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Research Article

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Vibration analysis of functionally graded tapered rotor shaft system

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Abstract: This investigation deals with the vibration analysis of a rotating tapered shaft in Functionally Graded Material (FGM). The dynamic system is modeled using the Timoshenko beam theory (FSDBT) with consideration of gyroscopic effect and rotary inertia. The equations of motion are expressed by the hierarchical finite element method based on bi-articulated boundary conditions. The material properties are continuously varied in the thickness direction of a hollow shaft according to the exponential law function (E-FGM). The presented model is validated by comparing the numerical results found with the available literature. Various analyses are carried out to determine the influence of taper angle and material distribution of the two extreme materials on the dynamic behavior of FGM conical rotors system.

Keywords: Vibration analysis, tapered shaft, gyroscopic effect, hierarchical finite element, rotors conicity

1 Introduction

Vibrations are one of the principle subjects of rotating shafts theory. Vibrations need to undergo the mastering process and we must also know their resonance frequencies to avoid the critical pulsation that causes a reduction of the yield and produces very high noise.

Indeed, vibrations can even lead to system instability and failure. Therefore, shaft dynamics is extremely relevant now. In the late 1980s, scientific researchers found the means for combining the particles of a structure according to a special method that led to a material with very specific properties, which vary according to a known function. The new material was named as Functionally Graded Material (FGM), and as the name indicates, this material is generally associated with composite particles where the volume fraction of the particles varies in one or more directions [1].

The invention of the new material (FGM) opened up new avenues by increasing the performance of industrial machines due to its intrinsic qualities such as lightness (combined with high strength characteristics) and good resistance to corrosion. The industrial applications of machines have been extended due to the development of this new material (FGM), which has been elaborated from a new design and manufacturing philosophies.

The first FGMs were used in the designing and manufacturing of mechanical parts for aeronautical, aerospace, maritime and construction structures and so on. Because of their excellent mechanical properties, they can be subjected to severe mechanical and/or thermal stress. Recently, these materials have found other uses in electrical appliances, energy transformation, biomedical engineering and optics [2].

Several studies have been carried out with a view to studying the thermo-mechanical behavior of FGMs [3]. However, research on dynamic studies of rotors in FGM is quite limited, and is particularly limited to theoretical [4]. He presented a model of an extended length cutting tool intended to be operated in high speed operational by the use of Galerkin's general method, based on Timoshenko's beam theory [5, 6]. Applied a membrane analogy to analyze plates and shells in FGM based on a third-order theory of plates [7]. He analyzed the free vibrations of a rotating composite shaft based on the p-version of the finite element method. [8] studied the free vibration analysis of thick FGM plates on elastic bases with two parameters [9]. He mainly deals with the dynamic analysis of a rotor made

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of FGM supported on two flexible bearings, he used the finite element method based on the theory of Timoshenko (TBT) [10]. He proposed a new element for analyzing conical beams with an arbitrarily variant cross-section of a functionally classified material, [11]. He has analyzed rotor vibration and stability in FGM and takes into account the internal damping of the shaft.

The objective of this work is to present an analytical model of the dynamic behaviors of functionally graded tapered rotor shaft subjected on bi-articulated boundary conditions. The vibration analysis is according to the first shear deformation beam theory (FSDBT) while including the gyroscopic effect and rotary inertia. The mechanical properties are continuously varied in the thickness direction of a hollow shaft according to the exponential law form (E-FGM). The government equations of motion are expressed using the finite element method combined by (p) version of the hierarchical method. The model presented is validated by comparing the numerical results found with those available in the literature. Several examples are examined in detail to determine the influence of the conical angle and the material distribution on the dynamic behavior of FGM tapered rotors shafts system.

2 Modeling of FGM tapered rotor shaft

Considering a conical elastic shaft of circular cross-section, the coordinates X and Y define the horizontal plane of the shaft, while the Z axis defines the vertical plane with the other axes, Figure 1.

The material properties of this shaft are: Young's modulus, density and Poisson coefficient. For current model, the volume fraction varies continuously in the thickness direction h.

Most researchers use the exponential function to describe the material properties of FGMs, defined by [12]. The exponential law (E-FGM) of the Functionally Graded tapered hollow shaft is modeling by:

$$p(r(x)) = p_A . e^{-\beta(x).(r(x)-R_i(x))}, \quad x \in [0, L] \tag{1}$$

With

$$\beta(x) = \frac{1}{(R_0(x) - R_i(x))} \ln(\frac{p_A}{p_B}) \tag{2}$$

where $r(x)$ presents the variation of radius r as a function of the position x, according to the conicity of the hollow rotor system. The inner surface of the shaft ($r(x) = R_i(x)$) consists of 100% material (i), while the outer surface of the shaft ($r(x) = R_0(x)$) has 100% material (o).

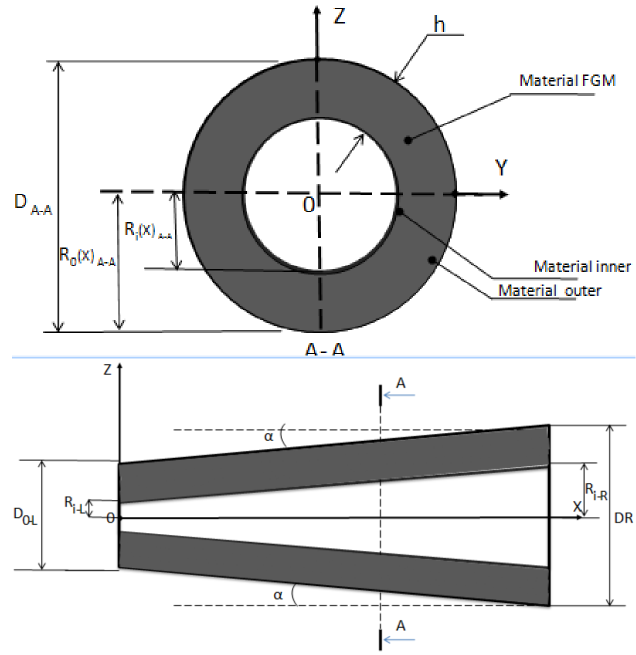

Figure 1: Modeling of FGM tapered rotor shaft based on the contained function of the material properties

3 Mathematical development

In Timoshenko's theory, the kinematic assumptions of any point on the tapered rotor shaft are adopted as follows:

$$\begin{cases} U(x,y,z,t) = U_0(x,t) + z\beta x(x,t) - y\beta y(x,t) \\ V(x,y,z,t) = V_0(x,t) - z\phi(x,t) \\ W(x,y,z,t) = W_0(x,t) + y\phi(x,t) \end{cases} \tag{3}$$

The formula of the deformation energy of the system is:

$$E_d = \frac{1}{2}\int (\sigma_{xx}\varepsilon_{xx} + 2\tau_{xr}\varepsilon_{xr} + 2\tau_{\theta x}\varepsilon_{x\theta}).dv \tag{4}$$

Eq. (4) in the developed form takes the following form:

$$E_d = \frac{1}{2}\int_0^L A_{11}(x).\left(\frac{\partial u_0}{\partial x}\right)^2 dx \tag{5}$$

$$+\left[\frac{1}{2}\int_0^L B_{11}(x).\left(\frac{\partial \beta_x}{\partial x}\right)^2 dx + \frac{1}{2}\int_0^L B_{11}(x).\left(\frac{\partial \beta_y}{\partial x}\right)^2 dx\right]$$

$$+\frac{1}{2}k_s\int_0^L B_{66}(x).\left(\frac{\partial \phi}{\partial x}\right)^2 dx$$

$$+\frac{1}{2}k_s\int_0^L (A_{55}(x) + A_{66}(x))$$

$$\left[\left(\frac{\partial v_0}{\partial x}\right)^2 + \left(\frac{\partial w_0}{\partial x}\right)^2 + \beta_x^2 + \beta_x^2 + 2\beta_x\frac{\partial w_0}{\partial x} - 2\beta_y\frac{\partial v_0}{\partial x}\right] dx$$

The expression of the kinetic energy of the shaft is given by the following equation:

$$E_c = \frac{1}{2}\int \rho(\vec{\dot{R}}_{p/o}.\vec{\dot{R}}_{p/o})dv \quad (6)$$

The kinetic energy in developed form is:

$$E_c = \frac{1}{2}\int_0^1 \Big[I_m(x)(\dot{u}_0^2 + \dot{v}_0^2 + \dot{w}_0^2) + I_d(x)(\dot{\beta}_x^2 + \dot{\beta}_y^2) \quad (7)$$
$$- 2\Omega I_p(x)\beta_x\dot{\beta}_y + 2\Omega I_p(x)\phi + I_p(x)\dot{\phi}^2 + \Omega^2 I_p(x)$$
$$+ \Omega^2 I_d(x)(\beta_x^2 + \beta_y^2)\Big] dx$$

With:

$$\begin{cases} A_{11}(x) = 2\pi \int_{R_i(x)}^{R_0(x)} Q_{11}(r).r.dr \\ A_{55}(x) = \pi \int_{R_i(x)}^{R_0(x)} Q_{55}(r).r.dr \\ A_{66}(x) = \pi \int_{R_i(x)}^{R_0(x)} Q_{66}(r).r.dr \\ B_{11}(x) = \pi \int_{R_i(x)}^{R_0(x)} Q_{11}(r).r^3.dr \\ B_{66}(x) = 2\pi \int_{R_i(x)}^{R_0(x)} Q_{66}(r).r^3.dr \\ I_m(x) = 2\pi \int_{R_i(x)}^{R_0(x)} \rho(r).r.dr \\ I_d(x) = \pi \int_{R_i(x)}^{R_0(x)} \rho(r).r^3.dr \\ I_p(x) = 2\pi \int_{R_i(x)}^{R_0(x)} \rho(r).r^3.dr \end{cases} \quad (8)$$

With:

$$\begin{cases} Q_{11} = \frac{E(r)}{1-\vartheta(r)^2} \\ Q_{55} = Q_{66} = \frac{E(r)}{2(1+\vartheta(r))} \end{cases} \quad (9)$$

$$\begin{cases} R_i(x) = \tan(\alpha).x + R_{i-L} \\ R_o(x) = tan(\alpha).x + (R_{i-L} + h) \end{cases} \quad (10a)$$

$$\tan(\alpha) = \frac{R_{i-R} - R_{i-L}}{L} \quad (10b)$$

With:
R_{i-L}: Left inner radius of the tapered shaft
R_{i-R}: Right inner radius of the tapered shaft

The displacement field of a point of the beam in free vibration is given by:

$$q_i(x,t) = [N_i(x)]^T \{q_i\ (t)\} = \sum_{n=1}^{p} g_{i,n}(x).q_{i,n}..e^{(j\omega t)} \quad (11)$$

Such as:
$q_i(x,t)$: Elemental displacement of the beam, with i= $U_0, V_0, W_0, \beta_x,\ \beta_y, \phi$
$N_i(x)$: Vector of the shape function
$\{q_i(t)\}$: Temporary displacement coefficient vector

We use the following Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial\{\dot{q}_i\}}\right) - \frac{\partial E_c}{\partial\{q_i\}} + \frac{\partial E_d}{\partial\{q_i\}} = \frac{\delta A}{\delta\{q_i\}} = F_i(t) \quad (12)$$

For the determination of the global equation of motion, Eqs. (5) and (7) are injected into the Lagrange formula (12):

$$[M]\{\ddot{q}\} + [G]\{\dot{q}\} + [K]\{q\} = \{0\} \quad (13)$$

4 Numerical results and discussion

The tapered rotor shaft is modeled by a single bi-supported element using the hierarchical finite element (h-p) method with trigonometric shape functions shown in Eq. (13) combined with the classical finite element method, indicated in Eq. (14). The mechanical properties and the geometric dimensions of the FGM shaft are illustrated in Table 1 [14].

$$g(x) = \begin{cases} g_1(x) = (l-x) \\ g_2(x) = x \\ g_{n+2}(x) = (n,\pi,x) \quad , n = 1,2,3,\ldots,p \end{cases} \quad (14)$$

Figures 2, 3 and 4 show the Campbell's diagram that superimposes the evolution of the first three bending modes depending on the rotational speed variation (Ω = 0 to 200 Hz) of a bi-supported hollow conical rotor shaft (A-A) in FGM. The conical angle (α) is varied by 0° and 7° for a slenderness ratio set at L / D-L = 10. The intersection points of the rotating speed line (Ω) with the fundamental frequencies curves indicate the values of the critical speeds (Ωcr).

From the analysis of these figures, it should be noted that the taper angle affects the critical speeds of the FGM shaft, taking the example of the first bending mode, when (α) varies from 0° to 7°, the first fundamental frequencies increase by 37.68% for Ω = 0 Hz and 48.82% for Ω = 200 Hz. So, we can deduce that the conical angle (α) has a significant effect on the increase of the critical speeds of FGM rotating shaft.

Table 1: Mechanical properties and geometric dimensions of the FGM shaft

Geometric parameters: h = 0.002 m, D_{-L}/ h = 500, L / D_{-L} = 20. The shear correction factor Ks = 0.5		
Property	FGM	
	Nickel (Ni)	Stainless Steel (SS)
E (GPa)	205,098	207,788
Poisson's ratio	0,31	0.317756
Density (kg/m^3)	8900	8166

Table 2: Variation of the fundamental frequencies (Hz) for extreme materials and FGM constant shaft with rotation speed Ω = 0.

		Fundamental frequencies (Hz)		
Material	Source	First frequency ($\omega 1$)	Second frequency ($\omega 2$)	Third frequency ($\omega 3$)
Nickel (Ni)	Present	6.9257	26.924	52.437
	Ref [14]	6.8577	-	-
	Ref [15]	6.9397	-	-
FGM (Ni-SS)	Present	7.0985	27.596	53.747
	Ref [14]	7.0972	-	-
	Ref [15]	7.0584	-	-
Stainless Steel (SS)	Present	7.2965	28.358	54.938
	Ref [14]	-	-	-
	Ref [15]	-	-	-

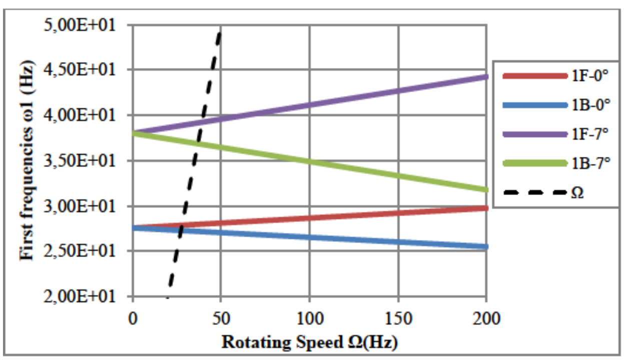

Figure 2: Campbell's diagram of the first-mode bending of FGM tapered rotor shaft bi-supported (A-A) ((F) direct modes (B) retrograde modes)

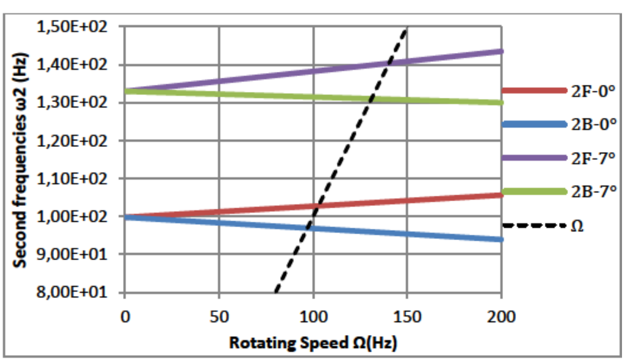

Figure 3: Campbell's diagram of the second-mode bending of FGM tapered rotor shaft bi-supported (A-A) ((F) direct modes (B) retrograde modes)

It may also be noted that the increase of the rotating speed (Ω) generates a remarkable separation of the fundamental frequencies into two branches, these two frequencies correspond to the mode in direct precession and mode

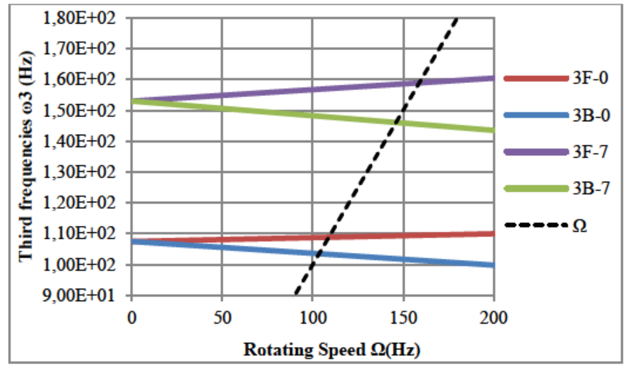

Figure 4: Campbell's diagram of the third-mode bending of FGM tapered rotor shaft bi-supported (A-A) ((F) direct modes (B) retrograde modes)

in retrograde precession; the direct modes increase with the increase of the rotating speed, and however, the reverse modes decrease. We have also observed that for a constant rotating shaft ($\alpha = 0°$), the difference between the direct and the retrograde frequencies of the first three bending modes is small compared to those of a tapered rotating shaft inclined by $\alpha = 7°$. We can therefore say that the gyroscopic effect of the FGM conical rotating shaft is important when the taper angle take higher values.

5 Conclusion

In this article, we studied the dynamic behavior of bi-supported Functionally Graded conical rotating shaft based on an exponential law between two extreme materials (E-FGM). Our results were calculated using the hierarchical finite element method using the Timoshenko beam theory (FSDBT) with consideration of gyroscopic effect and rotary inertia. Several examples were processed to determine the effect of the conical angle (α) and the internal properties on the fundamental frequencies of the FGM tapered rotor shafts. This work allowed us to reach the following conclusions:

- The results of the frequencies calculated by the continuous function E-FGM have reached an excellent agreement with the results found in the literature, which thus validates the accuracy of our model developed.
- The conical angle of the rotor shafts has an appreciable influence on the bending frequencies, and consequently, the critical speeds, whatever the speed of rotation.

- The gyroscopic effect of the conical rotating shaft is important when the taper angle take larger values. So, we can say that the change of the taper angle (α) has a significant effect on the dynamic behavior of the Functionally Graded conical rotor shafts system.

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