Research Article

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A thermoelastic microelongated layer immersed in an infinite fluid and subjected to laser pulse heating

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Abstract: The present investigation deals with the two-dimensional deformation because of laser pulse heating in a thermoelastic microelongated layer with a thickness of $2d$, which is immersed in an infinite nonviscous fluid. Normal mode analysis technique is applied to obtain the analytic expressions for displacement component, force stress, temperature distribution, and microelongation. The effect of elongation and laser pulse rise time on the derived components have been depicted graphically.

Keywords: thermoelasticity, microelongation, infinite fluid, laser pulse

1 Introduction

To model the behavior of materials having internal structure, classical theory is not sufficient. Eringen and Suhubi [1, 2] developed a nonlinear theory of microelastic solids. Later on, a theory in which material particles in solids can undergo macro-deformations as well as microrotations was formulated by Eringen [3–5], and this theory was named as "linear theory of micropolar elasticity." Then a theory of micropolar elastic solid with stretch was introduced by Eringen [6] in which he included axial stretch. Thermal effects were included in the micropolar theory by various authors [7–10]. Lord and Shulman [11] is one of the two important generalized theories of thermoelasticity, and the second one is the theory of temperature-rate-dependent thermoelasticity. In a review of thermodynamics of thermoelastic solids (TSs), Muller [12] proposed an entropy production inequality, with which he applied restrictions on a class of constitutive equations. A generalization of this inequality was proposed by Green and Laws [13]. Green and Lindsay [14] obtained another version of these constitutive equations. Suhubi [15] obtained these equations independently and explicitly, which contain two constants that act as relaxation times and transform all the equations of coupled theory.

Dhaliwal *et al.* [16] investigated thermoelastic interactions caused by a continuous line heat source in a homogeneous, isotropic, unbounded solid. Chandrasekharaiah and Srinath [17] studied thermoelastic interactions because of a continuous point heat source in a homogeneous and isotropic unbounded body. Sharma and Chauhan [18] discussed mechanical and thermal sources in a generalized thermoelastic half-space. Sarbani and Amitava [19] studied the transient disturbance in half-space because of moving internal heat source under L-S model and obtained the solution for displacements in the transformed domain. Youssef [20] solved the problem on a generalized thermoelastic infinite medium with a spherical cavity subjected to a moving heat source. Shaw and Mukhopadhyay [21, 22] discussed a few problems on thermoelastic microelongated medium.

Laser heating has become a very dominant aspect in the surface deformation of material in modern sciences. Laser is a very flexible device for carrying out change in the surfaces of materials.When the intensity is very high, laser interacts with the surface of solid and absorption takes place at the surface of solid. To modify the material as thin films, the microscopic two-step model is there, parabolic and hyperbolic. When a laser pulse heats a metal film, a thermoelastic wave is generated because of thermal expansion near the surface. Sun *et al.* [27] investigated laser-induced vibrations of microbeams in which he showed that a large thermal gradient exists at the boundaries for ultra-short-pulsed laser heating. Youssef and Al-Felali [28] discussed the effect of thermal loading because of laser pulse in generalized thermoelasticity problem. Youssef and El-Bary [29] studied the response

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owing to laser pulse heating in thermoelastic material. Othman *et al.* [30] discussed thermoelasticity under thermal loading caused by laser pulse. Othman and Hilal [31] discussed the influence of temperature dependent properties and gravity on porous TS because of laser pulse heating. Othman and Abd-Elaziz [32] studied the effect of thermal loading caused by laser pulse in generalized thermoelastic medium with voids in dual-phase lag model. Kumar *et al.* [33] discussed the thermomechanical interactions because of laser pulse in microstretch thermoelastic medium. Othman and Tantawi [34] investigated the effect of the gravitational field on a two-dimensional thermoelastic medium influenced by thermal loading caused by a laser pulse. Abbas and Marin [35] discussed applications of thermoelastic diffusion processes and the analytical solutions of a two-dimensional generalized thermoelastic diffusion problem because of laser pulse.

The field equation of motion for the displacement, microelongation, and temperature changes according to [23–25] is

$$(\lambda+\mu)u_{j,ij}+\mu u_{i,jj}-\beta_0\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T_{,i}+\lambda_0\phi_{,i} = \rho\ddot{u}_i \quad (1)$$

$$a_0\phi_{,ii}+\beta_1\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T-\lambda_1\phi-\lambda_0u_{j,j}=\frac{1}{2}\rho j_0\ddot{\phi} \quad (2)$$

$$K^{\star}T_{,ii}-\rho C^{\star}\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\dot{T} -\beta_0T_0\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\dot{u}_{k,k}-\dot{\beta}_1T_0\dot{\phi}+\rho\dot{Q}=0 \quad (3)$$

where Q is the heat input of the laser pulse that illuminates the plate surface and is given by $Q = \frac{I_0\gamma t}{2\pi r^2 t^{\star 2}}\exp\left(-\frac{y^2}{r^2}-\frac{t}{t^{\star}}\right)\exp\left(-\gamma x\right)$.

where I_0 is the energy absorbed, $t^{\star}$ is the pulse rise time, r is the beam radius, y is the heat deposition because of laser pulse that is assumed to decay exponentially within the solid, $\beta_0=(3\lambda+2\mu)\alpha_{t_1}$, $\beta_1=(3\lambda+2\mu)\alpha_{t_2}$, a_0, λ_0, λ_1 are microelongational constants, $C^{\star}$ is the specific heat at constant strain, $K^{\star}$ is the thermal conductivity, $\alpha_{t_1}$and α_{t_2} are the coefficent of linear thermal expansion where $T$is the temperature above reference temperature T_0, ϕ is the microelongational scalar, $\vec{u}=(u_i)$ is the displacement vector, and $k=2$ for Green–Lindsay (GL) theory.

The equations of motion and stress components in fluid are given by [26] as

$$\lambda^f u^f_{i,ij}=\rho^f\ddot{u}^f_i \quad (4)$$

$$t^f_{ij}=\lambda^f u^f_{r,r}\delta_{ij} \quad (5)$$

where $\vec{u}^f=(u^f_i)$ is the displacement vector; λ^f is the fluid constant, and ρ^f is the density of fluid.

We have restricted our analysis to the plane strain parallel to xy plane. A homogeneous isotropic, microelongated thermoelastic layer with a thickness of $2d$ occupying the region $-d\le x\le d$ with a displacement vector $\vec{u}$ given by $\vec{u}_i=(u_1,u_2,0)$, a displacement vector $\vec{u}^f$ for infinite nonviscous fluid as $\vec{u}^f_i=(u^f_1,u^f_2,0)$, is considered. The geometry of the problem is given in Figure 1.

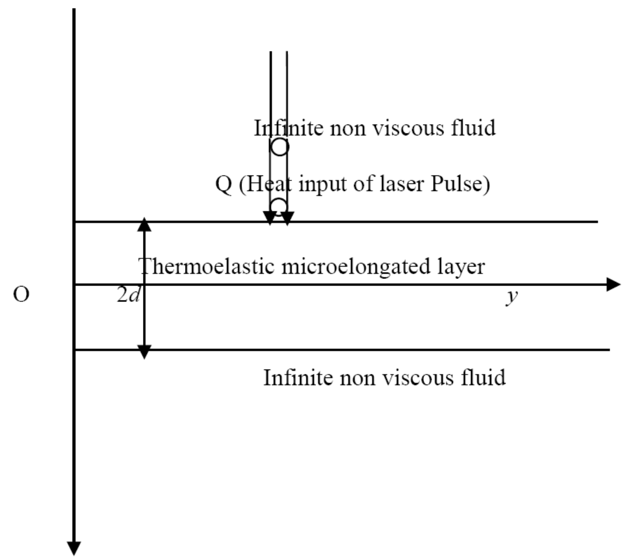

Figure 1: The geometry of the problem

Hence, equations (1)–(3) become

$$(\lambda+2\mu)\frac{\partial^2u_1}{\partial x^2}+(\lambda+\mu)\frac{\partial^2u_2}{\partial x\partial y}+\mu\frac{\partial^2u_1}{\partial y^2} -\beta_0\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}+\lambda_0\frac{\partial\phi}{\partial x}=\rho\frac{\partial^2u_1}{\partial t^2} \quad (6)$$

$$\mu\frac{\partial^2u_2}{\partial x^2}+(\lambda+\mu)\frac{\partial^2u_1}{\partial x\partial y}+(\lambda+2\mu)\frac{\partial^2u_2}{\partial y^2} -\beta_0\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}+\lambda_0\frac{\partial\phi}{\partial y}=\rho\frac{\partial^2u_2}{\partial t^2} \quad (7)$$

$$a_0\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial x^2}\right)+\beta_1\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T-\lambda_1\phi -\lambda_0\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)=\frac{1}{2}\rho j_0\frac{\partial^2\phi}{\partial t^2} \quad (8)$$

$$K^{\star}\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial x^2}\right)-\rho C^{\star}\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} -\beta_0T_0\left(\frac{\partial}{\partial t}+t_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}\right)-\dot{\beta}_1T_0\frac{\partial\phi}{\partial t}+\rho\frac{\partial Q}{\partial t} =0 \quad (9)$$

constitutive components of microelongational stress tensor are given by

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u_1}{\partial x} + \lambda\frac{\partial u_2}{\partial y} - \beta_0\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T + \lambda_0\phi \tag{10}$$

$$\sigma_{yy} = \lambda\frac{\partial u_1}{\partial x} + (\lambda + 2\mu)\frac{\partial u_2}{\partial y} - \beta_0\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T + \lambda_0\phi \tag{11}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right) \tag{12}$$

To simplify computation, we consider the following nondimensional variables:

$$x' = \frac{\omega^*}{c_1}x, \quad y' = \frac{\omega^*}{c_1}y, \quad u_i' = \frac{\omega^*\rho c_1}{\beta_0 T_0}u_i, \quad t' = \omega^* t, \tag{13}$$
$$t_0' = \omega^* t_0, \quad t_1' = \omega^* t_1, \quad \sigma_{ij}' = \frac{\sigma_{ij}}{\beta_0 T_0}, \quad \phi' = \frac{\lambda_0}{\beta_0 T_0}\phi,$$
$$T' = \frac{T}{T_0}, \quad Q' = \frac{1}{C^* T_0}Q.$$

where $\omega^* = \frac{\rho c_1^2 C^*}{K^*}$ and $c_1^2 = \frac{\lambda+2\mu}{\rho}$.

Using the above nondimensional variables given by (13) in (6)–(9) (after dropping superscripts), we get

$$\frac{\partial^2 u_1}{\partial x^2} + l_2\frac{\partial^2 u_2}{\partial x\partial y} + l_3\frac{\partial^2 u_1}{\partial y^2} - \left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} + \frac{\partial\phi}{\partial x} = \frac{\partial^2 u_1}{\partial t^2}, \tag{14}$$

$$l_3\frac{\partial^2 u_2}{\partial x^2} + l_2\frac{\partial^2 u_1}{\partial x\partial y} + \frac{\partial^2 u_2}{\partial y^2} - \left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y} + \frac{\partial\phi}{\partial y} = \frac{\partial^2 u_2}{\partial t^2}, \tag{15}$$

$$\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial x^2}\right) + l_4\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T - l_5\phi - l_6\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y}\right) = l_7\frac{\partial^2\phi}{\partial t^2}, \tag{16}$$

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial x^2}\right) - l_8\left(1 + t_0\delta_{1k}\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} - l_9\left(\frac{\partial}{\partial t} + t_0\delta_{1k}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y}\right) - l_{10}\frac{\partial\phi}{\partial t} + l_{11}\frac{\partial Q}{\partial t} = 0. \tag{17}$$

The constitutive relations (10)–(12) in dimensionless form reduces to

$$\sigma_{xx} = \frac{\partial u_1}{\partial x} + l_{12}\frac{\partial u_2}{\partial y} - \left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T + \phi, \tag{18}$$

$$\sigma_{yy} = l_{12}\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} - \left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)T + \phi, \tag{19}$$

$$\sigma_{xy} = l_3\left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right), \tag{20}$$

where $l_2 = \frac{(\lambda+\mu)}{\rho c_1^2}$, $l_3 = \frac{\mu}{\rho c_1^2}$, $l_4 = \frac{\beta_1\lambda_0 c_1^2}{a_0\omega^*\beta_0}$, $l_5 = \frac{\lambda_1 c_1^2}{a_0\omega^*}$, $l_6 = \frac{\lambda_0{}^2}{\rho a_0\omega^*}$, $l_7 = \frac{\rho j_0\omega^* c_1^2}{2a_0}$, $l_8 = \frac{\rho C^* c_1^2}{K^*\omega^*}$, $l_9 = \frac{\beta_0^2 T_0}{K^*\omega^*\rho}$, $l_{10} = \frac{\beta_0\beta_1 T_0 c_1^2}{K^*\omega^*\lambda_0}$, $l_{11} = \frac{\rho C^* c_1^2}{K^*\omega^*}$, and $l_{12} = \frac{\lambda}{\rho c_1^2}$.

2 Solution of the Problem

Here, we use normal mode analysis technique to find the solution of the considered physical variables in the following form:

$$(u_i, T, \phi, \sigma_{ij})(x, y, t) = (u_i^*, T^*, \phi^*, \sigma_{ij}^*)(x)e^{\omega t+iby} \tag{21}$$

where ω is the complex frequency, b is the wave number in y-direction, and $u_i^*(x)$, $T^*(x)$, $\phi^*(x)$, and $\sigma_{ij}^*(x)$ are the amplitudes of field quantities.

Using (21) in (14)–(20), we get

$$(D^2 - B_1)u_1^* + ibl_2 Du_2^* - B_2 DT^* + D\phi^* = 0, \tag{22}$$

$$ibl_2 Du_1^* + (l_3 D^2 - B_3)u_2^* - ibB_2 T^* + ib\phi^* = 0, \tag{23}$$

$$-l_6 Du_1^* - ibl_6 u_2^* + B_2 l_4 T^* + (D^2 - B_4)\phi^* = 0, \tag{24}$$

$$-l_9 B_6 Du_1^* - ibl_9 B_6 u_2^* + (D^2 - B_7)T^* - l_{10}\omega\phi^* = Q_1 F(y, t)\exp(-\gamma x), \tag{25}$$

$$\sigma_{xx}^* = Du_1^* + ibl_{12}u_2^* - B_2 T^* + \phi^*, \tag{26}$$

$$\sigma_{yy}^* = l_{12}Du_1^* + ibu_2^* - B_2 T^* + \phi^*, \tag{27}$$

$$\sigma_{xy}^* = l_3\left(ibu_1^* + Du_2^*\right), \tag{28}$$

where $D \equiv \frac{d}{dx}$, $F(y, t) = \left(1 - \frac{t}{t^*}\right)\exp\left(-\frac{y^2}{r^2} - \frac{t}{t^*} - \omega t - iby\right)$, $Q_1 = \frac{-l_{11}I_0\gamma}{2\pi r^2 t^{*2}}$, $B_1 = \omega^2 + l_3 b^2$, $B_2 = (1 + t_1\delta_{2k}\omega)$, $B_3 = \omega^2 + b^2$, $B_4 = b^2 + l_5 + l_7\omega^2$, $B_5 = (1 + t_0\delta_{1k}\omega)$, $B_6 = \omega(1 + t_0\delta_{1k}\omega)$, and $B_7 = b^2 + l_8 A_5\omega$.

Eliminating $u_2^*(x)$, $T^*(x)$, and $\phi^*(x)$ from equations (22)–(25), we get the differential equation for $u_1^*(x)$ as

$$(D^8 + AD^6 + BD^4 + CD^2 + E)u_1^*(x) = RF(y, t)\exp(-\gamma x) \tag{29}$$

where

$$A = \frac{-1}{l_3}\left[l_3(B_4+B_7) - B_3 + l_3B_1 + l_3l_6 + B_2l_3l_9B_6 + b^2l_2^2\right]$$

$$B = \frac{-1}{l_3}\Big[-B_2l_4l_{10}l_3\omega + l_3B_4B_7 + B_3(B_4+B_7) - b^2B_2B_6l_9 + b^2B_6 - B_1l_3(B_4+B_7) + B_1B_3 - b^2l_2^2(B_4+B_7) + l_3l_6l_{10}B_2\omega - l_3l_9B_2B_4B_6 - l_9B_2B_3B_6 - l_3l_6B_7 - l_3l_4l_9B_2B_6 - B_3l_6\Big]$$

$$C = \frac{-1}{l_3}\Big[B_2B_3l_4l_{10}\omega + B_3B_4B_7 - b^2l_6l_{10}B_2\omega + b^2l_9B_2B_4B_6 - b^2l_6B_7 - b^2l_4l_9B_2B_6 + B_1B_2l_3l_4l_{10}\omega^2 - l_3B_1B_4B_7 + B_1B_3(B_4+B_7) + b^2B_1B_2B_6l_9 + b^2l_2^2B_2l_4l_{10}\omega + b^2l_2^2B_4B_7 - 2b^2B_7l_2l_6 - 2b^2B_2B_6l_2l_4l_9 - l_6l_{10}B_2B_3\omega + B_2B_3B_4B_6l_9 + B_3B_7l_6 + B_2B_3B_6l_4l_9 - b^2l_6B_1\Big]$$

$$E = \frac{-1}{l_3}\Big[-l_4l_{10}B_1B_2B_3\omega - B_1B_3B_4B_7 + b^2l_6l_{10}B_1B_2\omega - b^2l_9B_1B_2B_4B_6 + b^2l_6B_1B_7 + b^2l_4l_9B_1B_2B_6\Big]$$

$$R = \frac{ibB_2Q_1}{l_3}\Big[-l_3\gamma^5 + \left\{(B_3 - b^2l_2) - l_3(l_4 - B_4)\right\}\gamma^3 + (l_4 - B_4)(B_3 - b^2l_2)\gamma\Big]$$

Equation (29) can be written as

$$(D^2-k_1^2)(D^2-k_2^2)(D^2-k_3^2)(D^2-k_4^2)u_1^\star(x) = RF(y,t)\exp(-\gamma x) \tag{30}$$

where k_n^2 is the roots of equation (29).

The solution of equation (29) can be considered in series form as

$$u_1^\star(x) = \sum_{n=1}^{4}[L_n(b,\omega)e^{-k_nx}] + \sum_{n=1}^{4}[L_{n+4}(b,\omega)e^{k_nx}] + \xi\exp(-\gamma x) \tag{31}$$

$$u_2^\star(x) = \sum_{n=1}^{4}[L_n'(b,\omega)e^{-k_nx}] + \sum_{n=1}^{4}[L_{n+4}'(b,\omega)e^{k_nx}] + \xi_1\exp(-\gamma x) \tag{32}$$

$$T^\star(x) = \sum_{n=1}^{4}[L_n''(b,\omega)e^{-k_nx}] + \sum_{n=1}^{4}[L_{n+4}''(b,\omega)e^{k_nx}] + \xi_2\exp(-\gamma x) \tag{33}$$

$$\phi^\star(x) = \sum_{n=1}^{4}[L_n'''(b,\omega)e^{-k_nx}] + \sum_{n=1}^{4}[L_{n+4}'''(b,\omega)e^{k_nx}] + \xi_3\exp(-\gamma x) \tag{34}$$

where $L_n(b,\omega)$, $L_n'(b,\omega)$, $L_n''(b,\omega)$, $L_n'''(b,\omega)$ are specific function depending on b and ω.

Using (31)–(34) in (22)–(25), we get

$$L_n'(b,\omega) = R_{1n}L_n(b,\omega);\, L_{n+4}'(b,\omega) = R_{1(n+4)}L_{n+4}(b,\omega) \tag{35}$$

$$L_n''(b,\omega) = R_{2n}L_n(b,\omega);\, L_{n+4}''(b,\omega) = R_{2(n+4)}L_{n+4}(b,\omega) \tag{36}$$

$$L_n'''(b,\omega) = R_{3n}L_n(b,\omega);\, L_{n+4}'''(b,\omega) = R_{3(n+4)}L_n(b,\omega) \tag{37}$$

Using (35)–(37), the solution of physical quantities in series form can be rewritten as

$$u_2^\star(x) = \sum_{n=1}^{4}[R_{1n}L_n(b,\omega)e^{-k_nx}] + \sum_{n=1}^{4}[R_{1(n+4)}L_{(n+4)}(b,\omega)e^{k_nx}] + \xi_1\exp(-\gamma x) \tag{38}$$

$$T^\star(x) = \sum_{n=1}^{4}[R_{2n}L_n(b,\omega)e^{-k_nx}] + \sum_{n=1}^{4}[R_{2(n+4)}L_{(n+4)}(b,\omega)e^{k_nx}] + \xi_2\exp(-\gamma x) \tag{39}$$

$$\phi^\star(x) = \sum_{n=1}^{4}[R_{3n}L_n(b,\omega)e^{-k_nx}] + \sum_{n=1}^{4}[R_{3(n+4)}L_{(n+4)}(b,\omega)e^{k_nx}] + \xi_3\exp(-\gamma x) \tag{40}$$

$$\sigma_{xx}^\star(x) = \sum_{n=1}^{4}[R_{4n}L_n(b,\omega)e^{-k_nx}] + \sum_{n=1}^{4}[R_{4(n+4)}L_{(n+4)}(b,\omega)e^{k_nx}] + \xi_4\exp(-\gamma x) \tag{41}$$

$$\sigma_{yy}^\star(x) = \sum_{n=1}^{4}[R_{5n}L_n(b,\omega)e^{-k_nx}] + \sum_{n=1}^{4}[R_{5(n+4)}L_{(n+4)}(b,\omega)e^{k_nx}] + \xi_5\exp(-\gamma x) \tag{42}$$

$$\sigma^{\star}_{xy}(x) = \sum_{n=1}^{4}[R_{6n}L_n(b,\omega)e^{-k_n x}] + \sum_{n=1}^{4}[R_{6(n+4)}L_{(n+4)}(b,\omega)e^{k_n x}] + \xi_6 \exp(-\gamma x) \quad (43)$$

where

$$R_{1n} = \frac{ib[B_1 - (l_2-1)k_n^2]}{[(B_3 - b^2 l_2)k_n - l_3 k_n^3]},$$

$$R_{1(n+4)} = \frac{ib[B_1 - (l_2-1)k_n^2]}{[(b^2 l_2 - B_3)k_n + l_3 k_n^3]},$$

$$R_{2n} = \frac{[l_3 k_n^4 - (B_4 l_3 + B_3)k_n^2 + (B_3 B_4 - b^2 l_6)]R_{1n}}{ib[B_2(k_n^2 - B_4) + B_2 l_4]} - \frac{ib[l_2 k_n^3 - (l_2 B_4 - l_6)k_n]}{ib[B_2(k_n^2 - B_4) + B_2 l_4]},$$

$$R_{3n} = \frac{(k_n^2 - B_1 - ibl_2 k_n R_{1n} + B_2 k_n R_{2n})}{k_n},$$

$$R_{3(n+4)} = \frac{(B_1 - k_n^2 - ibl_2 k_n R_{1(n+4)} + B_2 k_n R_{2(n+4)})}{k_n},$$

$$R_{4n} = ibl_{12}R_{1n} - B_2 R_{2n} + R_{3n} - k_n,$$

$$R_{4(n+4)} = ibl_{12}R_{1(n+4)} - B_2 R_{2(n+4)} + R_{3(n+4)} + k_n,$$

$$R_{5n} = ibR_{1n} - B_2 R_{2n} + R_{3n} - l_{12}k_n,$$

$$R_{6n} = l_3(ib - k_n R_{1n}),\ R_{6(n+4)} = l_3(ib + k_n R_{1(n+4)}),$$

$$\xi = \frac{RF(y,t)}{\gamma^8 + A\gamma^6 + B\gamma^4 + C\gamma^2 + E},$$

$$\xi_1 = \frac{ib[(1-l_2)\gamma^2 - B_1]\xi}{[(B_3 - b^2 l_2)\gamma - l_3\gamma^3]},$$

$$\xi_2 = \frac{[l_3\gamma^4 - (B_4 l_3 + B_3)\gamma^2 + (B_3 B_4 - b^2 l_6)]\xi_1}{ib[B_2(\gamma^2 - B_4) + B_2 l_4]} - \frac{ib[l_2\gamma^3 - (l_2 B_4 - l_6)\gamma]\xi}{ib[B_2(\gamma^2 - B_4) + B_2 l_4]},$$

$$\xi_3 = \frac{(\gamma^2 - B_1)\xi - ibl_2\gamma\xi_1 + B_2\gamma\xi_2}{\gamma},$$

$$\xi_4 = ibl_{12}\xi_1 - B_2\xi_2 + \xi_3 - \gamma\xi,$$

$$\xi_5 = ib\xi_1 - B_2\xi_2 + \xi_3 - l_{12}\gamma\xi,$$

$$\xi_6 = ibl_3\xi - \gamma l_3\xi_1.$$

Similarly, for infinite nonviscous fluid, the solutions are of the form

$$\overline{u}_1^f(z) = L_9(a,\omega)e^{-k_5 x} + L_{10}(a,\omega)e^{k_5 x}, \quad (44)$$

$$\overline{u}_2^{f(z)} = L_9'(a,\omega)e^{-k_5 x} + L_{10}'(a,\omega)e^{k_5 x}, \quad (45)$$

where $L_9(a,\omega)$, $L_{10}(a,\omega)$ and $L_9'(a,\omega)$, $L_{10}'(a,\omega)$ are specific function depending on a, ω, and k_5^2 are roots of the characteristic equation,

$$\left(D^2 - k_5^2\right)u_1^{f\star}(z) = 0 \quad (46)$$

where $k_5^2 = a^2 - l_1\omega^2$
and $l_1 = \frac{\rho^f c_1^2}{\lambda^f}$.

Thus we have

$$\overline{u}_2^{f^\star}(z) = R_{71}L_9(a,\omega)e^{-k_5 x} + R_{72}L_{10}(a,\omega)e^{k_5 x}, \quad (47)$$

$$\overline{\sigma}_{xx}^{f^\star}(z) = R_{81}L_9(a,\omega)e^{-k_5 x} + R_{82}L_{10}(a,\omega)e^{k_5 x}, \quad (48)$$

where $R_{71} = \frac{k_5^2 - l_1\omega^2}{ia(-k_5)}$, $R_{72} = \frac{k_5^2 - l_1\omega^2}{iak_5}$, $R_{81} = \frac{(\lambda^f)(iaR_{71}+k_5)}{\rho c_1^2}$, and $R_{82} = \frac{(\lambda^f)(iaR_{72}-k_5)}{\rho c_1^2}$.

3 Boundary Conditions

To determine the constants L_n ($n = 1, 2, \ldots, 10$), the boundary conditions at $x = \pm d$ are

$$\text{(i)}\quad (\sigma_{xx})_s = (\sigma_{xx}) \quad (49)$$

$$\text{(ii)}\quad (\sigma_{xy})_s = 0 \text{ at } x = \pm d,$$

$$\text{(iii)}\quad \left(\frac{\partial u_2}{\partial x}\right)_s = \left(\frac{\partial u_2}{\partial x}\right)_f \text{ at } x = \pm d,$$

$$\text{(iv)}\quad \left(\frac{\partial T}{\partial x}\right)_s = 0 \text{ at } x = \pm d,$$

$$\text{(v)}\quad \phi = 0 \text{ at } x = \pm d.$$

Using the expressions for $(\sigma_{xx})_s$, $(\sigma_{xy})_s$, $(u_2)_s$, ϕ, $(\sigma_{xy})_f$, $(u_2)_f$, and T in (49), we get

$$\sum_{n=1}^{4}[R_{4n}L_n e^{-k_n d} + R_{4(n+4)}L_{(n+4)}e^{k_n d}] - R_{81}L_9 e^{-k_5 d} = -\xi_4 e^{-\gamma d},$$

$$\sum_{n=1}^{4}[R_{4n}L_n e^{k_n d} + R_{4(n+4)}L_{(n+4)}e^{-k_n d}] - R_{82}L_{10}e^{-k_5 d} = -\xi_4 e^{\gamma d},$$

$$\sum_{n=1}^{4}[R_{6n}L_n e^{-k_n d} + R_{6(n+4)}L_{(n+4)}e^{k_n d}] = -\xi_6 e^{-\gamma d},$$

$$\sum_{n=1}^{4}[R_{6n}L_n e^{k_n d} + R_{6(n+4)}L_{(n+4)}e^{-k_n d}] = -\xi_6 e^{\gamma d},$$

$$\sum_{n=1}^{4}[R_{1n}L_n e^{-k_n d} + R_{1(n+4)}L_{(n+4)}e^{k_n d}] - R_{71}L_9 e^{-k_5 d} = -\xi_1 e^{-\gamma d},$$

$$\sum_{n=1}^{4}[R_{1n}L_n e^{k_n d} + R_{1(n+4)}L_{(n+4)}e^{-k_n d}] - R_{72}L_9 e^{-k_5 d} = -\xi_1 e^{\gamma d},$$

$$\sum_{n=1}^{4}[-k_n R_{2n}L_n e^{-k_n d} + k_n R_{2(n+4)}L_{(n+4)}e^{k_n d}] = -\xi_2\gamma e^{-\gamma d},$$

$$\sum_{n=1}^{4}[-k_n R_{2n}L_n e^{k_n d} + k_n R_{2(n+4)}L_{(n+4)}e^{-k_n d}] = -\xi_2\gamma e^{-\gamma d},$$

$$\sum_{n=1}^{4}[R_{3n}L_n e^{-k_n d} + R_{3(n+4)}L_{(n+4)}e^{k_n d}] = -\xi_3 e^{-\gamma d},$$

$$\sum_{n=1}^{4}[R_{3n}L_n e^{k_n d} + R_{3(n+4)}L_{(n+4)}e^{-k_n d}] = -\xi_3 e^{-\gamma d}.$$

Solving the above system of nonhomogeneous equations, we get the values of constants L_n (n = 1, 2, …, 10) and, hence, obtain the components of normal displacement, normal force stress, temperature distribution, and microelongation for microelongated thermoelastic layer under laser pulse heating.

4 Particular Case

If we neglect microelongation effect, that is, $\lambda_0 = \beta_1 = \lambda_1 = a_0 = j_0 = 0$, we obtain the results for TS.

5 Numerical Results, Discussion, and Conclusions

For numerical computations, we consider the values of constants for aluminum epoxy-like material as [22] $\lambda = 7.59 \times 10^{10}$ N/m^2, $\mu = 1.89 \times 10^{10}$ N/m^2, $a_0 = 0.61 \times 10^{-10}$ N, $\rho = 2.19 \times 10^3$ kg/m^3 $\beta_1 = 0.05 \times 10^5$ N/m^{2}K, $\beta_0 = 0.05 \times 10^5$ N/m^{2}K, $C_E = 966$ J/(kg K), $T_0 = 293$ K, $j_0 = 0.196 \times 10^{-4}$ m^2, $\lambda_0 = \lambda_1 = 0.37 \times 10^{10}$ N/m^2, $t_0 = 0.01$, $K = 252$ J/ms K.

The physical constants for water as nonviscous fluid are given by [26] $\lambda^f = 2.14 \times 10^9$ N/m^2 and $\rho^f = 10^3$ kg/m^3.

The computations are carried out for the value of nondimensional time $t = 0.2$ in the range $0 \le y \le 1.0$ and on the surface $x = 1.0$. The numerical values for normal displacement, normal force stress, temperature distribution, and microelongation are shown in Figures 2–5 for generalized theory (GL theory) by taking $\delta_{1k} = 0$, $\delta_{2k} = 1$, and r = 10 μm,I_0 = 10 J/m, γ = 50/m, $\omega = \omega_0 + \iota\zeta$, , $\omega_0 = -0.2$, $\zeta = 0.1$, and $b = 0.7$ for

(a) Thermoelastic microelongated solid (TMS) with a pulse rise time of $t^* = 0.1$ by solid line with dashed symbol ◊.
(b) TMS with a pulse rise time of $t^* = 0.01$ by dashed line with centered symbol ■.
(c) TS with a pulse rise time of $t^* = 0.1$ by dashed line with centered symbol ▲.
(d) TS with a pulse rise time of $t^* = 0.01$ by dashed line with centered symbol ×.

6 Discussions

The variations of normal displacement and temperature distribution are similar in nature for TMS and TS in the range $0 \le y \le 1.0$. The values for TS for a pulse rise time of $t^* = 0.1$ are more as compared to the values at $t^* = 0.01$ with the values decreasing very sharply in the range $0 \le y \le 0.2$, showing the appreciable effect of laser pulse heating, which approaches to zero with the increase in horizontal distance. The variations of normal displacement and temperature distribution are illustrated in Figures 2 and 3.

The variations of normal force stress are similar for TMS and TS for $t^* = 0.1$ as that of normal displacement and temperature distribution, with the values coinciding for $t^* = 0.01$ the variations of normal force stress are shown in Figure 4.

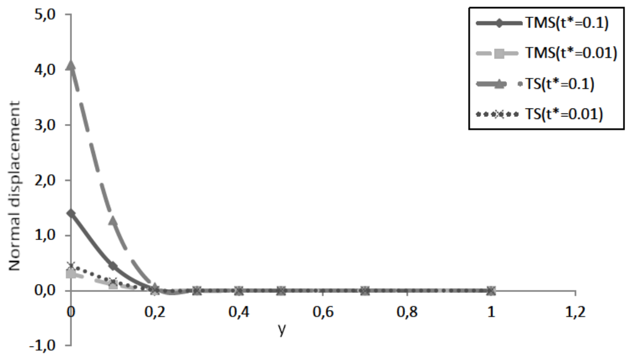

Figure 2: Variations of normal displacement with horizontal distance

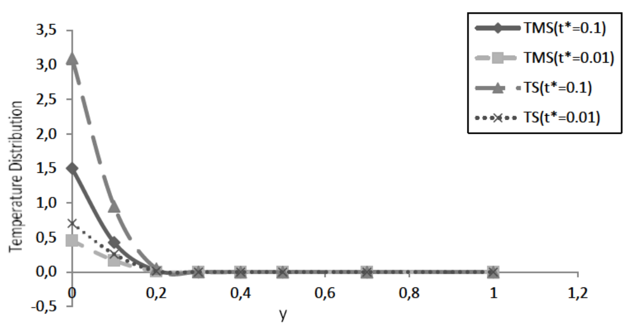

Figure 3: Variations of temperature distribution with horizontal distance

From Figure 5, it is clear that the variations of microelongation is more for TMS at $t^{*} = 0.01$ than that for which decreases sharply in the range $0 \leq y \leq 0.2$ and then converges to zero with the increase in horizontal distance. The variations of microelongation are shown in Figure 5.

7 Conclusion

(a) An analytical solution of the problem on thermoelastic microelongated layer surrounded by infinite fluid under the effect of laser pulse heating is developed.

(b) A significant effect of laser pulse heating and pulse rise time is observed on all the quantities, that is, normal displacement, temperature distribution, and normal force stress and microelongation.

(c) All the physical quantities, that is, normal displacement, temperature distribution, normal force stress, and microelongation approaches to zero very sharply as the horizontal distance increases, that is, as $y \to \infty$.

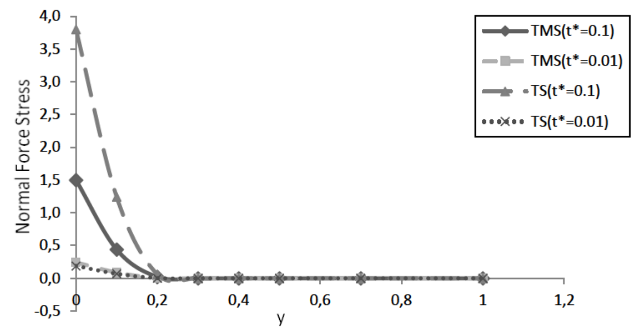

Figure 4: Variations of normal force stress with horizontal distance

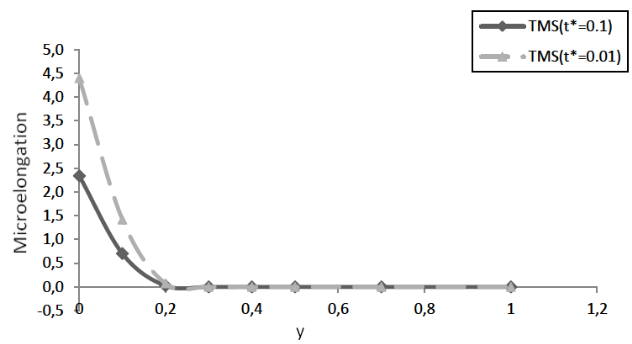

Figure 5: Variations of microelongation with horizontal distance

(d) Microelongation has an appreciable effect on all the considered physical quantities.

(e) The applied boundary conditions play an prominent role in deformation of the solid.

(f) The laser pulse heating is used in the geological treatments of the material particles and to modify the surface properties of the material.

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