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Research Article

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Mathematical study of Rayleigh waves in piezoelectric microstretch thermoelastic medium

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Abstract: This paper is concerned with the study of propagation of Rayleigh waves in a homogeneous isotropic piezo-electric microstretch-thermoelastic solid half-space. The medium is subjected to stress-free, isothermal boundary. After developing a mathematical model, the dispersion curve in the form of polynomial equation is obtained. Phase velocity and attenuation coefficient of the Rayleigh wave are computed numerically. The numerically simulated results are depicted graphically. Some special cases have also been derived from the present investigation.

Keywords: Rayleigh wave, microstretch-thermoelastic, phase velocity, attenuation coefficient, microtemperature

1 Introduction

Presently, smart materials have assumed great importance in engineering, technology, and sciences. The important features of smart materials are due to their internal molecular structures known as smart structure, for example sensors, actuators, etc. One of the smart materials currently under research applications are piezo-electric materials. The piezo-electric substances are those that generate electricity (known as piezo-electricity) in response to mechanical stress. Such type of materials are used in actuators and sensors due to their direct and converse piezo-electric effects. To ensure that the piezo-electric appliances are functional in extreme temperature conditions, the thermal effects are to be considered in mathematical model development. So, a result of these electrical-thermal-mechanical coupling thermoelastic theories of piezo-electric materials have been developed. First, a theory of piezo-electricity was developed by Mindlin [1]. Mindlin [2] proposed the basic governing relations for piezo-electric thermoelastic solid. Later Nowacki [3] deduced the physical laws and theorems for thermo-piezo-electric substances. Later Chandrasekharaiah [4] extended this theory also including the finite speed of thermal disturbances. The thermoelastic theory of piezo-electric materials was applied to composite plates by Tauchert [5].

Eringen [6] proposed the concepts of micropolar piezoelectricity and magneto-elasticity. Eringen [7] introduced the electromagnetic theory of microstretch thermoelasticity. The various applications of this theory are in porous elastic bodies, animal bones, and synthetic materials having microscopic components. The special cases of this theory are the theory of piezoelectricity and the theory of magneto-elasticity. The materials having linear coupling between mechanical and electric field are known as piezoelectric materials. These materials are widely used in intelligent structure systems, ultrasonic transducers, piezoelectric composite structures, and loudspeakers. Iesan [8] developed the linear theory of microstretch piezoelectricity and established the uniqueness theorem and reciprocity relation.

Eringen [9] developed the theory and basic equations of microstretch thermoelastic solids. Microstretch continuum is a model for Bravais lattice having its basis on the atomic level and two-phase dipolar substance having core at the macroscopic level. Examples of microstretch thermoelastic materials are composite materials filled with chopped elastic fibers, porous elastic fluids whose pores have gases or inviscid liquids, or other elastic inclusions and liquid–solid crystal. Kumar [10] discussed a dynamic problem in micropolar thermoelastic medium with mass diffusion. Kumar [11] also studied microstretch thermoelastic medium with the inclusion of Hall current.

Kumar and Gupta [12] presented the problem of Rayleigh wave propagation in generalized thermoelastic medium with mass diffusion. Kumar *et al.* [13] and Abd-Alla *et al.* [14, 15] recently discussed some problems related to Rayleigh waves. Singh [16] also investigated a

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Rayleigh wave problem in thermoelastic medium including the impedance boundary condition.

The present research is devoted to the study of behavior of Rayleigh waves in a piezo-electric microstretch thermoelastic solid half-space. Phase velocity and attenuation coefficients of Rayleigh wave propagation have been computed numerically, and graphical representation of their variations has been shown. Some particular cases of interest have also been discussed.

2 Basic equations

Following Iesan [8] and Iesan and Quintanilla [17], the field equations and constitutive relations for a homogeneous, isotropic piezo-electric thermo-microstretch solid are written as follows:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u}) + (\mu+K)\nabla^2\boldsymbol{u} + K\nabla\times\boldsymbol{\phi} + \lambda_0\nabla\phi^* - \beta_0\nabla\tau = \rho\ddot{\boldsymbol{u}}, \quad (1)$$

$$\left(\gamma\nabla^2 - 2K\right)\boldsymbol{\phi} + (\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi}) + K\nabla\times\boldsymbol{u} = \rho j\ddot{\boldsymbol{\phi}}, \quad (2)$$

$$\left(\alpha_0\nabla^2 - \lambda_3\right)\phi^* - \lambda_2\nabla^2\psi + \nu_1\nabla^2\tau - \lambda_0\nabla.\boldsymbol{u} + c_0\frac{\partial}{\partial t}\tau = \frac{\rho j_0}{2}\ddot{\phi}^* \quad (3)$$

$$\left(n_1K^* + n_2\frac{K_1}{T_0}\right)\nabla^2\tau - \beta_0(\nabla.\dot{\boldsymbol{u}}) - a\ddot{\tau} - c_0\dot{\phi}^* + \nu_1\nabla^2\phi^* - \nu_3\nabla^2\psi = 0, \quad (4)$$

$$\lambda_2\nabla^2\phi^* + \chi\nabla^2\psi + \nu_3\nabla^2\tau = 0, \quad (5)$$

$$E_i = -\psi_i, \quad (6)$$

$$t_{ij} = \left(\lambda_0\phi^* + \lambda u_{r,r}\right)\delta_{ij} + \mu\left(u_{i,j}+u_{j,i}\right) + K\left(u_{j,i} - \epsilon_{ijk}\phi_k\right) - \beta_0\delta_{ij}T, \quad (7)$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} + b_0\epsilon_{mji}\phi^*_{,m} + \lambda_1\epsilon_{ijk}E_k + \nu_2\epsilon_{ijk}\tau_k, \quad (8)$$

$$\lambda_i^* = \alpha_0\phi^*_{,i} + b_0\epsilon_{ijm}\phi_{j,m} + \lambda_2E_i + \nu_1\tau_i, \quad (9)$$

$$D_k = \lambda_1\epsilon_{ijk}\phi_{j,i} - \lambda_2\phi^*_{,k} - \nu_3\tau_{,k} + \chi E_k \quad (10)$$

λ, μ are Lame's constants, α, β, γ, λ_0, α_0, b_0 are microstretch constants, $K$is thermal conductivity, λ_1, λ_2, υ_1, υ_2, υ_3 are material constants, u is displacement vector, ϕ is the microrotation vector, ϕ^* is scalar microstretch, T represents temperature and $\dot{\tau} = T$, T_0 is reference temperature, K^* is the coefficient of thermal conductivity, c^* is specific heat at constant strain, j is the microinertia, j_0 is microinertia for the microelements, m_{ij} are components of couple stress, t_{ij} are components of stress, λ_i^* is microstress tensor, D_k is dielectric displacement vector, β_0 is the relaxation time, ψ is electric potential, n_1, n_2 are piezoelectric parameters, and χ represents the dielectric susceptibility.

In the above equations, the symbol (",") followed by a suffix denotes differentiation with respect to spatial coordinates and a superposed dot (" ") denotes the derivative with respect to time respectively.

3 Formulation of the problem

A rectangular Cartesian coordinate system $OX_1X_2X_3$ having origin on x_3-axis with x_3-axis pointing vertically downward the medium is considered.

Further, we consider the plane strain problem with all the field variables depending on (x_1, x_3, t). For such two-dimensional problems, we take

$$\boldsymbol{u} = (u_1, 0, u_3) \quad \boldsymbol{\phi} = (0, \phi_2, 0), \quad \boldsymbol{E} = (E_1, 0, E_3), \quad (11)$$

Also, it is convenient to define in equations (1)–(6) the following dimensionless quantities:

$$\left(x_1', x_3', u_1', u_3'\right) = \frac{1}{L_0}(x_1, x_3, u_1, u_3), \quad (12)$$

$$\phi_i' = \frac{\rho c_1^2}{\beta_1 T_0}\phi_i, \quad \phi^{*\prime} = \frac{\rho c_1^2}{\beta_1 T_0}\phi^*, \quad \tau' = \frac{c_1}{L_0 T_0}\tau, \quad t' = \frac{c_1}{L_0}t,$$

$$t_{ij}' = \frac{1}{\rho c_1^2}t_{ij}, \quad c_1^2 = \frac{\lambda+2\mu+K}{\rho}, \quad m_{ij}^* = \frac{1}{\rho c_1^2 L_0}m_{ij}$$

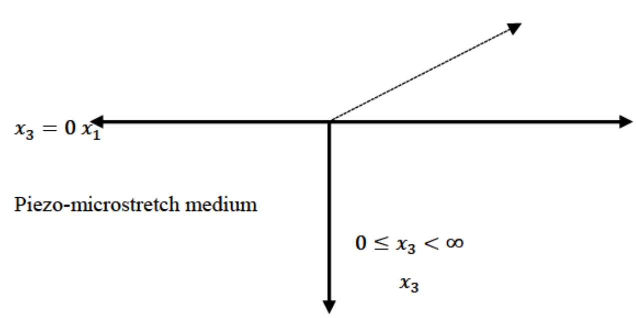

Figure 1: Geometry of the problem.

Making use of (12) in equations (1)–(6) and with the help of (11), we obtain the following component wise equations:

$$a_1\nabla^2 u_1 + (1-a_1)\frac{\partial}{\partial x_1}\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3}\right) - a_2\frac{\partial \phi_2}{\partial x_3} + a_3\frac{\partial \phi^*}{\partial x_1} - a_4\frac{\partial \tau}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2}, \tag{13}$$

$$a_1\nabla^2 u_3 + (1-a_1)\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3}\right) + a_2\frac{\partial \phi_2}{\partial x_1} + a_3\frac{\partial \phi^*}{\partial x_3} - a_4\frac{\partial \tau}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2}, \tag{14}$$

$$a_5\nabla^2\phi_2 + a_2\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right) - 2a_2\phi_2 = a_6\frac{\partial^2\phi_2}{\partial t^2}, \tag{15}$$

$$\left(a_7\nabla^2 - a_8\right)\phi^* - a_9\nabla^2\psi + a_{10}\nabla^2\tau - a_3\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3}\right) + a_{11}\frac{\partial\tau}{\partial t} = a_{12}\ddot{\phi}^*, \tag{16}$$

$$\left(n_1K_1 + n_2K_2\frac{\partial}{\partial t}\right)\nabla^2\tau - a_4\frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_3}{\partial x_3}\right) - a_{13}\frac{\partial^2\tau}{\partial t^2} - a_{11}\frac{\partial\phi^*}{\partial t} + a_{10}\nabla^2\phi^* - a_{14}\nabla^2\psi = 0 \tag{17}$$

$$a_9\nabla^2\phi^* + \upsilon\nabla^2\psi + a_{14}\nabla^2\tau = 0, \tag{18}$$

Here, $a_1 = \frac{\lambda+\mu}{\rho c_1^2}$, $a_2 = \frac{K}{\rho c_1^2}$, $a_3 = \frac{\lambda_0}{\rho c_1^2}$, $a_4 = \frac{\beta_0 T_0}{\rho c_1^2}$, $a_5 = \frac{\gamma_1}{\rho c_1^2 L_0^2}$, $a_6 = \frac{I_1}{\rho L_0^2}$, $a_7 = \frac{a_0}{\rho c_1^2 L_0^2}$, $a_8 = \frac{\lambda_3}{\rho c_1^2}$, $a_9 = \frac{\lambda_2\psi_0}{\rho c_1^2 L_0^2}$, $a_{10} = \frac{\upsilon_1 T_0}{\rho c_1^3 L_0^2}$, $a_{11} = \frac{c_0 T_0}{\rho c_1^2}$, $a_{12} = \frac{j_0}{\rho L_0^2}$, $a_{13} = \frac{a T_0^2}{\rho c_1^2}$, $a_{14} = \frac{\upsilon_{3\psi_0 T_0}}{\rho c_1^3 L_0}$, $K_1 = \frac{K^* T_0^2}{\rho c_1^4}$, $K_2 = \frac{K_1 T_0}{\rho c_1^3 L_0}$, $\nu = \frac{\chi\psi_0^2}{\rho c_1^2 L_0^2}$, $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$.

The relations connecting displacement components and microtemperature components to the potential functions in dimensionless form are:

$$u_1 = \frac{\partial\psi_1}{\partial x_1} + \frac{\partial\psi_2}{\partial x_3}, \quad u_3 = \frac{\partial\psi_1}{\partial x_3} - \frac{\partial\psi_2}{\partial x_1} \tag{19}$$

Using the relations defined by (19) in equations (13)–(18) and rewriting (after suppressing the primes), we obtain:

$$\left[\nabla^2 - \frac{\partial^2}{\partial t^2}\right]\psi_1 + a_3\phi^* - a_4\frac{\partial\tau}{\partial t} = 0, \tag{20}$$

$$\left(a_1\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\psi_2 - a_2\phi_2 = 0, \tag{21}$$

$$a_2\nabla^2\psi_2 + \left(a_5\nabla^2 - 2a_2 - a_6\frac{\partial^2}{\partial t^2}\right)\phi_2 = 0, \tag{22}$$

$$\left(a_{10}\nabla^2 - a_{11}\frac{\partial}{\partial t}\right)\phi^* - a_{14}\nabla^2\psi - a_4\frac{\partial}{\partial t}\nabla^2\psi_1 + \left[\left(n_1K_1 + n_2K_2\frac{\partial}{\partial t}\right)\nabla^2 - a_{13}\frac{\partial^2}{\partial t^2}\right]\tau = 0, \tag{23}$$

$$\left(a_7\nabla^2 - a_8 - a_{12}\frac{\partial^2}{\partial t^2}\right)\phi^* + \left(a_{10}\nabla^2 + a_{11}\frac{\partial}{\partial t}\right)\tau - a_9\nabla^2\psi - a_3\nabla^2\psi_1 = 0, \tag{24}$$

$$a_9\nabla^2\phi^* + \nu\nabla^2\psi + a_{14}\nabla^2\tau = 0. \tag{25}$$

4 Solution of the problem

The solution of the considered physical variables can be decomposed in terms of the normal modes as in the following form:

$$\left\{\psi, \psi_1, \phi^*, \tau, \psi_2, \phi_2\right\}(x_1, x_3, t) = \left\{\overline{\psi}, \overline{\psi}_1, \overline{\phi^*}, \overline{\tau}, \overline{\psi}_2, \overline{\phi}_2\right\}(x_3)\, e^{i(kx_1-\omega t)}, \tag{26}$$

Here ω is the angular frequency and k is wave number.

Utilizing the expression (26) in equations (20)–(25) yield the following equations:

$$\left(D^2 - k^2 + \omega^2\right)\overline{\psi_1} + i\omega a_4\overline{\tau} + a_3\overline{\phi^*} = 0, \tag{27}$$

$$\left[(n_1K_1 - i\omega n_2K_2)\left(D^2-k^2\right) + a_{13}\omega^2\right]\overline{\tau} + i\omega a_4\left(D^2-k^2\right)\overline{\psi_1} + \left(a_{10}\left(D^2-k^2\right) + i\omega a_{11}\right)\overline{\phi^*} - a_{14}\left(D^2-k^2\right)\overline{\psi} = 0, \tag{28}$$

$$\left[a_7\left(D^2-k^2\right) - a_8 + a_{12}\omega^2\right]\overline{\phi^*} + \left[a_{10}\left(D^2-k^2\right) - i\omega a_{11}\right]\overline{\tau} - \left(D^2-k^2\right)\left(a_3\overline{\psi_1} + a_9\overline{\psi}\right) = 0, \tag{29}$$

$$a_9\left(D^2-k^2\right)\overline{\phi^*} + \nu\left(D^2-k^2\right)\overline{\psi} + a_{14}\left(D^2-k^2\right)\overline{\tau} = 0, \tag{30}$$

$$\left(a_1\left(D^2-k^2\right) + \omega^2\right)\overline{\psi_2} - a_2\overline{\phi}_2 = 0, \tag{31}$$

$$a_2\left(D^2-k^2\right)\overline{\psi_2} + \left(a_5\left(D^2-k^2\right) - 2a_2 + a_6\omega^2\right)\overline{\phi_2} = 0. \tag{32}$$

These two system of equations (27)–(30) and (31) and (32) have a nontrivial solution if the value of the determinant of coefficients of field quantities is zero, resulting in the following characteristic equations:

$$\left[\boldsymbol{D}^8 + A\boldsymbol{D}^6 + B\boldsymbol{D}^4 + C\boldsymbol{D}^2 + E\right] = 0, \quad (33)$$

$$\left[\boldsymbol{D}^4 + F\boldsymbol{D}^2 + G\right] = 0. \quad (34)$$

Here, $\boldsymbol{D} = \frac{d}{dx_3}$ and A, B, C, E, F, G, are mentioned in Appendix 1.

Here, we are only interested in surface waves, so it is necessary that the movement must be confined to the free surface $x_3 = 0$ of the half-space. Therefore to satisfy the radiation condition $\left(\overline{\psi}, \overline{\psi}_1, \overline{\phi^\star}, \overline{\tau}, \overline{\psi}_2, \overline{\phi}_2\right) \to 0$ as $x_3 \to \infty$ are given as following:

$$\left(\overline{\psi}, \overline{\psi}_1, \overline{\phi^\star}, \overline{\tau}\right) = \sum_{i=1}^{4} (1, \alpha_{1i}, \alpha_{2i}, \alpha_{3i})\, C_i e^{-m_i x_3}, \quad (35)$$

$$\left(\overline{\psi}_2, \overline{\phi}_2\right) = \sum_{i=5}^{6} (1, \alpha_{4i})\, C_i e^{-m_i x_3}, \quad (36)$$

$C_i\ (i = 1, 2, \ldots, 6)$ are arbitrary constants.
$m_i^2\ (i = 1, \ldots, 4)$ are the roots of the equation (33) and $m_i^2 (i = 5, 6)$ are of equation (34).

Here, $\alpha_{1i} = \frac{D_{1i}}{D_{0i}}, \alpha_{2i} = \frac{D_{2i}}{D_{0i}}, \alpha_{3i} = \frac{D_{3i}}{D_{0i}} i = 1, 2, 3, 4, \alpha_{4i} = \frac{a_1(m_i^2-k^2)+\omega^2}{a_2}, i = 5, 6$
$D_{ji}, D_{0i} j = 1, 2, \ldots, 4$, are defined in Appendix 2.

5 Boundary conditions

We consider a stress-free insulated surface at $x_3 = 0$ along with vanishing of temperature gradient and dielectric displacement. Mathematically this can be written as

$$t_{33} = t_{31} = 0, \quad m_{32} = 0, \quad (37)$$
$$\lambda_3^\star = 0, \quad \frac{\partial T}{\partial x_3}, \quad D_3 = 0,$$

6 Derivation of the secular equations

Using equations (6)–(10), (19), and (26) in boundary conditions (37), we obtain following system of six simultaneous homogeneous linear equations:

$$\sum_{i,j=1}^{6} Q_{ij} C_j = 0 \quad (38)$$

This system of linear equations (34) has a nonvanishing/nontrivial solution if the determinant of the matrix of coefficients of amplitudes i.e. coefficients of C_j, $j = 1, 2, \ldots, 6$ vanishes. Mathematically, this concept can be presented by the following expression:

$$\left|Q_{ij}\right| = 0, \ i, j = 1, 2, \ldots, 6 \quad (39)$$

where

$$Q_{1j} = \begin{cases} \left[b_1\left(m_i^2 - k^2\right) + b_2 m_i^2\right]\alpha_{1i} & j = 1, 2, 3, 4 \\ \quad +b_3\alpha_{2i} + i\omega b_4 \alpha_{3i}, & \\ \iota k b_2 m_i, & j = 5, 6, \end{cases}$$

$$Q_{2j} = \begin{cases} \iota k b_2 \alpha_{1i} m_i, & j = 1, 2, 3, 4 \\ \left(b_5 m_l^2 - b_6 k^2\right) - b_5\alpha_{4i}, & j = 5, 6, \end{cases}$$

$$Q_{3j} = \begin{cases} -\iota k\left(b_9\alpha_{2i} + b_{10} - i\omega b_1 \alpha_{3i}\right), & j = 1, 2, 3, 4 \\ -b_8\alpha_{4i} m_i, & j = 5, 6, \end{cases}$$

$$Q_{4j} = \begin{cases} -m_i b_{12}\alpha_{2i} + m_i b_4 + i\omega m_i b_{15}\alpha_{3i}, & j = 1, 2, 3, 4 \\ -ikb_{13}\alpha_{4i}, & j = 5, 6, \end{cases}$$

$$Q_{5j} = \begin{cases} -i\omega\alpha_{3i}, & j = 1, 2, 3, 4 \\ 0, & j = 5, 6, \end{cases}$$

$$Q_{6j} = \begin{cases} b_{17} m_i \alpha_{2i} + m_i b_{18}, & j = 1, 2, 3, 4 \\ ikb_{16}\alpha_{4i}, & j = 5, 6, \end{cases}$$

$$b_1 = \frac{\lambda}{\rho c_1^2}, \quad b_2 = \frac{2\mu + K}{\rho c_1^2}, \quad b_3 = \frac{\lambda_0 L_0^2}{j_0^2 \rho c_1^2}, \quad b_4 = \frac{\beta_0 T_0}{\rho c_1^2},$$
$$b_5 = \frac{\mu + K}{\rho c_1^2}, \quad b_6 = \frac{\mu}{\rho c_1^2}, \quad b_7 = \frac{K L_0^2}{j^2 \rho c_1^2}, \quad b_8 = \frac{\gamma}{j^2 \rho c_1^2},$$
$$b_9 = \frac{b_0}{j_0^2 \rho c_1^2}, \quad b_{10} = \frac{\lambda_1 \psi_0}{L_0^2 \rho c_1^2}, \quad b_{11} = \frac{\nu_2 T_0}{L_0 \rho c_1^3},$$
$$b_{12} = \frac{\alpha_0}{j_0^2 \rho c_1^2}, \quad b_{13} = \frac{b_0}{j^2 \rho c_1^2}, \quad b_{14} = \frac{\lambda_2 \psi_0}{L_0^2 \rho c_1^2},$$
$$b_{15} = \frac{\nu_1 T_0}{L_0 \rho c_1^3}, \quad b_{16} = \frac{\lambda_1 L_0}{j^2}, \quad b_{17} = \frac{\lambda_2 L_0}{{j_0}^2}, \quad b_{18} = \frac{\chi \psi_0}{L_0}.$$

7 Particular case

Microstretch thermoelastic medium: If the piezo-electric parameters are neglected, then this problem reduces to that of microstretch thermoelastic medium.

8 Numerical Results and Discussions

In order to illustrate the theoretical results obtained in the previous sections, some numerical results are presented. For numerical computation, the values for relevant parameters are taken for aluminum epoxy-like material, the values of physical parameters are given below: $\lambda = 7.59\times10^9$ Nm^{-2}, $\mu = 1.89 \times 10^9$ Nm^{-2}, $K = 1.49 \times 10^7$ Nm^{-2}, $\rho = 2190$ kgm^{-3}, $j = 0.2 \times 10^{-19}$ m^2, $\gamma = 2.63 \times 10^3$ N, $\lambda_1 = 0.5 \times 10^{10}$ Nm^{-2}, $T_0 = 298$ K, $I = 19.6 \times 10^{-8}$ m^2, $K^* = 1.7 \times 10^6$ $Jm^{-1}s^{-1}K^{-1}$, $a = 9.6 \times 10^2$ $m^2s^{-2}K^{-1}$, $b = 32\times10^2$ $kg^{-1}m^5s^{-2}$, $j_0 = 0.19\times10^{-6}$ m^2, $\alpha_0 = 0.9\times10^3$ N, $b_0 = 9.1\times10^2$ N, $\lambda_0 = 0.5\times10^9$ Nm^{-2}, $\lambda_1 = .5\times10^9$ Cm^{-1}, $\lambda_2 = 1.7 \times 10^4$ Cm^{-1}, $\lambda_3 = 0.7 \times 10^9$ Nm^{-2}, $\nu_1 = 0.3 \times 10^6$ Ns^{-1}, $\nu_2 = 0.457 \times 10^9$ $NK^{-1}s^{-1}$, $\nu_3 = 2.4 \times 10^3$, $Cm^{-1}s^{-1}$, $\chi = 318$, $L_0 = 1m$, $\psi_0 = 1NmC^{-1}$.

The analysis of dimensionless field quantities has been made and the graphs have been plotted with respect to wave number. The variation of $|Q_{ij}|$, Rayleigh wave velocity and amplitude ratios is shown in figures below. The variation of $|Q_{ij}|$, Rayleigh wave velocity and amplitude ratios have been shown for different values of angular frequency as well as for different values of piezo-electric parameter n_1. The different values of angular frequency ω chosen for analysis are $\omega = 0.1$, $\omega = 0.11$ and $\omega = 0.12$. Also, the different values of piezo-electric parameter n_1 are $n_1 = 0.001$, $n_1 = 0.01$ and $n_1 = 0.03$.

The different plots of variation of $|Q_{ij}|$, Rayleigh wave velocity and amplitude ratios for these different values are compared in graphs to study the influence of angular frequency and piezo-electric parameter in the present investigation of Rayleigh waves.

The variation of secular equations $|Q_{ij}|$ with respect to wave number k under the influence of angular frequency ω is shown in Figure 2. The significant variation is shown in range $0.2 \le k \le 1$. Also it is noted from the figure that the highest variation is shown in the curve corresponding to the value $\omega = 0.1$. The variation of $|Q_{ij}|$ w.r.t. wave number exhibits a similar trend for different values of ω.

Figure 3 shows the variation of Rayleigh wave velocity with respect to wave number k for different values of angular frequency ω. For $\omega = 0.1$, the Rayleigh wave velocity initially decreases uniformly and then decreases sharply near $k = 0.7$. After reaching a minimum value, the Rayleigh wave velocity increases and approaches the boundary surface for $k \ge 1$. For other values of ω, the variation of Rayleigh wave velocity shows an oscillatory trend.

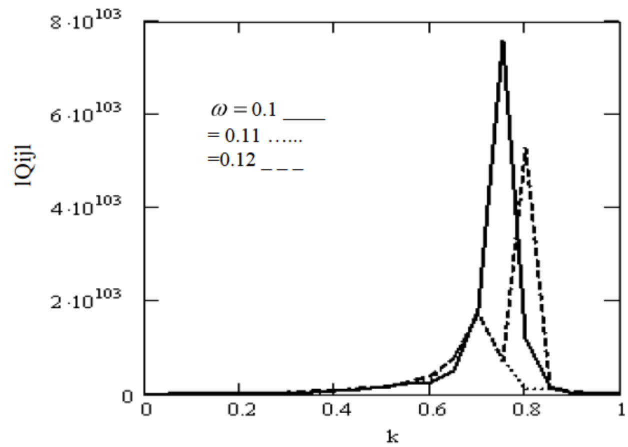

Figure 2: Variation of $|Q_{ij}|$ w.r.t. wave number under the influence of angular frequency.

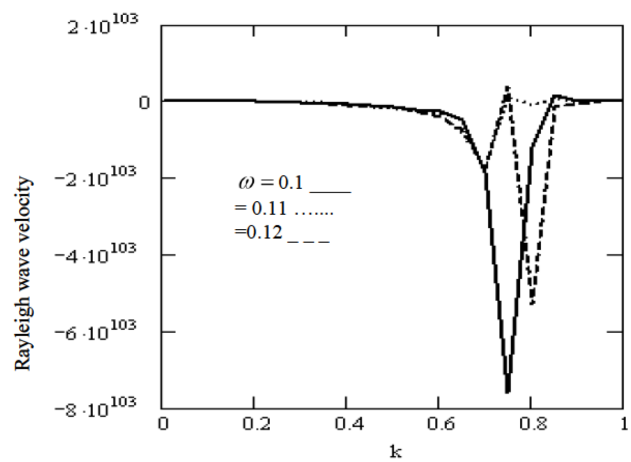

Figure 3: Variation of Rayleigh wave velocity w.r.t. wave number under influence of ω.

Figure 4 presents the variation of attenuation coefficient with respect to wave number k for different assignments of angular frequency ω. It is observed from the graph that maximum variation in attenuation coefficient corresponds to $\omega = 0.11$ and minimum variation is seen for $\omega = 0.1$.

Figure 5 exhibits the trend of variation of $|Q_{ij}|$ with respect to wave number under the influence of piezo-electric parameter n_1. It is observed from the graph that the variation in $|Q_{ij}|$ in the case of $n_1 = 0.03$ is maximum and is minimum for $n_1 = 0.001$. So, it can be stated that the variation in $|Q_{ij}|$ is large in those materials having high value of piezo-electric parameters.

Figure 6 shows the variation of Rayleigh wave velocity with respect to wave number $k$for different values of piezo-electric factor n_1. The variation is maximum in case of $n_1 =$ 0.03 and is minimum in case of $n_1 = 0.001$.

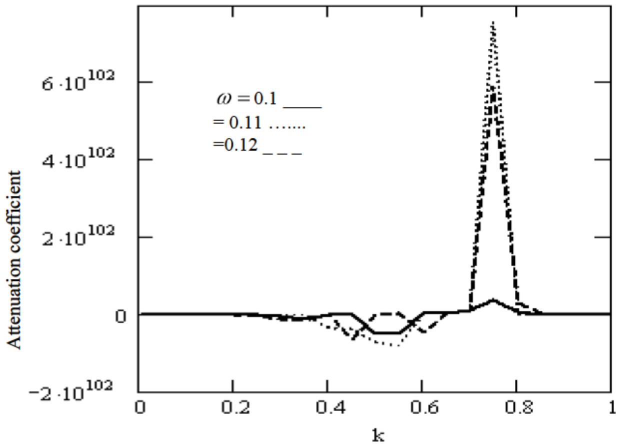

Figure 4: Variation of attenuation coefficient w.r.t. wave number under influence of frequency ω.

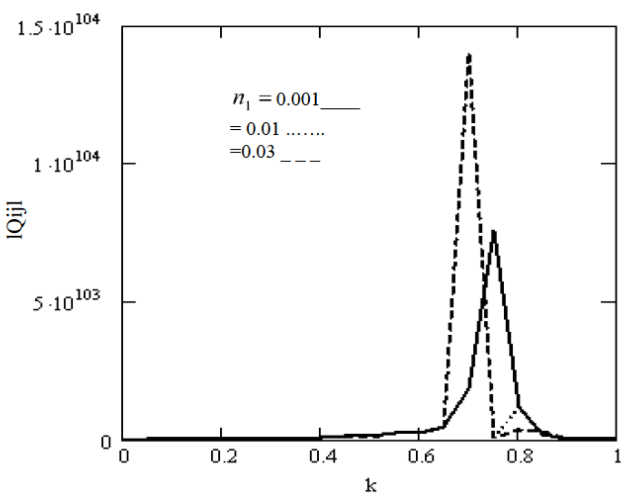

Figure 5: Variation of $|Q_{ij}|$ w.r.t. wave number under the influence of n_1.

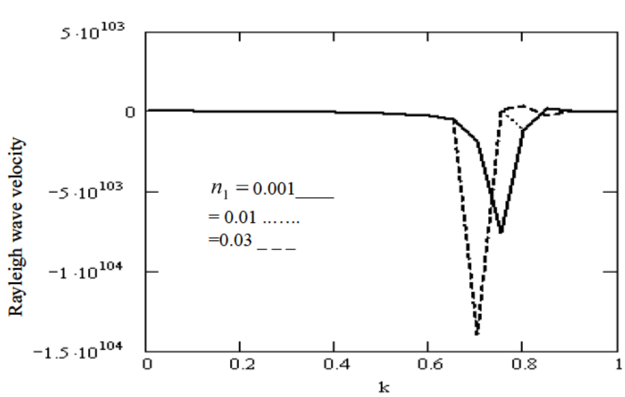

Figure 6: Variation of Rayleigh wave velocity w.r.t. wave number under influence of n_1.

In Figure 7, the variation of attenuation coefficient with respect to the wave number is shown for three different values of piezo-electric parameter n_1. The trend of variation of attenuation coefficient is shown for the range $0 \le k \le 1$. There is significant variation only in the subrange $0.2 \le k \le 0.8$. The variation of attenuation coefficient shows oscillatory behavior in this region.

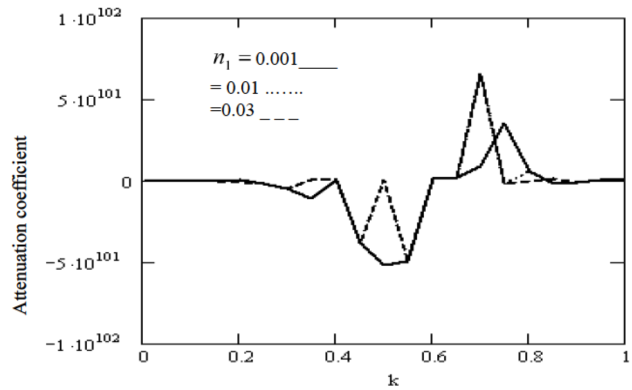

Figure 7: Variation of attenuation coefficient w.r.t. wave number under influence of n_1.

9 Conclusions

The study of the Rayleigh waves in a piezo-electric microstretch thermoelastic medium was done in this paper. The boundary surface of the medium is stress free and thermally insulated. The secular equation for the mathematical model of Rayleigh waves was written.

The graphs showing the variation of Rayleigh wave velocity and attenuation coefficient with respect to wave number have been plotted. The influence of angular frequency and piezo-electric parameter is also shown in graphs. All the physical quantities show variation only in a specified range of wave number, which is in accordance with various theories of thermoelasticity.

The present research has practical usefulness in rock mechanics and seismological research. Such type of studies may have applications in study of properties of valuable deposits under the earth's surface and even useful for detection of these useful smart materials.

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Appendix A:

$$A = \frac{\left(k_5 + k_4\left(\omega^2 - 2k^2\right) + a_3 k_7 + k_9\right)}{k_4},$$

$$B = \frac{X}{k_4},$$

where $X = \left(k_4\left(k^2 - \omega^2\right) + k_5\left(\omega^2 - 2k^2\right) + k_6 + a_3\left(k_8 - 2k_7k^2\right) + k_{10} - 2k^2 k_9\right)$

$$C = \frac{Y}{k_4},$$

where $Y = \left(k_6\left(\omega^2 - 2k^2\right) + k_5\left(k^2 - \omega^2\right) + a_3k_7k^4 - 2a_3k_8k^2 + k_9k^4 - 2k_{10}k^2\right)$

$$E = \frac{\left(k_6k^2\left(k^2 - \omega^2\right) + a_3k_8k^4 + k_{10}k^4\right)}{k_4},$$

$$F = \frac{\left(a_1l_5 + a_5\left(\omega^2 - a_1k^2\right) + a_2^2\right)}{a_1a_5},$$

$$G = \frac{l_5\left(\omega^2 - a_1k^2\right) - a_2^2k^2}{a_1a_5}$$

Here,

$$l_6 = n_1k_1 - i\omega n_2k_2,$$

$$K^\star = a\frac{\lambda + 2\mu}{4},$$

$$l_1 = i\omega a_{11} - a_{10}k^2,$$

$$l_2 = a_{13}\omega^2 - l_6k^2,$$

$$l_3 = a_{12}\omega^2 - a_8 - a_7k^2,$$

$$l_4 = -a_{10}k^2 - i\omega a_{11},$$

$$l_5 = a_6\omega^2 - 2a_2 - a_5k^2$$

$$k_4 = a_{14}^2\left(a_7 - a_9\right) - a_9a_{10}a_{14} + a_9^2l_6 - \nu a_{10}^2 + \nu m_3a_7,$$

$$\begin{aligned} k_5 &= a_{14}^2\left(l_3 - a_7k^2 + a_9k^2\right) - l_4a_9a_{14} \\ &+ a_9\left(l_2a_9 - l_6a_9k^2 - l_1a_{14} + a_{10}k^2\right) \\ &+ \nu\left(l_2a_7 + l_3l_6 - l_1a_{10} - l_4a_{10}\right), \end{aligned}$$

$$\begin{aligned} k_6 &= a_{14}a_9k^2\left(l_1 + l_4\right) - a_9^2l_2k^2 - a_{14}^2l_3k^2 \\ &- \nu\left(l_2\, l_3 - \, l_1\, l_4\right), \end{aligned}$$

$$k_7 = i\nu\omega a_{10}a_4 + \nu l_6a_3 + a_{14}\left(a_{14}a_3 + i\omega a_4a_9\right),$$

$$k_8 = i\nu\omega a_4l_4 + \nu a_3l_2 - a_{14}k^2\left(a_3a_{14} + i\omega a_4a_9\right),$$

$$k_9 = -i\omega a_4\left(i\omega\nu a_4a_7 + \nu a_3a_{10} + a_9\left(a_{14}a_3 + i\omega a_4a_9\right)\right),$$

$$k_{10} = i\omega a_4\left(a_9k^2\left(a_3a_{14} + i\omega a_4a_9\right) - i\nu\omega a_4l_3 - \nu a_3l_1\right)$$

Appendix B:

$$D_{0i} = \left(m_i^2 - k^2\right)^2 \begin{vmatrix} i\omega a_4 & a_{10}m_i^2 + l_1 & l_6 m_i^2 + l_2 \\ -a_3 & a_7 m_i^2 + l_3 & a_{10}m_i^2 + l_4 \\ 0 & a_9 & a_{14} \end{vmatrix},$$

$$D_{1i} = \left(m_i^2 - \xi^2\right) \begin{vmatrix} -a_{14} & a_{10}m_i^2 + l_1 & l_6 m_i^2 + l_2 \\ -a_9 & a_7 m_i^2 + l_3 & a_{10}m_i^2 + l_4 \\ \nu & a_9\left(m_i^2 - k^2\right) & a_{14}\left(m_i^2 - k^2\right) \end{vmatrix},$$

$$D_{3i} = \left(m_i^2 - \xi^2\right)^2 \begin{vmatrix} -a_{14} & i\omega a_4 & l_6 m_i^2 + l_2 \\ -a_9 & -a_3 & a_{10}m_i^2 + l_4 \\ \nu & 0 & a_{14} \end{vmatrix},$$

$$D_{4i} = \left(m_i^2 - \xi^2\right)^2 \begin{vmatrix} -a_{14} & i\omega a_4 & a_{10}m_i^2 + l_1 \\ -a_9 & -a_3 & a_7 m_i^2 + l_3 \\ \nu & 0 & a_9\left(m_i^2 - k^2\right) \end{vmatrix},$$