative or Recursive Quadratic Programming and Constrained Variable Metric [32].

3. Numerical Procedure

3.1. The FEM Models

As told before, two different models were designed: an internally reinforced plate, (model one) and a thin-walled beam (model two). Each of them has 3 geometric variables and 3 material properties were subject of the study. Because the nature of the two models is dissimilar, the chosen geometric variables were different in each case. However, the material properties – the Poisson ratio, density and Young's modulus were identical. Table 1 shows the variables of the FEM models for an internally reinforced plate and Table 2 for thin-walled beam. Figures 1 and 2 demonstrate the areas of these models, and also the variables used in simulations.

Table 1 Variables applied for the FEM model of an internally reinforced plate

Geometry		Material	
LG1	distance of inner ribs location to the center	EX	Young's modulus
LG2	rib height	PXY	Poisson ratio
LG3	plate thickness	DNS	density

Table 2 Variables used for the the FEM model of a thin-walled beam

Geometry		Material	
LG4	distance from center to corner at segment	EX	Young's modulus
LG5	half width of the central segment	PXY	Poisson ratio
LG6	height of the central part on the wall	DNS	density

In order to quantify the improvement in terms of displacement that was originated from the optimization routines, the mechanical behaviour of the final models was compared with that of the initial ones. This was done for both internally reinforced plate model and thin-walled beam model and for both maximum global

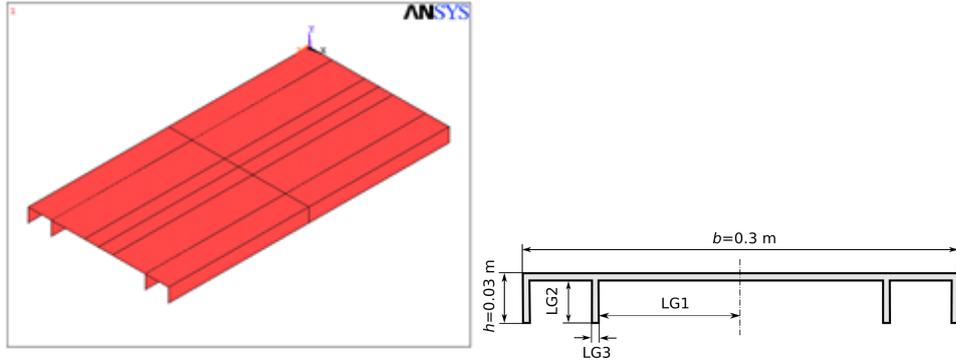


Figure 1 Areas, at the left and variables, at the right of the internally reinforced plate FEM model

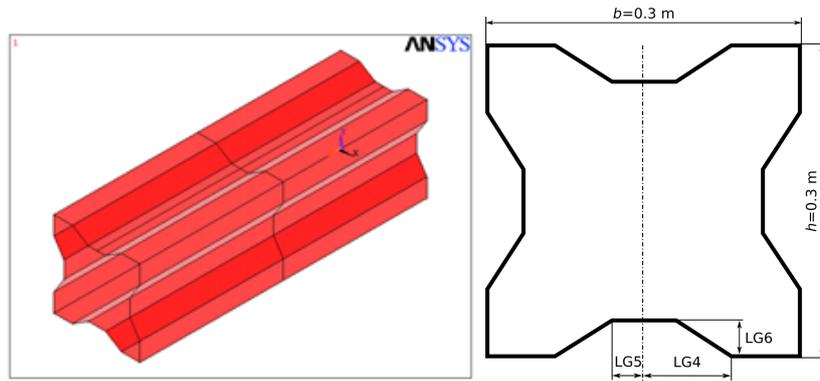


Figure 2 Areas at the left, and variables at the right applied for the thin-walled beam FEM model

negative and maximum global positive values. The formulae used are shown next in Eqs. (7–10).

$$Imp_{pnp}[\%] = \frac{Max_{nip} - Max_{nfp}}{Max_{nip}} \quad (7)$$

$$Imp_{ppp}[\%] = \frac{Max_{pip} - Max_{pfp}}{Max_{pip}} \quad (8)$$

$$Imp_{pnb}[\%] = \frac{Max_{nib} - Max_{nfb}}{Max_{nib}} \quad (9)$$

$$Imp_{ppb}[\%] = \frac{Max_{pib} - Max_{pfb}}{Max_{pib}} \quad (10)$$

where: Max_{nip} is the maximum negative value of the initial model for the internally reinforced plate, Max_{nfp} is the maximum negative value of the final (optimized) model for the internally reinforced plate, Max_{pib} is the maximum positive value of the initial model for the thin-walled beam and Max_{pfb} is the maximum positive value of the final (optimized) model for the thin-walled beam.

Imp_{pnp} [%] is defined by (7) and is the improvement of the maximum negative value of the final model in relation to the initial model for the plate, Imp_{ppp} [%] is defined by (8) and is the improvement of the maximum positive value of the final model in relation to the initial model for the plate, Imp_{pnb} [%] is defined by (9) and is the improvement of the maximum negative value of the final model in relation to the initial model for the internally reinforced beam and Imp_{ppb} [%] is defined by (10) and is the improvement of the maximum positive value of the final model in relation to the initial model for the internally reinforced beam.

3.2. Optimization Model

The developed *MATLAB* programming code was coupled with the Finite Element software *ANSYS*. *MATLAB* manages the optimization procedure and *ANSYS* calculates results regarding the stiffness behaviour of the two tested models. *ANSYS* creates and fills the displacement values to specific nodes in form of a text file (output file). *MATLAB* then reads and interprets the previously obtained results. In the next stage, *MATLAB* determines the new value of the objective function, updates variable values and sends the model to *ANSYS*, for calculating the displacements in a new iteration and for the updated values of the design variables. The objective function $q(U)$ is defined as:

$$q(U) = \sum_i |U_i| . \tag{11}$$

Herein, $q(U)$ describes the objective function, U_i is the displacement at each of the chosen nodal point and $| |$ is the absolute value of an argument. The objective function $q(U)$ was minimized by the *MATLAB* optimization toolbox function `fmincon` which performs a nonlinear constrained minimization process. Therefore, the obtained value is considered as better as closer it is to zero. Fig. 3 shows the flow of tasks between *ANSYS* and the *MATLAB*:

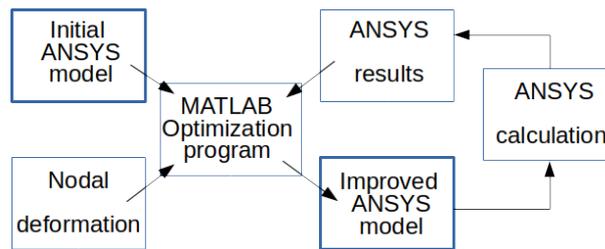


Figure 3 Scheme of the optimization procedures [2, 10, 11]

4. Results

4.1. Tensile Tests

To determine Young’s modulus value, the stress-strain chart in the elastic domain has to be plotted. From the obtained results a linear fit approximates the desired value. 0.2% offset has been assumed in the yield strength calculations. The obtained yield strength and Youngs modulus charts for both sample are shown in Figs. 4

and 5 in their left and right parts, respectively. The results were collected earlier by Authors in [10,11]. Numerical values are gathered in Table 3.

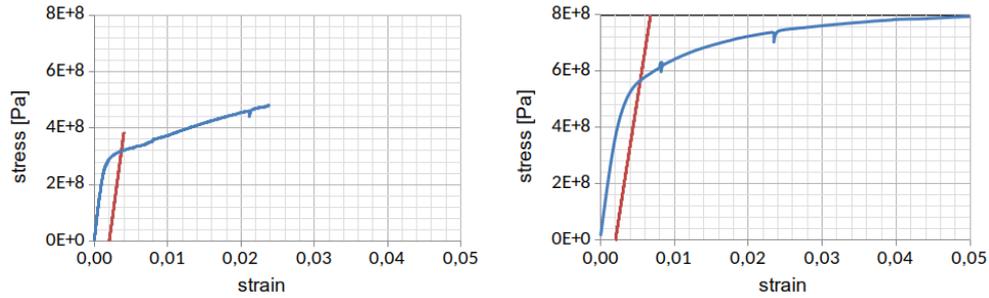


Figure 4 Results gathered for the yield strength for the sample 1 (left) and 2 (right) (stress-strain chart)

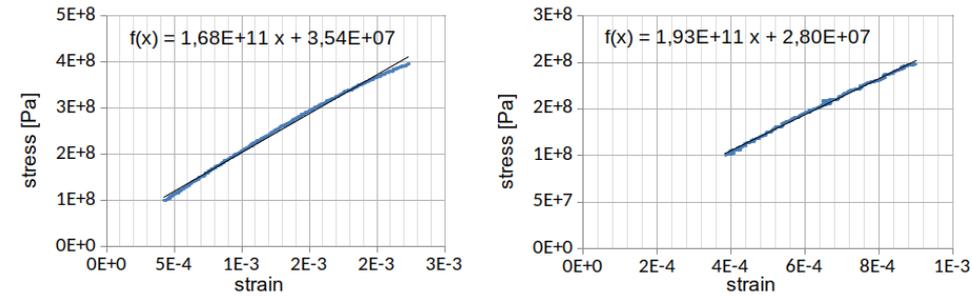


Figure 5 Linear fit applied to the stress-strain curve in the elastic domain determining the Young’s modulus value for the sample 1 (left) and 2 (right)

Table 3 Values of the Young’s modulus and yield strength obtained from tensile tests

sample	sample designation	Young’s modulus [GPa]	yield strength [MPa]
No. 1	DOCOL 600 DL Long	193.2	309.12
No. 2	DOCOL 800 DP Long	168.2	555.06

4.2. Elastic-only Tensile Test Results

Tables 4–10 show values of Young’s modulus range and the mean Poisson ratio for the samples 4, 5, 6 and 7, respectively, obtained during elastic-only tensile test by Authors in [11].

Table 4 Material properties values recorded for sample 4 in the 1st test

Load [N]	ϵ_{long}	ϵ_{trans}	Poisson cf. [-]	stress [MPa]	Young mod. [GPa]
2066	424	-124	0.2925	83.3	196.48
3043	624	-126	0.2011	123.0	196.64
4028	834	-248	0.2974	162,0	194.75
5000	1103	-335	0.3037	202.0	182.79

Table 5 Material properties values recorded for sample 4 in the 2nd test

Load [N]	ϵ_{long}	ϵ_{trans}	Poisson cf. [-]	stress [MPa]	Young mod. [GPa]
2010	416	-123	0.2957	81.0	194.83
3100	634	-187	0.2950	125. 0	197.16
4040	821	-243	0.2960	163.0	198.42
5010	1044	-311	0.2979	202.0	193.50

Table 6 Material properties values recorded for sample 5 in the 1st test

Load [N]	ϵ_{long}	ϵ_{trans}	Poisson cf. [-]	stress [MPa]	Young mod. [GPa]
2060	459	-129	0.2810	83.1	180,97
3010	663	-187	0.2821	121.0	183,06
4060	885	-252	0.2847	164.0	184,98
5020	1148	-333	0.2901	202.0	176,32

Table 7 Material properties values recorded for sample 5 in the 2nd test

Load [N]	ϵ_{long}	ϵ_{trans}	Poisson cf. [-]	stress [MPa]	Young mod. [GPa]
2100	465	-131	0.2817	85.0	182.10
3100	676	-191	0.2825	130.0	184.91
4045	873	-247	0.2829	160.0	186.83
5050	1107	-315	0.2846	200.0	183.95

Table 8 Material properties values recorded for sample 6 in the 1st test

Load [N]	ϵ_{long}	ϵ_{trans}	Poisson cf. [-]	stress [MPa]	Young mod. [GPa]
2018	437	-126	0.2883	81.4	186.20
4030	859	-244	0.2841	163.0	189.17
6020	1285	-369	0.2872	243.0	188.90

Table 9 Material properties values recorded for sample 6 in the 2nd test

Load [N]	ϵ_{long}	ϵ_{trans}	Poisson cf. [-]	stress [MPa]	Young mod. [GPa]
2040	428	-126	0.2944	82.3	192.19
4050	852	-243	0.2852	163.0	191.67
6100	1277	-365	0.2858	246.0	192.61

Table 10 Material properties values recorded for sample 7 in the 1st test

Load [N]	ϵ_{long}	ϵ_{trans}	Poisson cf. [-]	stress [MPa]	Young mod. [GPa]
2120	420	-117	0.2786	85.5	203.53
4060	867	-234	0.2699	164.0	188.82
6060	1325	-355	0.2679	244.0	184.42

Table 11 Material properties values recorded for sample 7 in the 2nd test

Load [N]	ϵ_{long}	ϵ_{trans}	Poisson cf. [-]	stress [MPa]	Young mod. [GPa]
2140	435	-118	0.2713	86.3	198.37
4120	879	-235	0.2673	166.0	189.00
6100	1313	-351	0.2673	246.0	187.33

4.3. Results of the Optimization Process

Table 13 presents the final values of the variables for the internally reinforced plate model. The objective function was determined as $q(x) = 0.92 \cdot 10^{-3}$ m for the geometric variables optimization process.

Table 12 Geometric variables after the optimization process for internally reinforced plate

geometric variables			desired material properties		
name	initial	final	name	initial	final
LG1 [m]	0.100	0.0250	EX [Pa]	$2.10 \cdot 10^{11}$	$2.31 \cdot 10^{11}$
LG2 [m]	0.030	0.0507	PXY [-]	0.29	0.435
LG3 [m]	0.002	0.0046	DNS [kg/m ³]	7890	7890

The final values of the variables for the model of the thin-walled beam are presented in Table 13. The objective function was resolved as $q(x) = 0.53 \cdot 10^{-3}$ m at the geometric variables optimization procedure.

Table 13 Geometric variables after the optimization process for thin-walled beam

geometric variables optimization			desired material properties		
name	initial	final	name	initial	final
LG4 [m]	0.085	0.0213	EX [Pa]	$2.10 \cdot 10^{11}$	$2.31 \cdot 10^{11}$
LG5 [m]	0.030	0.0076	PXY [-]	0.29	0.292
LG6 [m]	0.035	0.0805	DNS [kg/m ³]	7890	7890

In Figs. 6 and 7, results of calculated y -displacements for both initial and final (optimized) models are presented. Fig. 6 shows the internally reinforced plate model, and Fig. 7 shows the thin-walled beam model.

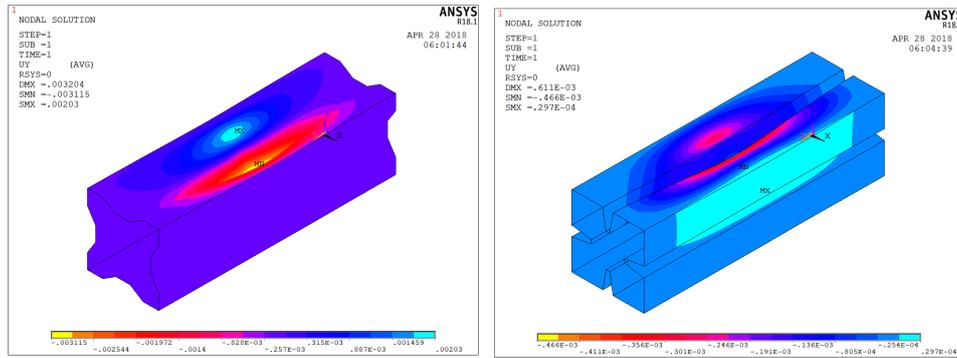


Figure 6 y -displacement results for the internally reinforced plate model: initial (left) and optimized (right)

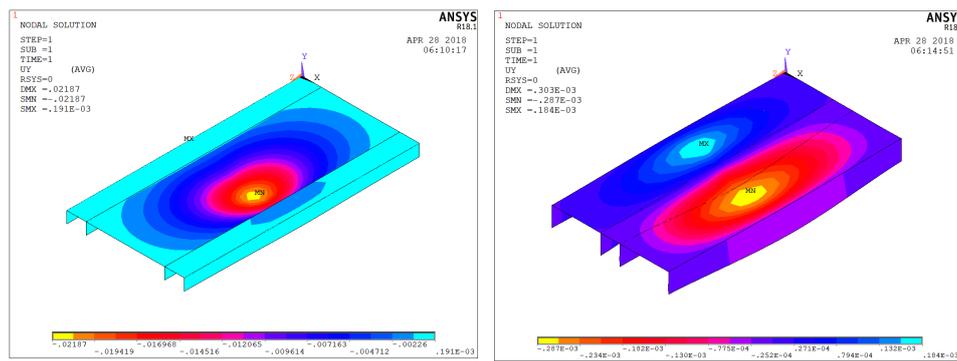


Figure 7 y -displacement results for the thin-walled beam model: initial (left) and optimized (right)

Tables 14 and 15 contains displacement results taken from Fig. 6, Tab. 14 and from Fig. 7, Tab. 15. The improvement measures were calculated using Eqs. (7–8), Tab. 14 and Eqs. (9–10), Tab. 15.

Table 14 displacement values and respective improvements for the plate model

	Max_{nlp}	Max_{nfp}	Imp_{pnp} [%]	Max_{pip}	Max_{pfp}	Imp_{ppp} [%]
Plate	0.003115	$0.466E^{-3}$	85.04	0.00203	$0.297E^{-4}$	98.54

Table 15 displacement values and respective improvements for the thin-walled beam model

	Max_{nib}	Max_{nfb}	Imp_{pnb} [%]	Max_{pib}	Max_{pfb}	Imp_{pnb} [%]
Beam	0.02187	$0.287E^{-3}$	98.69	$0.191E^{-3}$	$0.184E^{-3}$	3.66

5. Discussion and Conclusions

During the tensile tests, sample 1 has been detected as having the highest Young's modulus value equal to 193 GPa. Sample 2 had lower value of about 168 GPa. Elastic-only tensile tests results ranged for this variable between 180 and 200 GPa. These numbers are smaller than those for typical steel properties, being 210 GPa. The reason of such results may be mainly related to the production processes. In fact, during the phase when the material is formed and also as a consequence of gain in plasticity, the Young's modulus may become lower. The sample 1 shown a smaller value of yield strength, which can be understood as its designation was 600 and the sample 2 was 800. The above numbers should be recognized in relative terms for the yield strength, which means that a smaller number designation relates to a lower yield strength in comparison of the two samples. The Poisson ratio in elastic-only tensile test was about 0.29, which is typical value for steel. An except was the sample 6 presenting value of 0.27 only.

The internally reinforced plate model is shown to have both its maximum negative and maximum positive values significantly improved. The thin-walled beam model has a significant maximum negative improvement, while the maximum positive improvement is very mild. This can be seen by the values of the improvements shown in Tabs. 14 and 15.

In terms of geometric changes to the model's resultant from optimization processes, the optimized parts are geometrically more compact than the initial models. The variation of the geometric variables during optimization routines originates, therefore, models with better structural efficiency. Considering the optimization results, the obtained objective function value presents a significant improvement in the geometric variables optimization.

Therefore the conclusion of this work can be stated as, the developed Finite Element Model Updating Code can be utilized in process of optimization of geometric variables chosen in the problem solution. The tested samples present high yield strength values, which means the material can become a subject of application of high loadings before entering the plastic zone.

In a possible practical utilization of these results (beam elements of laser cutting machines or plotters), it is of utmost importance that that the material works al-

ways in the elastic zone. If not, the permanent deformation can affect the machine operation. Due to the consideration, the dual phase steel can be a good choice for a potential target application. It is shown that the selected material does not contribute in significant displacement reduction, because it possess a lower Young's modulus than what is theoretically expected. However, it is also shown that the displacement values can be significantly corrected through the optimization procedure of the geometry of the applied beam.

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