Research Article

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Elastodynamical disturbances due to laser irradiation in a microstretch thermoelastic medium with microtemperatures

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Abstract: This research is concerned with the study of mechanical disturbances due to presence of ultra-short laser pulse as input heat source in a microstretch thermoelastic medium with microtemperatures. The medium is subjected to normal and tangential forces. The solution of the problems is developed in terms of normal modes. Mathematical expressions have been obtained for normal stress, tangential stress, microstress and temperature change. The numerically computed results are shown graphically. A mathematical model has been developed and various stress quantities have been analyzed. Some particular cases are also derived from the present investigation.

Keywords: microstretch-thermoelastic, laser heat source, normal mode analysis, microtemperature

1 Introduction

Eringen [1] developed the theory and basic equations of microstretch thermoelastic solids. Microstretch continuum is a model for Bravais lattice having basis on the atomic level and two-phase dipolar substance having a core on macroscopic level. Examples of microstretch thermoelastic materials are composite materials filled with chopped elastic fibers, porous elastic fluids whose pores have gases or inviscid liquids, or other elastic inclusions and liquid–solid crystal. Marin [2, 3] established the solution of equations in microstretch thermoelasticity and in elasticity of dipolar bodies with voids.

Laser technologies have a lot of utilities in medical science, industries, metallurgies, and nondestructive testing and evolution. High-rated thermal processes are interesting in the development of theories of thermoelasticity, due to thermal-mechanical coupling. The thermal shock creates very fast movements in the internal molecular particles, which causes an increase in very significant inertial forces and vibrations. The ultra-short lasers have pulse durations ranging from nanoseconds to femto seconds. In irradiation of ultra-short pulsed laser, the high-intensity energy flux and ultrashort duration lead to a very large thermal gradient. So, in these cases, Fourier law of heating is no longer valid. Rose [4] developed an analytical mathematical basis for point laser source. Scruby *et al.* [5] studied the point source ultrasonic generation by lasers. A new laser generation model was presented by Spicer [6] and McDonald [7]. Al-Huniti and Al-Nimr [8] studied a problem related to laser ultrasound in thermoelastic materials. Thermoelastic behavior in metal plates due to laser interactions using fractional theory of thermoelasticity was studied by Ezzat *et al.* [9]. A comparison in context of four theories of thermoelasticity was presented by Youssef *et al.* [10]. A generalized thermoelastic diffusion problem for a thick plate irradiated by thermal laser was discussed by Elhagary [11]. Kumar, Kumar and Singh [12] recently studied the thermomechanical interactions of an ultra-laser pulse with microstretch thermoelastic medium.

Grot [13] developed a theory of thermoelasticity of elastic solids with microelements possessing microtemperatures. The Clausius–Duhemin relation is modified to include microtemperatures, and the first-order moment of the energy relations are included. Riha [14] discussed heat conduction in solids with microtemperatures. The linear theory of thermoelasticity with microtemperatures for elastic materials was derived by Iesan and Quintanilla [15]. Iesan and Quintanilla [16] proposed the theory of micromorphic elastic solids with microtemperatures. Different type of problems in thermoelasticity with microtemperatures were discussed by Iesan [17]. Chirita *et al.* [18] discussed some important aspects in the theory of thermoelasticity with microtemperatures. Iesan [19] extended ther-

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moelasticity of bodies with microstructures and microtemperatures.

In this research, a model has been developed for microstretch thermoelastic solid with microtemperature in the presence of input ultra-short laser pulse. The stress components and temperature distribution have been computed numerically. The resulting expressions are then applied to the problem of a microstretch thermoelastic medium with microtemperatures whose boundary is subjected to two types of loads, *i.e.*, normal force and tangential load. The resulting quantities are shown graphically to show the effect of microtemperature and input laser heat source.

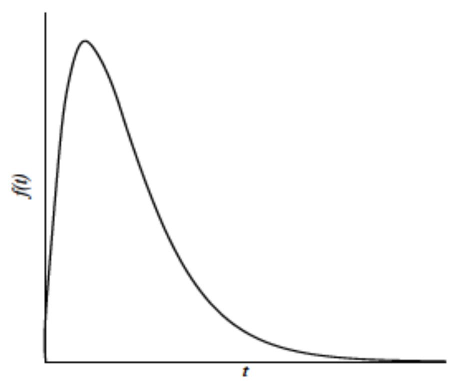

Figure 1: Temporal profile of $f(t)$

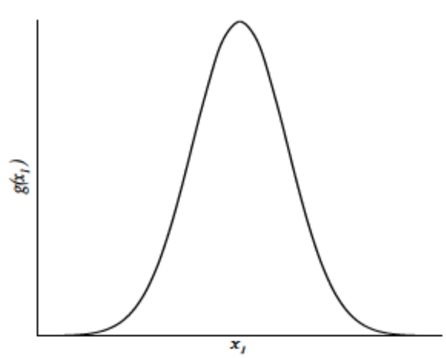

Figure 2: Profile of $g(x_1)$

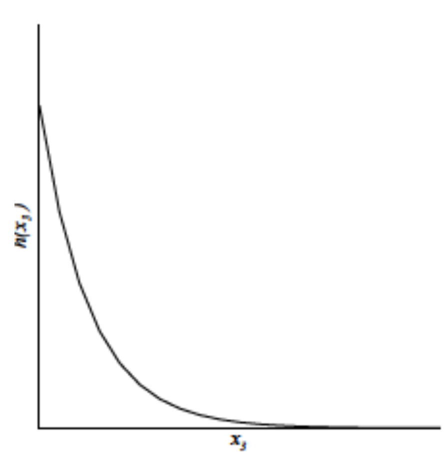

Figure 3: Profile of $h(x_3)$

2 Basic equations

Following Eringen [1], Iesan [16], and Al Qahtani and Dutta [20], the basic equations and constitutive equations for microstretch thermoelastic medium with microtemperatures are

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}+\lambda_0\nabla\phi^* - \nu\nabla T=\rho\ddot{\boldsymbol{u}}, \tag{1}$$

$$\left(\gamma\nabla^2-2K\right)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u} - \mu_1\nabla\times\boldsymbol{w}=\rho j\ddot{\boldsymbol{\phi}}, \tag{2}$$

$$\left(\alpha_0\nabla^2-\lambda_1\right)\phi^*-\lambda_0\nabla.\boldsymbol{u}+\nu_1 T-\mu_2\nabla.\boldsymbol{w}=\frac{\rho j_0}{2}\ddot{\phi}^*, \tag{3}$$

$$K^*\nabla^2 T=\rho c^*\frac{\partial T}{\partial t}+\upsilon T_0(\nabla.\boldsymbol{u}-\boldsymbol{Q})+\nu_1 T_0\frac{\partial\phi^*}{\partial t} - K_1(\nabla.\boldsymbol{w}), \tag{4}$$

$$K_6\nabla^2\boldsymbol{w}+(K_4+K_5)\nabla(\nabla.\boldsymbol{w})+\mu_1\frac{\partial}{\partial t}(\nabla\times\boldsymbol{\phi}) - \mu_2\frac{\partial}{\partial t}\left(\nabla\phi^*\right)-b\frac{\partial\boldsymbol{w}}{\partial t}-K_2\boldsymbol{w}-K_3\nabla T=0, \tag{5}$$

$$t_{ij}=\left(\lambda_0\phi^*+\lambda u_{r,r}\right)\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right) + K\left(u_{j,i}-\epsilon_{ijk}\phi_k\right)-\nu\delta_{ij}T, \tag{6}$$

$$m_{ij}=\alpha\phi_{k,k}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\epsilon_{mji}\phi^*_{,m}, \tag{7}$$

$$\lambda_i^*=\alpha_0\phi^*_{,i}+b_0\epsilon_{ijm}\phi_{j,m}, \tag{8}$$

$$q_{i,j}=-K_4 w_{k.k}\delta_{ij}-K_5 w_{i,j}-K_6 w_{j,i},\ i,j,m=1,2,3 \tag{9}$$

The surface of the medium is irradiated by laser heat input (following Al Qahtani and Dutta [20]):

$$Q=I_0 f(t)\,g(x_1)\,h(x_3), \tag{10}$$

$$f(t)=\frac{t}{t_0^2}e^{-\left(\frac{t}{t_0}\right)}, \tag{11}$$

$$g(x_1) = \frac{1}{2\pi r^2} e^{-\left(\frac{x_1^2}{r^2}\right)}, \tag{12}$$

$$h(x_3) = \gamma^* e^{-\gamma^* x_3}, \tag{13}$$

Here I_0 is the energy absorbed, t_0 is the pulse rise time, and r is the beam radius.

Here λ, μ, α, β, γ, K, λ_0, λ_1, α_0, b_0 are constants with values depending on the nature of material, ρ is density of the medium, $\boldsymbol{u} = (u_1, u_2, u_3)$, $\boldsymbol{w} = (u_1, u_2, u_3)$ and $\boldsymbol{\phi} = (\phi_1, \phi_2, \phi_3)$ are respectively the displacement vector, microtemperature vector the microrotation vector, T represents temperature, ϕ^* is the scalar microstretch, T_0 is the reference temperature of the medium, c^* is the specific heat at constant strain, t_{ij} are components of stress, K^* is the coefficient of the thermal conductivity, λ_i^* is the microstress tensor, j is the microinertia, j_0 is the microinertia for the microelements, m_{ij} are components of couple stress, e_{ij} is the strain tensor, e_{ii} is the dilatation, and δ_{ij} is Kroneker delta function.

3 Formulation of the problem

A microstretch thermoelastic medium with microtemperatures irradiated by ultra-short laser pulse as input heat source is considered. The origin of the Cartesian coordinate system $Ox_1x_2x_3$ is taken on a point of the x_1x_2-plane and x_3-axis points vertically downwards into the medium.

For two-dimensional problem, the displacement vector $\boldsymbol{u}$, microtemperature vector $\boldsymbol{w}$, microrotation vector $\boldsymbol{\phi}$ can be written mathematically as:

$$\boldsymbol{u} = (u_1, 0, u_3), \quad \boldsymbol{\phi} = (0, \phi_2, 0), \quad \boldsymbol{w} = (w_1, 0, w_3), \tag{14}$$

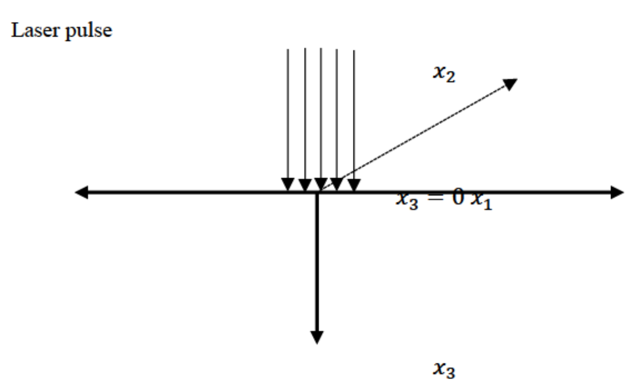

Figure 4: Geometry of the problem

We introduce the following non-dimensional quantities:

$$u_i' = \frac{u_i}{L},\ x_i' = \frac{x_i}{L},\ t' = \omega^* t,\ \phi^{*\prime} = \frac{j_0^2}{L^2}\phi^*,\ T' = \frac{T}{T_0}, \tag{15}$$

$$t_{ij}' = \frac{1}{\upsilon T_0} t_{ij},\ \phi_i' = \frac{j^2}{L^2}\phi_i,\ c_1^2 = \frac{\lambda + 2\mu + k}{\rho},$$

$$m_{ij}^* = \frac{1}{L\upsilon_1 T_0} m_{ij},\ q_{ij}' = \frac{1}{Lc_1 \upsilon T_0} q_{ij},\ L = \frac{K^*}{\rho c^* c_1},\ w_i' = Lw_i,$$

$$t' = \frac{c_1}{L}t,\ \lambda_i^{*\prime} = \frac{1}{L\upsilon T_0}\lambda_i^*,\ Q' = \frac{Q}{\omega^* T_0 c^*}$$

Also, it is appropriate to introduce the scalar potentials ϕ, ϕ_1 and vector potential ψ, ψ_1 through the Helmholtz representation of vector fields u and w as:

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3},\ u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1} \tag{16}$$

$$\text{and } w_1 = \frac{\partial\phi_1}{\partial x_1} - \frac{\partial\psi_1}{\partial x_3},\ w_3 = \frac{\partial\phi_1}{\partial x_3} + \frac{\partial\psi_1}{\partial x_1}$$

Making use of Equations (14) and (15) and then using the potential functions defined in (16), in relations (1)–(5), yield:

$$\left[(a_1 + 1)\nabla^2 - a_5\frac{\partial^2}{\partial t^2}\right]\phi + a_3\phi^* - a_4 T = 0, \tag{17}$$

$$\left(\nabla^2 - a_5\frac{\partial^2}{\partial t^2}\right)\psi + a_2\phi_2 = 0, \tag{18}$$

$$\left(\nabla^2 - 2a_6 - a_8\frac{\partial^2}{\partial t^2}\right)\phi_2 - a_6\nabla^2\psi + a_7\nabla^2\psi_1 = 0, \tag{19}$$

$$\left(\nabla^2 - a_{10} - a_{13}\frac{\partial^2}{\partial t^2}\right)\phi^* - a_{11}\nabla^2\phi - a_{12}\nabla^2\phi_1 + a_9 T = 0, \tag{20}$$

$$-\left(\nabla^2 - a_{14}\frac{\partial}{\partial t}\right)T - a_{15}\frac{\partial\phi^*}{\partial t} - a_{16}\nabla^2\phi + a_{17}\nabla^2\phi_1 = Q_0 f^*(x_1, t)\, e^{-\gamma^* x_3}, \tag{21}$$

$$\left[\nabla^2(1 + a_{18}) - a_{21} - a_{23}\frac{\partial}{\partial t}\right]\phi_1 - a_{20}\frac{\partial\phi^*}{\partial t} - a_{22}T = 0, \tag{22}$$

$$\left[\nabla^2(1 + a_{18}) - a_{21} - a_{23}\frac{\partial}{\partial t}\right]\psi_1 + a_{19}\frac{\partial\phi_2}{\partial t} = 0 \tag{23}$$

Here, $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$ is the Laplacian operator and $f(x_1, t) = \left[t + \epsilon\tau_0\left(1 - \frac{t}{t_0}\right)\right] e^{-\left(\frac{x_1^2}{r^2} + \frac{t}{t_0}\right)}$, $Q_0 = \frac{a_{14} I_0 \gamma^*}{2\pi r^2 t_0^2}$

Here, a_i are defined in Appendix A.

4 Solution of the problem:

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:

$$\left\{\phi, \phi_1, \phi^\star, T, \psi, \psi_1, \phi_2\right\}(x_1, x_3, t) = \left\{\overline{\phi}, \overline{\phi_1}, \overline{\phi^\star}, \overline{T}, \overline{\psi}, \overline{\psi_1}, \overline{\phi_2}\right\}(x_3)\, e^{i(kx_1-\omega t)}, \tag{24}$$

Here ω is the angular frequency and k is the wave number.

Making use of (24) in Eqs. (17), (20), (21), and (22) and eliminating $\left(\overline{\phi_1}, \overline{\phi^\star}, \overline{T}\right)$, $\left(\overline{\phi}, \overline{\phi^\star}, \overline{T}\right)$, $\left(\overline{\phi}, \overline{\phi_1}, \overline{T}\right)$ and $\left(\overline{\phi}, \overline{\phi_1}, \overline{\phi^\star}\right)$ respectively we obtain the following equations:

$$\left[D_1\boldsymbol{D}^8 + D_2\boldsymbol{D}^6 + D_3\boldsymbol{D}^4 + D_4\boldsymbol{D}^2 + D_5\right]\overline{\phi} = f_1\left(\gamma^\star, x_1, t\right) e^{-\gamma^\star x_3}, \tag{25}$$

$$\left[D_1\boldsymbol{D}^8 + D_2\boldsymbol{D}^6 + D_3\boldsymbol{D}^4 + D_4\boldsymbol{D}^2 + D_5\right]\overline{\phi_1} = f_2\left(\gamma^\star, x_1, t\right) e^{-\gamma^\star x_3} \tag{26}$$

$$\left[D_1\boldsymbol{D}^8 + D_2\boldsymbol{D}^6 + D_3\boldsymbol{D}^4 + D_4\boldsymbol{D}^2 + D_5\right]\overline{\phi^\star} = f_3\left(\gamma^\star, x_1, t\right) e^{-\gamma^\star x_3}, \tag{27}$$

$$\left[D_1\boldsymbol{D}^8 + D_2\boldsymbol{D}^6 + D_3\boldsymbol{D}^4 + D_4\boldsymbol{D}^2 + D_5\right]\overline{T} = f_4\left(\gamma^\star, x_1, t\right) e^{-\gamma^\star x_3} \tag{28}$$

Also using (24) in relations (18), (19), and (23) and simplifying the resulting equations we obtain:

$$\left[D_6\boldsymbol{D}^6 + D_7\boldsymbol{D}^4 + D_8\boldsymbol{D}^2 + D_9\right]\overline{\psi} = 0, \tag{29}$$

Here, $g_1 = a_1 + 1$, $g_2 = a_{18} + 1$, $k_1 = a_5\omega^2 - k^2 g_1$, $k_2 = a_{13}\omega^2 - a_{10} - k^2$, $k_3 = i\omega a_{14} - k^2$, $k_4 = i\omega a_{23} - k^2 g_2 - a_{21}$, $k_5 = k^2 - a_5\omega^2$, $k_6 = a_8\omega^2 - k^2 - 2a_6$,
Here, $\boldsymbol{D} = \frac{d}{dx_3}$ in Eqs. (25)–(29), and

$$\begin{aligned}
D_1 &= g_1 g_2, \\
D_2 &= g_1\left(i\omega a_{20} a_{12} + k_{10}\right) + k_1 g_2 + g_2\left(a_{11}a_3 - a_4 a_{16}\right), \\
D_3 &= k_1\left(i\omega a_{20} a_{12} + k_{10}\right) + g_1\left(k_8 a_{12} + k_{11} + k_{13}\right) + k_{15}a_3 \\
&\quad + k_{18}a_3 + k_{19}a_4 - k_{23}a_4, \\
D_4 &= g_1\left(k_{12} + k_{14} - k_9 a_{12}\right) + k_1\left(k_{11} + k_{13} + k_8 a_{12}\right) \\
&\quad + k_{16}a_3 + k_{20}a_4 + 2k^2\left(k_{23}a_4 - k_{18}a_3\right), \\
D_5 &= k_1\left(k_{12} + k_{14} - k_9 a_{12}\right) - k^4\left(k_{23}a_4 - k_{18}a_3\right) - k_{17}a_3 \\
&\quad + k_{21}a_4, \\
D_6 &= -g_2, \quad D_7 = k_{24}, \quad D_8 = k_{25}, \quad D_9 = k_{26},
\end{aligned}$$

$k_i\,(i = 1, \ldots, 26)$ are defined in Appendix B.

It is desired to satisfy the radiation conditions, *i.e.*, $\left(\overline{\phi}, \overline{\phi_1}, \overline{\psi}, \overline{\psi_1}, \overline{T}, \overline{\phi_2}, \overline{\phi^\star}\right) \to 0$as $x_3 \to \infty$, the solutions of Eqs. (25)–(29) can be considered in the following form:

$$\overline{\phi} = \sum_{i=1}^{4} c_i e^{-m_i x_3} + \frac{f_1}{f_5} e^{-\gamma^\star x_3}, \tag{30}$$

$$\overline{\phi_1} = \sum_{i=1}^{4} \alpha_{1i} c_i e^{-m_i x_3} + \frac{f_2}{f_5} e^{-\gamma^\star x_3}, \tag{31}$$

$$\overline{\phi^\star} = \sum_{i=1}^{4} \alpha_{2i} c_i e^{-m_i x_3} + \frac{f_3}{f_5} e^{-\gamma^\star x_3}, \tag{32}$$

$$\overline{T} = \sum_{i=1}^{4} \alpha_{3i} c_i e^{-m_i x_3} + \frac{f_4}{f_5} e^{-\gamma^\star x_3}, \tag{33}$$

$$\left(\overline{\psi}, \overline{\psi_1}, \overline{\phi_2}\right) = \sum_{i=5}^{7} \left(1, \alpha_{4i}, \alpha_{5i}\right) C_i e^{-m_i x_3}, \tag{34}$$

$C_i\,(i = 1, 2, \ldots, 7)$ are arbitrary constants.
$m_i^2 (i = 1, 2, 3, 4)$ are the roots of the characteristic equation (25) and $m_i^2 (i = 5, 6, 7)$ are the roots of characteristic equation (29).

$$\alpha_{1i} = \frac{D_{1i}}{D_{0i}}, \; \alpha_{2i} = \frac{D_{2i}}{D_{0i}}, \; \alpha_{3i} = \frac{D_{3i}}{D_{0i}} i = 1, 2, 3, 4$$
$$\& \; \alpha_{4i} = \frac{D_{4i}}{\Delta_{0i}}, \; \alpha_{5i} = \frac{D_{5i}}{\Delta_{0i}} i = 5, 6, 7$$

D_{ji}, Δ_{0i} and D_{0i} $j = 1, 2, \ldots, 5$, are defined in Appendix C.

Substituting the values of $\left(\overline{\phi}, \overline{\phi_1}, \overline{\psi}, \overline{\psi_1}, \overline{T}, \overline{\phi_2}, \overline{\phi^\star}\right)$ from the Eqs. (30)–(34) in (6)–(9), and using (14)–(16), (24) and solving the resulting equations, we obtain:

$$\bar{t}_{33} = \sum_{i=1}^{7} G_{1i} e^{-m_i x_3} + M_1 e^{-\gamma^\star x_3}, \tag{35}$$

$$\bar{t}_{31} = \sum_{i=1}^{7} G_{2i} e^{-m_i x_3} + M_2 e^{-\gamma^\star x_3}, \tag{36}$$

$$\overline{m}_{32} = \sum_{i=1}^{7} G_{3i} e^{-m_i x_3} + M_3 e^{-\gamma^\star x_3}, \tag{37}$$

$$\overline{\lambda_3^\star} = \sum_{i=1}^{7} G_{4i} e^{-m_i x_3} + M_4 e^{-\gamma^\star x_3}, \tag{38}$$

$$\overline{T} = \sum_{i=1}^{7} G_{5i} e^{-m_i x_3} + M_5 e^{-\gamma^* x_3}, \tag{39}$$

$$\overline{q_{33}} = \sum_{i=1}^{7} G_{6i} e^{-m_i x_3} + M_6 e^{-\gamma^* x_3}, \tag{40}$$

$$\overline{q_{31}} = \sum_{i=1}^{7} G_{6i} e^{-m_i x_3} + M_6 e^{-\gamma^* x_3}, \tag{41}$$

Here, $G_{mi} = g_{mi} C_i$, $i, m = 1, 2, \ldots, 7$
G_{rs}, $(r, s = 1, 2, \ldots, 7)$ and M_r, $(r = 1, 2, 3, \ldots, 7)$ are described in Appendix D.

5 Boundary conditions

We consider normal and tangential force acting on the surface $x_3 = 0$ along with vanishing of couple stress, microstress, and temperature gradient with insulated and impermeable boundary at $x_3 = 0$ and $I_0 = 0$. Mathematically this can be written as:

$$t_{33} = -F_1 e^{i(kx_1 - \omega t)}, \; t_{31} = -F_2 e^{i(kx_1 - \omega t)}, \; m_{32} = 0, \tag{42}$$
$$\lambda_3^*, \; \frac{\partial T}{\partial x_3} = 0, \; q_{33} = q_{31} = 0,$$

Substituting the expression of the variables considered into these boundary conditions, we can obtain the following equations:

$$\sum_{i=1}^{7} G_{1i} c_i = -F_1, \tag{43}$$

$$\sum_{i=1}^{7} G_{2i} c_i = -F_2, \tag{44}$$

$$\sum_{i=1}^{7} G_{3i} c_i = 0, \tag{45}$$

$$\sum_{i=1}^{7} G_{4i} c_i = 0, \tag{46}$$

$$\sum_{i=1}^{7} m_i G_{5i} c_i = 0, \tag{47}$$

$$\sum_{i=1}^{7} G_{6i} c_i = 0, \tag{48}$$

$$\sum_{i=1}^{7} G_{7i} c_i = 0, \tag{49}$$

Eqs. (43)–(49) can be written in matrix form in the following manner:

$$\begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} & g_{16} & g_{17} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} & g_{26} & g_{27} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35} & g_{36} & g_{37} \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{45} & g_{46} & g_{47} \\ g_{51} & g_{52} & g_{53} & g_{54} & g_{55} & g_{56} & g_{57} \\ g_{61} & m_2 & g_{63} & m_4 & g_{65} & g_{66} & g_{67} \\ g_{71} & g_{72} & g_{73} & g_{74} & g_{75} & g_{76} & g_{77} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} = \begin{bmatrix} -F_1 \\ -F_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{50}$$

Equation (50) is solved by using the matrix method as follows:

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} & g_{16} & g_{17} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} & g_{26} & g_{27} \\ g_{31} & g_{32} & g_{33} & g_{34} & g_{35} & g_{36} & g_{37} \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{45} & g_{46} & g_{47} \\ g_{51} & g_{52} & g_{53} & g_{54} & g_{55} & g_{56} & g_{57} \\ g_{61} & m_2 & g_{63} & m_4 & g_{65} & g_{66} & g_{67} \\ g_{71} & g_{72} & g_{73} & g_{74} & g_{75} & g_{76} & g_{77} \end{bmatrix}^{-1} \begin{bmatrix} -F_1 \\ -F_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{51}$$

5.1 Particular case

5.1.1 Microstretch thermoelastic medium:

In absence of microtemperature effect, *i.e.*,

$$\mu_1 = \mu_2 = K_1 = K_2 = K_3 = K_4 = K_5 = K_6 = 0$$

in Eqs. (43)–(49), we obtain the dispersion equation for microstretch thermoelastic medium.

5.1.2 Generalized thermoelastic medium:

In the absence of microtemperature and microstretch effect in Eqs. (43)–(49), we obtain the dispersion equation for microstretch thermoelastic medium.

6 Numerical results and discussions:

The following values of relevant parameters are taken for numerical computations. Following Eringen [22], the values of micropolar constants are:
$\lambda = 9.4 \times 10^{10}$N.m^{-2}, $\mu = 4.0 \times 10^{10}$N.m^{-2}, $K = 1.0 \times 10^{10}$N.m^{-2}, $\rho = 1.74 \times 10^{3}$Kg.m^{-3}, $j = 0.2 \times 10^{-19}$m^2, $\gamma = 0.779 \times 10^{-9}$N

Thermal parameters are given by (following Dhaliwal [23]):
$c^{\star} = 1.04 \times 10^3 \text{J.Kg}^{-1}.\text{K}^{-1}$, $K^{\star} = 1.7 \times 10^6 \text{J.m}^{-1}.\text{s}^{-1}.\text{K}^{-1}$, $a = 2.9 \times 10^4 \text{m}^2.\text{s}^{-2}.\text{K}^{-1}$, $T_0 = 298\text{K}$, $\tau_1 = 0.613 \times 10^3 \text{s}$

Microstretch and microtemperature parameters are taken as (following Kumar and Kaur [24]):
$j_0 = 0.000019 \times 10^{-13} \text{m}^2$, $\alpha_0 = 0.8 \times 10^{-9} \text{N}$, $\lambda_0 = 2.1 \times 10^{10} \text{N.m}^{-2}$, $\lambda_1 = 0.7 \times 10^{10} \text{N.m}^{-2}$, $b = 1.5 \times 10^{-9} \text{Kg}^{-1}.\text{m}^5.\text{s}^{-2}$, $b_0 = .5 \times 10^{-9} \text{Kg}^{-1}.\text{m}^5.\text{s}^{-2}$, $K_1 = .0035 \text{Ns}^{-1}$, $K_2 = .045 \text{Ns}^{-1}$, $K_3 = 0.055 \text{NK}^{-1}\text{s}^{-1}$, $K_4 = 0.065 \text{Ns}^{-1}\text{m}^2$, $K_5 = 0.076 \text{Ns}^{-1}\text{m}^2$, $K_6 = 0.096 \text{Ns}^{-1}\text{m}^2$, $\mu_1 = 0.0085\text{N}$, $\mu_2 = 0.0095\text{N}$

A comparison of the dimensionless form of the field variables for the cases of microstretch thermoelastic medium with microtemperature and ultra-short laser pulse as input heat source (MTPL), microstretch thermoelastic medium with microtemperature but without laser pulse (MTP) and microstretch thermoelastic medium, *i.e.*, without microtemperature effect and without laser pulse (MSTH) subjected to normal force is presented in Figures 5–11. The values of all physical quantities for all cases are shown in the range $0 \le x_3 \le 20$.

Solid lines, small dash lines, and large dash lines including a dot corresponds to microstretch thermoelastic solid with microtemperature and laser pulse (MTPL), microstretch thermoelastic solid with microtemperature (MTP), and microstretch thermoelastic solid without microtemperature and laser pulse (MSTH) respectively.

The computations were carried out in the absence and presence of laser pulse ($I_0 = 10^5, 0$) and on the surface of plane $x_1 = 1$, $t = .1$

Figure 5 presents the variation of normal stress t_{33} with the distance x_3. It is noticed that for MTPL, MTP and MSTH, the normal stress t_{33} shows a similar behavior. The value of normal stress for MTPL and MSTH monotonically decreases as x_3 increases. MTP shows a similar behavior followed by some oscillatory trend. The values of normal stress approaches the boundary surface at large distance from the point of application of source.

Figure 6 shows the change in tangential stress t_{31} with respect to distance x_3. It is noticed that initially the trend of t_{31} for the two cases, *i.e.* MTPL and MSTH is monotonically increasing but in case of MTP it is decreasing. After this, *i.e.*, $x_1 \ge 4$*i.e.* far away from the point of application of ultra-laser heat source and normal force the values of t_{31} for all the cases tends to approach value zero.

Figure 7 exhibits the trend of couple stress m_{32} w.r.t. the displacement x_3. For MTPL and MTP, the initial behav-

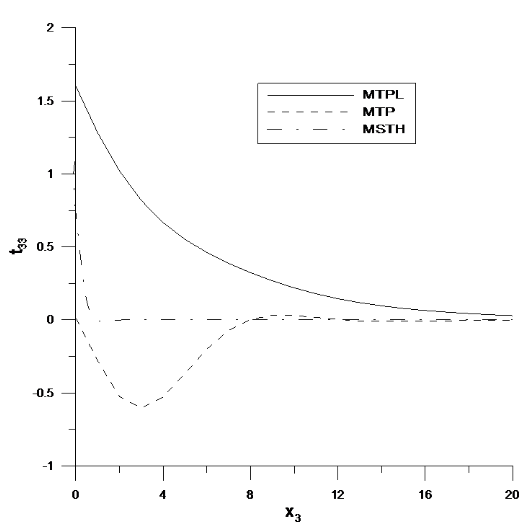

Figure 5: Variation of normal stress

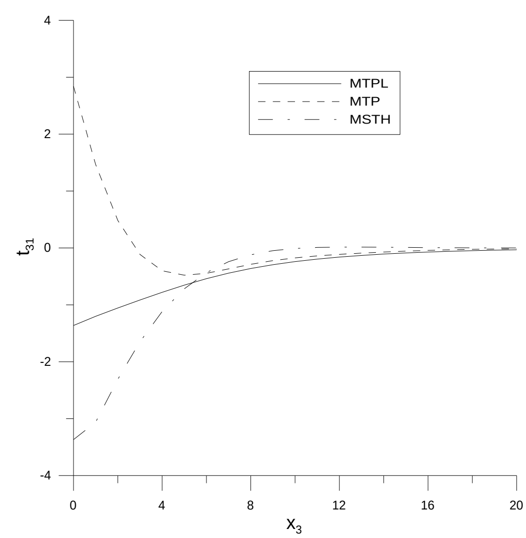

Figure 6: Variation of tangential stress

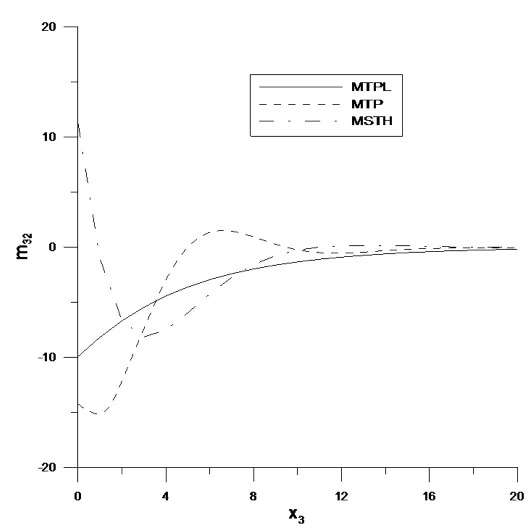

Figure 7: Variation of coupled tangential stress

ior of couple stress is monotonically increasing but behavior of MSTH is opposite to it. For higher values of x_1 the value of m_{32} is approaching to boundary surface for all the three cases.

Figure 8 shows the trend of micro stress λ_3^* with distance x_3. The trend and variation of λ_3^* is oscillatory for MTP indicating that the micro-stress is much affected by including the microtemperature effect. In case of MTPL and MSTH the micro-stress approaches to the boundary surface uniformly.

Figure 9 shows the behavior of temperature distribution T with distance x_3. The values of temperature for MTPL, MTP, and MTH decreases monotonically and approaches to the boundary surface away from the point of application of source.

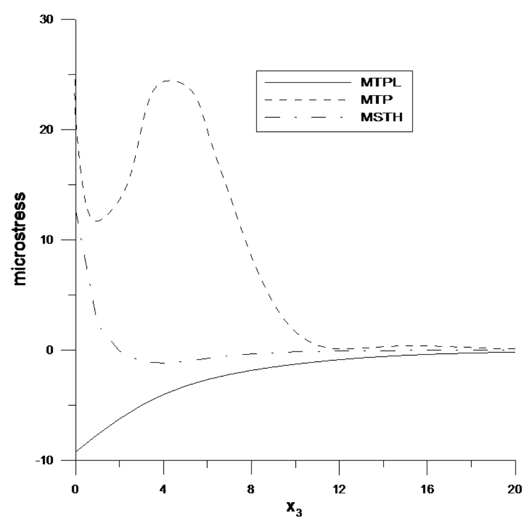

Figure 8: Variation of micro-stress

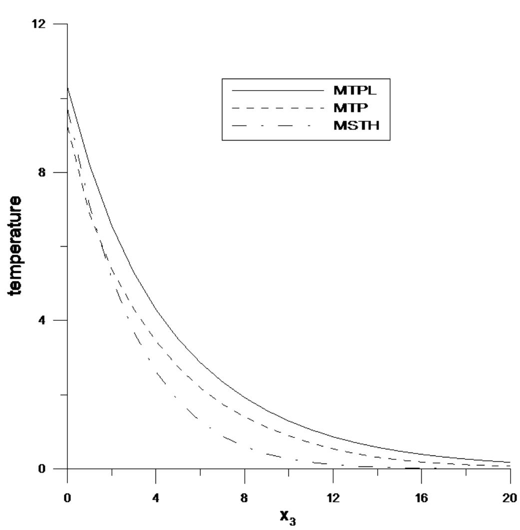

Figure 9: Variation of temperature distribution

Figure 10 shows the behavior of q_{33} with distance x_3. The values of q_{33} for MTPL and MTP decreases monotonically and approaches to the boundary surface away from the point of application of source. Here we observe that the magnitude of q_{33} in case of MTP is larger than the magnitude of q_{33} in MTPL.

Figure 11 shows the behavior of q_{31} with distance x_3. The values of q_{31} for MTPL, MTP increases monotonically and approaches to the boundary surface away from the point of application of source.

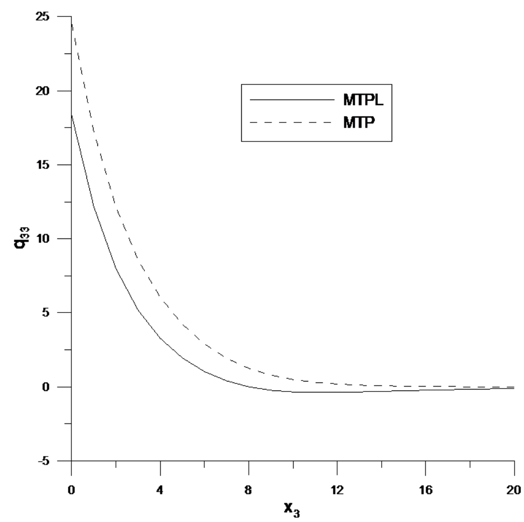

Figure 10: Variation of q_{33}

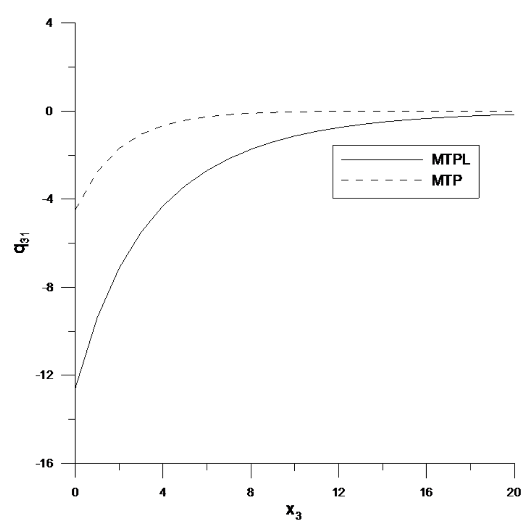

Figure 11: Variation of q_{31}

Variation of temperature with respect to time:

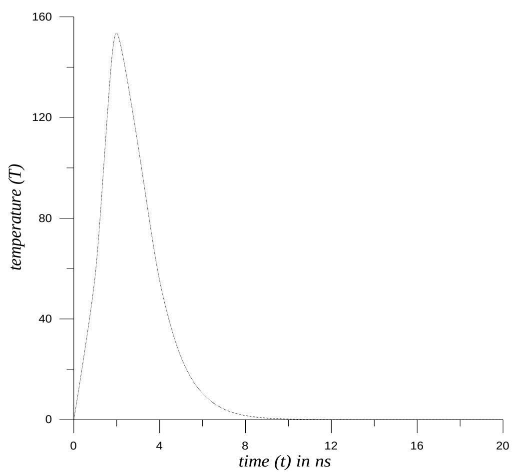

Figure 12: Variation of temperature with respect to time

Figure 12 shows the variation of temperature distribution with respect to time. It is clear from the figure that initial trend of variation is monotonically increasing until the temperature reaches a maximum value. After that the trend of variation of temperature change is monotonically decreasing and approaches boundary surface away from the point of laser heat irradiation.

7 Conclusions

In this problem, we have investigated the displacement components, stress components, and temperature change in a microstretch thermoelastic medium with microtemperature. The solution of the physical variables has been obtained in terms of normal modes. Theoretically computed variables are also discussed graphically.

This analysis of the results obtained give the following conclusions:

(1) It can be concluded from Figures 5–11 that all the physical variables have nonzero values only in the bounded region. This indicates that all the results obtained here are in agreement with the generalized theory of thermoelasticity.
(2) It is clear from the results that the input laser heat source (value of I_0) has a significant role in the variation of all field quantities.
(3) If the microtemperature parameters are absent, then the results are obtained for generalized thermoelastic problem, which are in agreement with Kumar, Kumar and Singh [12].
(4) The variation of various stress components differs significantly due to the presence of normal force / thermal source.
(5) The temperature change is also affected due to input laser heat source as well as load/source applied.
(6) The microtemperature effect has also significant effect on the physical quantities.

The new model is employed in a microstretch thermoelastic medium with microtemperatures as a new concept in the field of thermoelasticity. The subject becomes more interesting due to presence of an ultra-short input laser heat source. The method of solution in this research can be applied to a large number of problems in engineering and science. It is hoped that this model will serve as more realistic model and will motivate the other authors to solve problems in microtemperature thermoelasticity. Solutions to such problems also have utilities in geophysical mechanics.

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Appendix A:

$$a_1 = \frac{\lambda+\mu}{\mu+K},\ a_2 = \frac{K}{\mu+K},\ a_3 = \frac{\lambda_0}{\mu+K},\ a_4 = \frac{\nu T_0}{\mu+K},$$
$$a_5 = \frac{\rho c_1^2}{\mu+K},\ a_6 = \frac{KL^2}{\gamma},\ a_7 = \frac{\mu_1}{\gamma},\ a_8 = \frac{\rho j c_1^2}{\gamma},$$
$$a_9 = \frac{\upsilon_1 T_0 L^2}{\alpha_0},\ a_{10} = \frac{\lambda_0 L^2}{\alpha_0},\ a_{11} = \frac{\lambda_1 L^2}{\alpha_0},\ a_{12} = \frac{\mu_2}{\alpha_0},$$
$$a_{13} = \frac{\rho j_0 c_1^2}{2\alpha_0},\ a_{14} = \frac{\rho c^\star c_1 L}{K^\star},\ a_{15} = \frac{\upsilon_1 c_1 L}{K^\star},\ a_{16} = \frac{\nu c_1 L}{K^\star},$$
$$a_{17} = \frac{k_1}{K^\star T_0},\ a_{18} = \frac{k_4+k_5}{k_6},\ a_{19} = \frac{\mu_1 c_1 L}{k_6},\ a_{20} = \frac{\mu_2 c_1 L}{k_6},$$
$$a_{21} = \frac{k_2 L^2}{k_6},\ a_{22} = \frac{k_3 T_0 L^2}{k_6},\ a_{23} = \frac{b c_1 L^2}{k_6}$$

Appendix B:

$$k_1 = a_5\omega^2 - k^2 g_1,\ k_2 = a_{13}\omega^2 - a_{10} - k^2,$$
$$k_2 = i\omega a_{14} - k^2,\ k_4 = i\omega a_{23} - a_{21} - k^2 g_2,$$
$$k_5 = k^2 - a_5\omega^2,\ k_6 = a_8\omega^2 - k^2 - 2a_6,$$
$$k_6 = i\omega a_{23} - a_{21} - k^2 g_2,$$
$$k_8 = \left(i\omega a_5 a_{22} + i\omega a_{20} k_3 - i\omega a_{20} k^2\right),$$
$$k_9 = k^2\left(i\omega a_5 a_{22} + i\omega a_{20} k_3\right),$$
$$k_{10} = \left(g_2 k_3 + k_4 + a_{17} a_{22} + g_2 k_2\right),$$
$$k_{11} = \left(k_3 k_4 - a_{17} a_{22} k^2 + k_2 k_3 g_2 + k_2 k_4 + k_2 a_{17} a_{22}\right),$$
$$k_{12} = k_2\left(k_3 k_4 - a_{17} a_{22} k^2\right),$$
$$k_{13} = i\omega a_9\left(a_{17} a_{20} - a_5 g_2\right),$$
$$k_{14} = -i\omega a_9\left(a_{17} a_{20} k^2 + a_5 k_4\right)$$
$$k_{15} = g_2\left(a_{11}\left(a_{11} k_3 - k^2\right) - a_9 a_{16}\right) + a_{11} k_4,$$
$$k_{18} = a_{22}\left(a_{11} a_{17} + a_{12} a_{16}\right),$$
$$k_{16} = g_2\left(a_9 a_{16} k^2 - k^2 k_3\right)$$
$$+ k_3\left(a_{16}\left(a_{16} a_{16} - k^2\right) - a_9 a_{16}\right),$$
$$k_{17} = k_4 k^2\left(a_9 a_{16} - k_3\right),$$
$$k_{19} = g_2\left(i\omega a_5 a_{11} - a_{16}\left(k_2 - k^2\right)\right) - a_{16} k_4,$$
$$k_{23} = i\omega a_{20}\left(a_{17} a_{11} + a_{12} a_{16}\right)$$
$$k_{20} = g_2\left(k_2 k^2 a_{16} - i\omega a_5 a_{11} k^2\right)$$
$$+ k_4\left(i\omega a_5 a_{11} - a_{16}\left(k_2 - k^2\right)\right),$$
$$k_{21} = k_4\left(k_2 a_{16} k^2 - i\omega a_5 a_{11} k^2\right)$$
$$k_{24} = -i\omega a_7 a_{19} - \left(k_7 + g_2 k_6\right) + g_2 k_5 - a_2 a_6 g_2,$$
$$k_{25} = i\omega a_7 a_{19}\left(k_5 + k^2\right) - k_6 k_7 + k_5\left(k_7 + g_2 k_6\right)$$
$$- a_2 a_6\left(k_7 - g_2 k^2\right),$$
$$k_{26} = -i\omega a_7 a_{19} k_5 k^2 + k_5 k_6 k_7 + a_2 a_6 k_7 k^2$$

Appendix C:

For $i = 1, 2, 3, 4$

$$D_{0i} = \begin{vmatrix} -a_{12}\left(\boldsymbol{D}^2 - k^2\right) & \boldsymbol{D}^2 + k_2 & a_9 \\ a_{17}\left(\boldsymbol{D}^2 - k^2\right) & i\omega a_5 & \boldsymbol{D}^2 + k_3 \\ g_2\boldsymbol{D}^2 + k_4 & i\omega a_{20} & -a_{22} \end{vmatrix},$$

$$D_{1i} = \begin{vmatrix} -a_{11}\left(\boldsymbol{D}^2 - k^2\right) & \boldsymbol{D}^2 + k_2 & a_9 \\ -a_{16}\left(\boldsymbol{D}^2 - k^2\right) & i\omega a_5 & \boldsymbol{D}^2 + k_3 \\ 0 & i\omega a_{20} & -a_{22} \end{vmatrix}$$

$$D_{2i} = \begin{vmatrix} -a_{11}\left(\boldsymbol{D}^2 - k^2\right) & -a_{12}\left(\boldsymbol{D}^2 - k^2\right) & a_9 \\ -a_{16}\left(\boldsymbol{D}^2 - k^2\right) & a_{17}\left(\boldsymbol{D}^2 - k^2\right) & \boldsymbol{D}^2 + k_3 \\ 0 & g_2\boldsymbol{D}^2 + k_4 & -a_{22} \end{vmatrix},$$

$$D_{3i} = \begin{vmatrix} -a_{11}\left(\boldsymbol{D}^2 - k^2\right) & -a_{12}\left(\boldsymbol{D}^2 - k^2\right) & \boldsymbol{D}^2 + k_2 \\ -a_{16}\left(\boldsymbol{D}^2 - k^2\right) & a_{17}\left(\boldsymbol{D}^2 - k^2\right) & i\omega a_5 \\ 0 & g_2\boldsymbol{D}^2 + k_4 & i\omega a_{20} \end{vmatrix}$$

For $i = 5, 6, 7$

$$\varDelta_{0i} = \begin{vmatrix} a_7\left(\boldsymbol{D}^2 - k^2\right) & \boldsymbol{D}^2 + k_6 \\ 0 & -i\omega a_{19} \end{vmatrix},$$

$$D_{4i} = \begin{vmatrix} -a_6\left(\boldsymbol{D}^2 - k^2\right) & \boldsymbol{D}^2 + k_6 \\ g_2\boldsymbol{D}^2 + k_7 & -i\omega a_{19} \end{vmatrix},$$

$$D_{5i} = \begin{vmatrix} -a_6\left(\boldsymbol{D}^2 - k^2\right) & a_7\left(\boldsymbol{D}^2 - k^2\right) \\ g_2\boldsymbol{D}^2 + k_7 & 0 \end{vmatrix}$$

and

$$g_{1i} = \imath k b_3 m_i,\ g_{2i} = -\left(b_4 m_l^2 + b_5 k^2\right) - b_6\alpha_{5i},\ g_{3i} = -b_7\alpha_{5i}m_i,$$
$$g_{4i} = -ikb_{10}\alpha_{5i},\ g_{5i} = 0,\ g_{6i} = ikm_i\left(k_5 + k_6\right)\alpha_{4i},$$
$$g_{7i} = \imath k m_i\left(k_5 k^2 + k_6 m_l^2\right)\alpha_{4i} \text{ for } i = 5, 6, 7$$

$$M_1 = \left(\frac{\left[b_2\left(\gamma^{\star 2} - k^2\right) - b_3\gamma^{\star 2}\right] f_1 + b_1 f_3 - f_4}{f_5}\right),$$

$$M_2 = \frac{-\imath k b_3\gamma^{\star} f_1}{f_5},\ M_3 = \frac{\imath k b_8 f_3}{f_5},\ M_4 = \frac{-\gamma^{\star} b_9 f_3}{f_5},$$

$$M_5 = \frac{-\gamma^{\star} f_4}{f_5},\ M_6 = \frac{\left(-k_4\left(\gamma^{\star 2} - k^2\right) - \left(k_5 + k_6\right)\gamma^{\star 2}\right) f_2}{f_5},$$

$$M_7 = \frac{\left(\imath k\gamma^{\star}\left(k_5 + k_6\right) f_2\right)}{f_5}.$$

Appendix D:

$$d_1 = \frac{\rho c_1^2}{\nu T_0},\ d_2 = \frac{\lambda}{\nu T_0},\ d_3 = \frac{L^2\lambda_0}{j_0^2\nu T_0},\ d_4 = \frac{\mu}{\nu T_0},$$

$$d_5 = \frac{L^2}{j^2}\frac{K}{\nu T_0},\ d_6 = \frac{L^2}{j^2}\frac{\gamma}{L^2\nu T_0},\ d_7 = \frac{b_0}{j_0^2\nu T_0},\ d_8 = \frac{\alpha_0}{j_0^2\nu T_0},$$

$$d_9 = \frac{L^2}{j^2}\frac{b_0}{L^2\nu T_0},\ d_{10} = \frac{K_4}{L^3 c_1\nu T_0},\ d_{11} = \frac{\left(K_5 + K_6\right)}{L^3 c_1\nu T_0},$$

$$d_{12} = \frac{K_5}{L^3 c_1\nu T_0},\ d_{13} = \frac{K_6}{L^3 c_1\nu T_0},\ d_{17} = -\frac{\mu_1}{L^2\nu T_0},$$

$$d_{19} = \frac{\mu_2}{L^2\nu T_0}$$

$$g_{1i} = \left[b_2\left(m_i^2 - k^2\right) - b_3 m_i^2\right] + b_1\alpha_{2i} - \alpha_{3i},$$
$$g_{2i} = -\imath k b_3 m_i,\ g_{3i} = \imath k b_8\alpha_{2i}\ , g_{4i} = -m_i b_9\alpha_{2i},$$
$$g_{5i} = -m_i\alpha_{3i},\ g_{6i} = -k_4\left(m_i^2 - k^2\right) - \left(k_5 + k_6\right) m_i^2,$$
$$g_{7i} = \imath k m_i\left(k_5 + k_6\right)\alpha_{1i},\ \text{for } i = 1, 2, 3, 4$$