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Research Article

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Energy analysis of a real suction-pressure unit

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Abstract: The article presents an energy analysis of a real suction-pressure unit of a high-efficiency radial piston pump. This pump can be used in small hydropower plants, where, together with a variable displacement hydraulic motor, it would form a hydrostatic transmission. The main advantage of this solution is the possibility of a stepless, automatic change of gear ratio during its operation, which allows an increase of the efficiency of power generation in a hydropower plant. Mathematical models of friction and resistance of flow through self-acting and geometrical deformations of chambers and piston under stress field are presented. In addition, mass, volume, and energy balance calculations were computed for the ideal unit and pressure losses were determined. Derived indicator diagrams for each chamber of the ideal suction-pressure unit served to analyze the technical work of occurring processes. The paper also presents the formulation of total efficiency of the unit. Lastly, the experimental procedure is presented, according to which it is possible to register the most important parameters of the proposed model.

Keywords: Hydraulics, pump, hydrostatic gear, renewable energy

Nomenclature

Δp_K – increase in pressure in working chambers A and B
Δp_{zs} – pressure drop in suction valve
Δp_{zw} – pressure drop in discharge valve
η – total efficiency coefficient
ν_s – Poisson's ratio for steel
ε_B – friction al energy losses generated by external leaks at the piston rods
ε_s – energy of pressed out working fluid
ε_U – friction energy losses
ε_w – energy of sucked working fluid
ε_{AB} – frictional energy losses generated by internal leaks between the piston and cylinder
ε_{us} – energy of the working fluid pushed by the suction-pressure unit
ε_{UT} – friction energy for the piston per cycle
ε_{Ut} – friction energy for the piston rod per cycle
ε_{uV} – isochoric transformation energy
D_t – diameter of the piston rod
D_w – internal diameter of the cylinder
D_z – external diameter of the cylinder
e – eccentricity of the drive unit
E_S – Young modulus for steel
f_T, f_t – coefficients of friction for the piston and piston rod
$f_{T0}, f_{T1}, f_{T2}, f_{T3}$ – empirical coefficients for the piston
$f_{t0}, f_{t1}, f_{t2}, f_{t3}$ – empirical coefficients for the piston rod
F_{UT} – friction in the piston seals
F_{Ut} – friction in the piston rod seals
i_s – specific enthalpy of the working fluid in the suction channel
i_w – specific enthalpy of the working fluid in the discharge (pressure) channel
j_{t0}, j_{t1}, j_{t2} – empirical exponents for the piston rod
k_{AB}, k_B – coefficients related to leaks in the sealing nodes
K_{Qs}, K_{Qw} – coefficients of flow rates through valves
k_{rT}, k_{rt} – coefficients that involve radial load
L_A – internal work in chamber A
l_A – distance from the piston to the bottom of chamber A
L_B – internal work in chamber B
l_B – distance from the piston to the bottom of chamber B
l_C – total length of cylinder
l_t – length of the piston rod
L_y – mechanical energy supplied to the suction-pressure unit
l_{UT}, l_{Ut} – lengths of sealing for the piston and rod

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p_A – pressure in chamber A
P_y – mechanical power supplied through piston rod
P_{us} – useful power of working fluid
Q_B – external leaks at the piston rods
Q_{AB} – internal leaks between the piston and cylinder
Q_{ps} – fluid returning through the suction inlet and check valves to the suction channel
Q_{pw} – fluid returning through the discharge channel to the suction channel
S_A – area of the piston cross-section in chamber A
S_{zw}, S_{zs} – area of flow gaps in valves
t_c – time of cycle
u_s – specific internal energy of working fluid in the suction channel
u_w – specific internal energy of working fluid in the pressure channel
V_A – volume of the chamber
v_s – specific volume of working fluid in the suction channel
v_w – specific volume of working fluid in the pressure channel
$V_{sA}, V_{sB}, V_{wA}, V_{wB}$ – volumes of self-acting valves
V_{zs} – total volume of suction losses
y_T – displacement of the piston
ΔQ_K – decrease of the volume flow caused by an increase of pressure of the fluid in the chamber
Δy_{CB} – increase of cylinder length
Δy_{DA} – additional change in the displacement of the piston
Δy_{DB} – increase of cylinder diameter
Δy_{lA} – spring reduction in piston length
Δy_{lB} – elastic extension of the piston rod
Δy_{TA} – virtual displacement of the piston rod
Δy_{TB} – virtual displacement of the piston
$\dot{\varepsilon}_{zs}$ – energy losses caused by volumetric and pressure losses
$\dot{\varepsilon}_{zw}$ – energy losses caused by pressure drop in the discharge valve
$\dot{m}_B$ – external mass losses at the piston rods
$\dot{m}_s$ – mass flow sucked in by the unit
$\dot{m}_w$ – mass flux flowing out from the unit
$\dot{m}_{AB}$ – internal mass losses between the piston and cylinder
$\dot{m}_{ps}$ – mass flux caused by leaks in the suction valves
$\dot{m}_{pw}$ – mass flux caused by leaks in the discharge valves
$\dot{m}_{zs}, \dot{m}_{zw}$ – mass flux of internal mass losses

1 Introduction

There is a current increase in the energy production from renewable energy sources. This is caused by the necessity for reduction of emission of greenhouse gases, produced as a result of burning fossil fuels. The energy of the flowing water has been used by humans for a long time, initially to replace the muscle power of humans and animals to drive equipment and then, after the industrial revolution, to produce electricity. Former locations in which water wheels were operated could be used to generate electricity [1]. The reasons for not exploiting such places are most often negligence, resulting from the centralization of the energy production, and costly technologies that make the operation of the power plant unprofitable. Currently, small hydros mostly use turbines. Their advantages over the water wheels include higher rotational speeds and less sensitivity to changing work conditions [2, 3]. Turbines require a greater head than water wheels to operate. This involves significant interference with the natural environment, which often excludes location of new power plants. Due to lesser environment interference and a large number of locations, including historical ones [1], the technology of production of electricity from the energy of flowing water using a water wheel still deserves further development.

A hydrostatic transmission with a low-speed radial piston pump can be used to transfer the drive from the water wheel to the asynchronous generator. Due to the low rotational speed of the water wheels, it is necessary to multiply the speed transferred to the generator whose speed ranges from 750 to 3000 rpm [2–4]. Such generators are applied in low-power small hydros (up to several kilowatts) due to their lower price than low-speed generators. It is also necessary to maintain the speed of the generator in order to synchronize it with the power grid. An important element of a plant is the power transmission system. Most common solutions involve belt transmission. Gear transmission is applied less frequently. With such gearboxes, it is not possible to adapt the gear ratio during operation to the current working conditions, for example, the variable intensity of the flowing water. In the available solutions, this involves stopping work and changing the gear ratio manually. This requires an additional service, which results in power plant downtime and thus reduces the profitability of small hydros operation [2, 3]. The use of hydrostatic transmission, which includes a radial piston pump, will allow a stepless, maintenance-free gear change during operation. It will also reduce power plant and maintenance costs. Visualization of the power plant with such a transmission is shown in Figure 1, together with the input parameters.

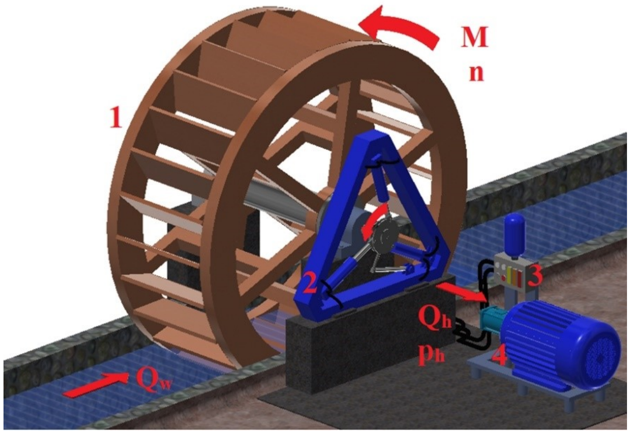

Figure 1: Visualization of hydrostatic transmission in a small hydropower plant with input parameters: 1 – water wheel, 2 – radial piston pump, 3 – hydraulic equipment, 4 – hydraulic motor connected to the asynchronous generator.
Q_w – flow rate of water that drives water wheel, M – water wheel torque, n – water wheel speed, Q_h – flow rate of hydraulic piston pump, p_h – pressure of fluid in pump's output line.

Hydrostatic transmission is widely used in wind turbine, where it transfers power from the turbine to the generator. This technology is constantly developed, and new concepts of use appear frequently in renewable energy application [5–7]. The hydrostatic transmission system can provide multiple benefits over the mechanical power transmission system due to its compact design and an ability to transmit high power and vary its speed continuously. The purpose of this paper is to demonstrate an energy analysis of the real suction-pressure unit of a high-efficiency radial piston pump which can be applied in the renewable energy field. This pump can be used in small hydropower plants, where, together with variable displacement hydraulic motor, it forms a hydrostatic transmission. Its main task is to provide transmission from a water wheel to the shaft of an asynchronous generator connected to a power grid, with the possibility of stepless gear ratio change. The main advantages of hydraulic transmission, of which the radial piston pump of high efficiency is a part, in comparison with the currently used solutions are:

- transfer of high torque,
- simple construction,
- possibility of stepless change of gear ratio during work, and
- lower construction and operating costs (in small hydros).

In the hydraulic piston pumps, building a mathematic model requires to correctly define losses (both hydraulic and mechanical) and to consider variable operating parameters. The correct kinematic analysis of the system is important as it has a significant impact on the mathematical model. The models that can be found in literature are developed in order to understand what effect the loss components have on the overall efficiency, and to estimate the pump characteristics [8–10]. The models are also used to analyze the entire system with hydrostatic transmission in an attempt to compare the characteristics of hydrostatic transmission working in closed loop or open loop hydraulic circuit [11]. Al-Qattan *et al.* presented a model of the piston compressor, of which the design and principle of operation are similar to the radial piston pump presented in this article [12]. It was used to monitor the technical condition of the compressor in order to prevent unplanned repairs. Another important aspect of modeling is its validation by comparing the simulation results obtained using the model with the experimental results of a physical unit [9].

In a previously published paper [13], an ideal suction-pressure unit that was a component of a low-speed piston pump, which can be potentially applied to small hydros [2, 3], was described. This cited article presents an energy, mass, and volume balance of a hydraulic medium in the ideal unit. It also shows indicator diagrams and describes the energy transformations that occur during operation. Shown transformation models of working medium do not take into account numerous important factors which can influence the performance of the real suction-pressure unit. Thus, it is necessary to build a new theoretical model that incorporates those essential accompanying phenomena that influence the process.

Hence, there is a need to build a theoretical model that takes into account the most important phenomena affecting the operation of the unit. The theoretical model of a real suction-pressure unit can be called a system of equations describing its work, involving: friction, leakage, resistance of the working fluid flow, and deformations of mechanical components. Considering such a large number of nonlinear factors, which cannot always be described accurately, it requires to simplify some assumptions. In this paper, an attempt is made to mathematically describe the energy flow and the method of determining the efficiency of the suction-pressure unit, including the actual physical phenomena that occur during the operation. In order to verify the presented mathematical model of the suction-pressure unit, an experimental test should be performed. A flow chart accompanied by a test procedure is presented in this study.

2 Factors influencing the performance of the suction-pressure unit and their mathematical models

2.1 Deformations of components under force and pressure

Changes in the pressure of unit chambers cause deformation of the cylinder as well as changes in piston rod length. These deformations depend also on fixturing, the dimensions of the suction-pressure unit, and the actual pressure present. Under the influence of periodic changes in the sign and value of force transmitted through the piston rod, its length will change, which will lead to an additional change in the volume of working chambers. These changes can be considered in the model by adjusting the displacement of the y_T piston. It should be remembered, however, that these deformations will change when the piston rod is extended or inserted. For the example shown in Figure 2, while compressing the working fluid in the piston chamber A, the axial force in the piston rod increases, causing a spring reduction in its length by:

$$\Delta y_{lA} = \frac{4 \cdot p_A \cdot S_A \cdot l_t}{E_S \cdot \pi \cdot {D_t}^2}, \tag{1}$$

where l_t is the length of piston rod, D_t the diameter of piston rod, S_A the area of the piston cross-section in chamber A, E_S the Young modulus for steel, and p_A is the pressure in chamber A.

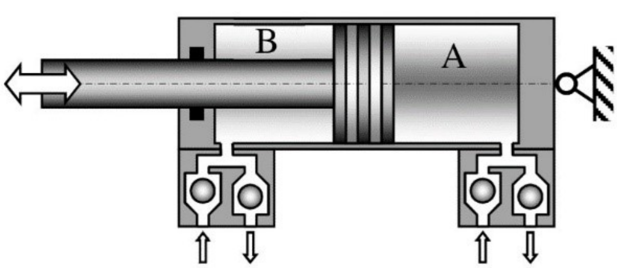

Figure 2: Schematic view of a suction-pressure unit [13].

The increase in pressure inside chamber A also causes an increase in the internal diameter of the cylinder D_w, which can be reduced (while maintaining the equivalent volume of the chamber V_A) to an additional change in the displacement of the piston [14]. This can be expressed by:

$$\Delta y_{DA} = \frac{2 \cdot \Delta p_A \cdot l_A}{E_s} \cdot \left(\frac{{\chi_s}^2 + 1}{{\chi_s}^2 - 1} + \nu_s \right), \tag{2}$$

where l_A is the distance from the piston to the bottom of chamber A, ν_s the Poisson's ratio for steel, and $\chi_s = \frac{D_z}{D_w}$, D_z, D_w are the external and internal diameters of a cylinder.

Total change of volume of chamber A can be replaced by a virtual displacement of piston rod by:

$$\begin{aligned} \Delta y_{TA}(t) &= \Delta y_{lA} + \Delta y_{DA} \\ &= \frac{2 \cdot \Delta p_A}{E_s} \cdot \left(\frac{2 \cdot \Delta p_A \cdot S_A \cdot l_A}{\pi \cdot {D_t}^2} \right. \\ &\quad \left. + l_A \left(\frac{{\chi_s}^2 + 1}{{\chi_s}^2 - 1} + \nu_s \right) \right). \end{aligned} \tag{3}$$

During compression in chamber B, there will be an elastic extension of the piston rod

$$\Delta y_{lB}(t) = \frac{4 \cdot \Delta p_B \cdot S_B \cdot l_t}{E_s \cdot \pi \cdot {D_T}^2} \tag{4}$$

and an increase of cylinder diameter [14]

$$\Delta y_{DB}(t) = \frac{2 \cdot \Delta p_B \cdot l_B}{E_s} \cdot \left(\frac{{\chi_s}^2 + 1}{{\chi_s}^2 - 1} + \nu_s \right) \tag{5}$$

and its length

$$\Delta y_{CB}(t) = \frac{\Delta p_B \cdot l_C}{\left({\chi_s}^2 - 1\right) \cdot E_s}, \tag{6}$$

where l_B is the distance from the piston to the bottom of chamber B and l_C is the total length of cylinder.

Virtual displacement of piston $\Delta y_{TB}(t)$ caused by the increase in pressure in chamber B can be expressed by:

$$\begin{aligned} \Delta y_{TB}(t) &= \left(\frac{\Delta p_B}{E_s} \frac{4 \cdot S_B \cdot l_t}{{D_t}^2} + \frac{1}{{\chi_s}^2 - 1} \right. \\ &\quad \left. + 2 \cdot l_B \left(\frac{{\chi_s}^2 + 1}{{\chi_s}^2 - 1} + \nu_s \right) \right). \end{aligned} \tag{7}$$

The described deformations of mechanical elements are elastic; therefore, it can be assumed that the energy balance of the deformations themselves during the entire cycle remains equal to zero. The geometrical changes, that is, virtual displacement of the piston, not only affect the volume of the working chambers, and thus the specific efficiency of the entire suction-pressure unit, but also, only indirectly, influence the unit efficiency.

2.2 Substantial and volumetric losses generated at self-acting valves

Losses in flow machines are caused by pressure drops which occur in pipes and valves. The suction-pressure unit is used for automatic timing. Pressure drops will, therefore, occur in inlet and outlet channels as well as check (one-way) valves. As a result of these losses, the increase in pressure Δp_K in working chambers A and B must be greater than the pressure difference before and after the considered unit. Pressure drops in valves and channels depend on their geometrical parameters, flow rate, and properties of the working fluid. Assuming negligible length and relatively large cross-sectional dimensions of the channels, the total pressure drop can be reduced to a local pressure drop occurring in the valves. The velocity of the piston, and hence the flow rate as well as the pressure drops in the valves, will also change. Considering a turbulent flow, the pressure drop in the suction valve can be described by:

$$\Delta p_{zs} = \left(\frac{Q_s}{K_{Qs} \cdot S_{zs}}\right)^2, \tag{8}$$

while in the pressure (discharge) valve,

$$\Delta p_{zw} = \left(\frac{Q_s}{K_{Qs} \cdot S_{zw}}\right)^2, \tag{9}$$

where K_{Qs} and K_{Qw} are the coefficients of flow rates through the valves and S_{zw} and S_{zs} are the areas of flow gaps in the valves.

After the suction is completed, the piston begins to compress the working fluid. As a result, it flows out of the check valves, forcing them to close. The volume of fluid flowing out of the valve can be called the volume of valve losses. The total volume of suction valve losses ($V_{zs} = V_{sA} + V_{sB}$) flowing out of the working chambers to the suction hose, related to the duration of a single cycle, can be treated as a flux of internal mass losses $\dot{m}_{zs}$:

$$\dot{m}_{zs} = \frac{Q_{zs}}{v_s} = \frac{V_{sA} + V_{sB}}{v_s \cdot t_c}. \tag{10}$$

Analogically, when closing pressure check valves in the suction-pressure unit, a flux of internal mass losses is created, $\dot{m}_{zs}$. It can be described by the following relation:

$$\dot{m}_{zw} = \frac{Q_{zw}}{v_w} = \frac{V_{wA} + V_{wB}}{v_w \cdot t_c}, \tag{11}$$

where V_{sA}, V_{sB}, V_{wA}, and V_{wB} are the volumes of self-acting valves and t_c is the time of cycle.

Energy losses occurring in check valves triggered by the flow of working fluid during their closing and pressure drops must be included in the indicator diagram of the suction-pressure unit. Their impact on the energy balance will be taken into account in the technical work as determined on the basis of indicator changes.

2.3 Friction at sealing nodes

In the analyzed suction-pressure unit, friction occurs in the sealing nodes of piston and piston rod. These nodes serve for both sealing and guiding. Determining the exact value of the friction force is practically impossible because it depends on the slip speed, clamping force, type of materials in contact, surface roughness, lubricating material, temperature, rest time, etc. In the literature [14–17], there are models that can help to roughly estimate the value of the friction force. A comprehensive study on friction in hydraulic sealing was presented by Dindorf [18]. He showed the mechanism of the phenomena occurring and proposed an accurate modeling technique. Based on these considerations, friction in the piston seals (F_{UT}) and piston rods (F_{Ut}) of the suction-pressure unit can be described as follows:

$$F_{UT} = f_{T0} exp\left(-j_{T0}\left|\frac{dy_T}{dt}\right|\right) + f_{T1}\left|\frac{dy_T}{dt}\right|^{j_{T1}} + f_{T2}|p_A - p_B|^{j_{T2}} + f_{T3} \tag{12}$$

and

$$F_{Ut} = f_{t0} exp\left(-j_{t0}\left|\frac{dy_T}{dt}\right|\right) + f_{t1}\left|\frac{dy_T}{dt}\right|^{j_{t1}} + f_{t2}|p_A - p_B|^{j_{t2}} + f_{t3}, \tag{13}$$

where f_{T0}, f_{T1}, f_{T2}, and f_{T3} are the empirical coefficients for the piston, j_{T0}, j_{T1}, and j_{T2} the empirical exponents for the piston, f_{t0}, f_{t1}, f_{t2}, and f_{t3} the empirical coefficients for the piston rod, and j_{t0}, j_{t1}, and j_{t2} are the empirical exponents for the piston rod.

The presented model has a large number of empirically determined coefficients, which significantly complicates its usability in theoretical and simulation research stages. Therefore, it may be more convenient to use a friction model presented by Palczak [19] in sealing nodes, which in the presented unit takes the form of:

$$F_{UT} = f_T \cdot k_{rT} \cdot l_{UT} \cdot D_w \cdot (p_A - p_B) \tag{14}$$

and

$$F_{Ut} = f_T \cdot k_{rt} \cdot l_{Ut} \cdot D_w \cdot (p_B - p_o), \tag{15}$$

where f_T and f_t are the coefficients of friction for the piston and rod, k_{rT} and k_{rt} the coefficients that involve radial load, k_r = 0.15–0.2 (please note that k_r can take a lower value for greater l), and l_{UT} and l_{Ut} are the lengths of sealing for the piston and rod.

The frictional forces, derived from the above dependences, are responsible for the creation of mechanical energy losses. These losses also depend on the geometrical parameters, pressure in the working chambers A and B, and on the velocity of piston with the piston rod (in the case of using the equations (12) and (13)). The energy lost in the sealing nodes as a result of friction is converted into heat.

The amount of friction energy losses can be calculated by the following equation:

$$\varepsilon_U = \varepsilon_{UT} + \varepsilon_{Ut} = \int_0^{2e} y\, dF_{Ut} + \varepsilon_{2e}^0 - y\, dF_{Ut} + \int_0^{2e} y\, dF_{Ut} + \varepsilon_{2e}^0 - y\, dF_{Ut}, \tag{16}$$

where ε_{UT} is the friction energy for the piston per cycle, $\varepsilon_{Ut}$the friction energy for the piston rod per cycle, and $e$is the eccentricity of the drive unit.

2.4 Leakage through piston and piston rod sealing

The reasons for energy losses in flow machines are also losses due to leakage in the sealing nodes. In the suction-pressure unit, there may be internal leaks between the piston and cylinder Q_{AB} and the external one at the piston rod Q_B. With the use of typical sealing applied in hydraulic cylinders, these losses are associated only with the lubrication of these nodes and are therefore negligible small [14, 17, 20–22]. However, it is possible to use solutions that allow leakages. This is explained by the fact that durability can be increased and friction is reduced in these nodes, while external leaks may be discharged into the tank. By formulating a general model of the real sucking-pressing unit, this type of loss should be taken into account. Leaks will depend on the pressure difference between the two sides of the seal. The character of this phenomenon is initially presumed as the root relation (or generally, power characteristics) and is described as follows:

$$\dot{m}_{AB} = \frac{Q_{AB}}{v_w} = \frac{k_{AB} \cdot \sqrt{|p_A - p_B|}}{v_w} \tag{17}$$

and

$$\dot{m}_B = \frac{Q_B}{v_w} = \frac{k_B \cdot \sqrt{p_B}}{v_w}, \tag{18}$$

where k_{AB} and k_B are the coefficients related to leaks in sealing nodes.

In the case of leakage, losses are generated. These can be included in the overall energy balance. During a single cycle, their values can be defined as:

$$\varepsilon_{AB} = m_{AB} \cdot (i_w - i_s) \tag{19}$$

and

$$\varepsilon_B = m_B \cdot i_w, \tag{20}$$

where i_w is the specific enthalpy of the working fluid in the discharge (pressure) channel and i_s is the specific enthalpy of the working fluid in the suction channel.

3 Balance of the real suction-pressure unit

In the analysis of energy losses of the unit (Figure 3a), it is necessary to study volumetric (Figure 3b) and mass (Fig-

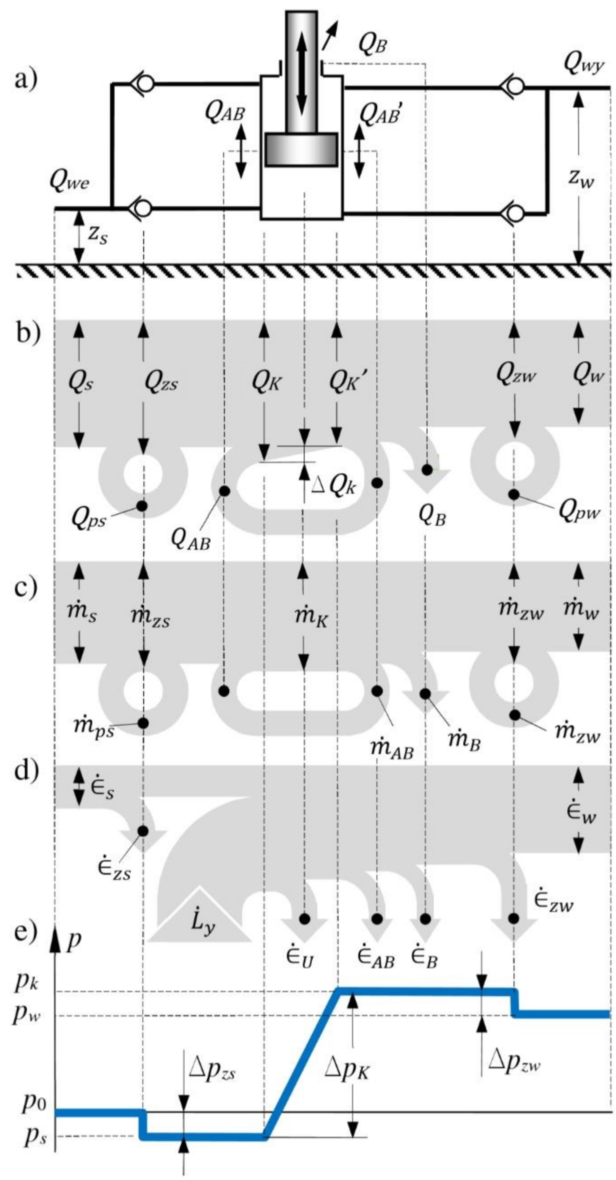

Figure 3: Suction-pressure unit: a) functional diagram, b) band chart of volume flux, c) band chart of mass flux, d) band chart of energy, e) pressure change.

ure 3c) balance, energy flow (Figure 3d), and pressure diagram (Figure 3e). The working fluid flows through the suction inlet and check valves, where it partially returns Q_{ps} ($\dot{m}_{ps}$) to the suction channel during closing. The occurance of flow resistance leads to pressure drop Δp_{zs}. The volumetric and pressure losses are the cause of energy losses $\dot{\varepsilon}_{zs}$. In order for the suction-pressure unit to suck in, compress, and pressurize the working fluid, the $\dot{L}_y$ power is fed, which is transferred to the working fluid by the reciprocating motion of the piston and also converted into friction in the sealing nodes $\dot{\varepsilon}_U$. As a result of the increase in the pressure of the fluid Δp_K in the chambers, its volume decreases, and thus the volume flow Q_K also decreases by the value ΔQ_K. There may also be leakages in the sealing nodes of the piston Q_{AB} ($\dot{m}_{AB}$) and piston rods Q_{AB} ($\dot{m}_{AB}$), causing further energy losses, $\dot{\varepsilon}_{AB}$ and $\dot{\varepsilon}_B$, respectively. During pressing, the working fluid flows out through the discharge port and check valves, in which during its closing, it partially returns Q_{pw} ($\dot{m}_{pw}$) from the discharge channel to the chambers. As a result of the flow resistance, there is also a pressure drop Δp_{zw}. The volumetric and pressure losses created are the cause of energy losses $\dot{\varepsilon}_{zw}$.

The mass flux $\dot{m}_w$ (Figure 3c) flowing out from the unit corresponds to the mass flow $\dot{m}_s$ sucked in, but is reduced by the mass flux of possible external leakages $\dot{m}_s$ occurring in the sealing node of the piston rod. The mass flux constituting leaks in the suction valves $\dot{m}_{ps}$ and discharge valves $\dot{m}_{pw}$ and the leakages on the piston $\dot{m}_{AB}$ are internal leakages, therefore they do not cause mass losses in the system. However, they affect the reduction of specific mass efficiency and volume. In the steady state of the suction-pressure unit, it is possible to balance the energy that has been brought in and out of the system. The energy flux flowing into the system is the sum of:

- energy flux of the working fluid sucked into the system and
- work supplied through the piston rod drive.

Energy losses are mainly caused by:

- flow resistance in suction valves (section 2.2),
- losses of the working liquid during the closing of suction valves (section 2.2),
- friction in the sealing nodes of the piston and piston rod (section 2.3),
- losses of liquid at the piston sealing (section 2.4),
- losses of liquid at the piston rod sealing (section 2.4),
- flow resistance in pressure check valves (section 2.2), and
- losses of the working fluid when closing the discharge check valves (section 2.2).

4 Internal work

The term "internal work of a real suction-pressure unit" is defined as the work transferred to the working fluid through the piston. In the p–V plot, this work is determined by the area covered by the thermodynamic curves of the indicator diagrams for chambers A and B (Figure 4). Knowing the equations describing these curves, it is possible to determine the internal work of chambers, and thus the whole unit. As the starting point for each circuit, points 2' and 6' can be assumed, which correspond respectively to the piston chamber DMP and the piston rod chamber GMP. At these points, the direction of the piston motion changes as suction ends and compression of the working fluid commences. During the automatic closing of the suction check valves (point 2.2), the working fluid flows out isobarically ($2'$–$2''$, $6'$–$6''$) from the chambers to the suction outlet. Further movement of the piston causes the isentropic pressure to increase ($2''$–$3'$, $6''$–$7'$) until the pressure, p_w, existing in the discharge connector (through which the working

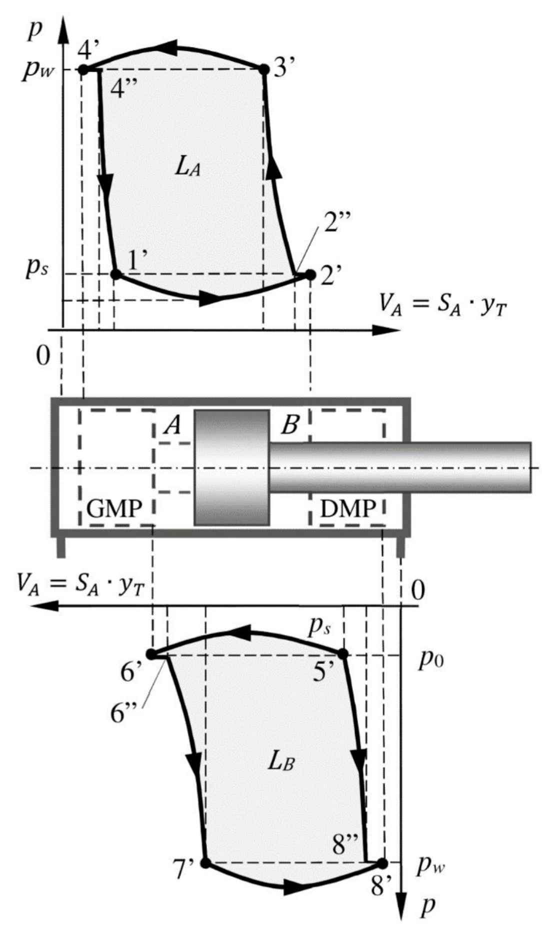

Figure 4: Indicator diagrams of real suction-pressure unit.

fluid is removed) is reached. The flow through the pressure check valves results in a pressure drop, which depends on the flow rate, that is, the speed of the piston. The piston generates, accordingly, higher pressure in the chambers (3′–4′, 7′–8′). After another change in the direction of the piston motion, during the automatic closing of the pressure check valves, isobaric outflow of the working liquid (4′–4″, 8′–8″) occurs from the pressure connector into the working chambers. Once the pressure check valves are closed, an adiabatic expansion of the working liquid (4″–1′, 8″–5′) occurs in the idle space of the chambers and channels. When the pressure in the chambers is balanced against the pressure existing in the suction hose, the automatic valves are opened and after a further pressure drop, depending on the piston speed, the working fluid (1′–2′, 5′–6′) is sucked. This flow causes a pressure drop at the intake valves. The internal work in the chambers can be described by equations:

$$\begin{aligned} L_A &= \varepsilon_{1'}^{4''} V dp \\ &= L_{1'-2'} + L_{2'-2''} + L_{2''-3'} + L_{3'-4'} + L_{4'-4''} + L_{4''-1'} \end{aligned} \tag{21}$$

and

$$\begin{aligned} L_B &= \varepsilon_{1'}^{4''} V dp \\ &= L_{5'-6'} + L_{6'-6''} + L_{6''-7'} + L_{7'-8'} + L_{8'-8''} + L_{8''-5'}. \end{aligned} \tag{22}$$

5 Efficiency

5.1 Hydraulic useful energy

Considering the efficiency of the suction-pressure unit, it should be treated as a hydraulic pump, for which the useful energy of the working fluid pushed through the outlet nozzle should be defined. Most often, in hydraulics, it is referred to as isochoric transformation energy [14], which can be expressed by the area covered by the 1–2a–3–4–1 curves on the indicator diagram (shown in Figure 5) and the relation:

$$\varepsilon_{uV} = m_w \cdot v_w \cdot (p_w - p_s), \tag{23}$$

where v_w is the specific volume of the working fluid in the pressure channel.

In fact, the energy determined by equation (23) is accompanied by the energy of the working fluid pushed by the suction-pressure unit, defined by the area 2b–2b–3–2a on the indicator diagram, expressed by the equation:

$$\begin{aligned} \varepsilon_{us} &= m_w \cdot (u_w - u_s + p_w \cdot v_w - p_s \cdot v_s) \\ &= m_w \cdot (i_w - i_s), \end{aligned} \tag{24}$$

where u_s is the specific internal energy of working fluid in the suction channel, u_w the specific internal energy of working fluid in the pressure channel, and v_s is the volume of working fluid in the suction channel.

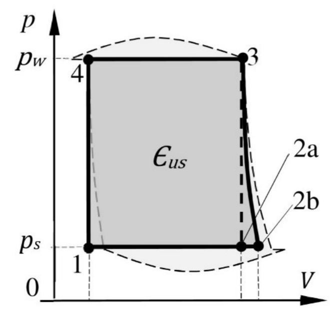

Figure 5: Indicator diagram of useful energy.

The useful energy of the working fluid can also be denoted as the difference between the energy of sucked and pressed out working fluid:

$$\varepsilon_{us} = \varepsilon_w - \varepsilon_s. \tag{25}$$

5.2 Supplied mechanical energy

In the presented suction-pressure unit, mechanical energy is supplied through a reciprocating piston rod. Its value is the sum of the hydraulic useful energy of the pressed out working liquid and the energy losses generated in the suction-pressure unit. It can be expressed by a relationship expressing energy as:

$$L_y = \varepsilon_{zs} + \varepsilon_U + \varepsilon_{AB} + \varepsilon_B + \varepsilon_{zw}. \tag{26}$$

5.3 Total efficiency

The degree of energy conversion in the suction-pressure unit is characterized by the total efficiency coefficient, which is defined [23] as the ratio of the obtained hydraulic power to the external mechanical output or as the ratio of mechanical work to the energy of the working fluid:

$$\eta = \frac{P_{us}}{P_y} = \frac{\dot{\varepsilon}_{us}}{\dot{L}_y} = \frac{\varepsilon_{us}}{L_y}, \tag{27}$$

where P_{us} is the useful power of the working fluid and P_y is the mechanical power supplied through piston rod.

In the unit, when the factors influencing its operation are taken into account, its total efficiency can be determined by following equation:

$$\eta = \frac{\varepsilon_w - \varepsilon_s}{\varepsilon_{zs} + \varepsilon_U + \varepsilon_{AB} + \varepsilon_B + \varepsilon_{zw}}. \tag{28}$$

6 Experimental procedure

Figure 6 shows a flowchart of the test stand. The suction-pressure unit (1), with a valve timing system consisting of non-return valves (2), is connected to the hydraulic oil tank (3). Pressure relief valve (4) is used to set pressure in the system. There is also a flow meter (5) in the output line of the unit. The piston rod of the unit (1) is connected via a force sensor (6) to the piston rod of the drive actuator (7), which is additionally equipped with a position sensor (8). The drive actuator (7) is powered from separate hydraulic power supply (9) and is controlled by a servo valve (10). A heater (11) is installed in the oil tank for the tested suction-pressure unit in order to heat the test oil. In addition to the force and position of the piston rod, pressure in the unit's output line (12) is registered. Also, each chamber, under and over the piston, is equipped with separate pressure sensors (13, 14). The control system and measuring system are connected to a computer (15), whose task is to control the stand and save data.

Registration of the above-mentioned parameters of the suction-pressure unit operation will enable the following tests to be carried out:

- Tests simulating operation of the unit as a part of the pump; control of the drive actuator which simulates the operation of the unit by making reciprocating motions with the sinusoidal movement of the piston speed, recording of force, position of the piston, and pressures will enable determination of:
  - (i) pressure differences between the piston and the piston rod chambers;
  - (ii) piston speeds; and
  - (iii) frictional force of the seals during insertion and ejection of the piston rod of the suction-pressure unit.
- Leakage test on the sealing node of the piston, change in pressure differential on the two sides of the sealing node, recording the position of the piston and the pressures in both chambers of the unit will enable determination of:
  - (i) pressure differences between the piston and the piston rod chamber;
  - (ii) pressure difference between the piston rod chamber and the environment; and
  - (iii) leakage rate through piston seals.
- Testing of the total efficiency of the pump; control of the drive actuator which simulates operation of the unit by making reciprocating motions with the sinusoidal movement of the piston, recording the force, displacement of the piston rod, and the pressure as well as the flow rate in the output line of the suction-pressure line will enable determination of:
  - (i) energy supplied by the driving actuator (product of force and displacement);
  - (ii) hydraulic power of the suction-pressure unit (product of flow rate and pressure); and
  - (iii) total efficiency of the unit (quotient of the hydraulic power generated by the suction-pressure unit and the energy supplied by the driving actuator).

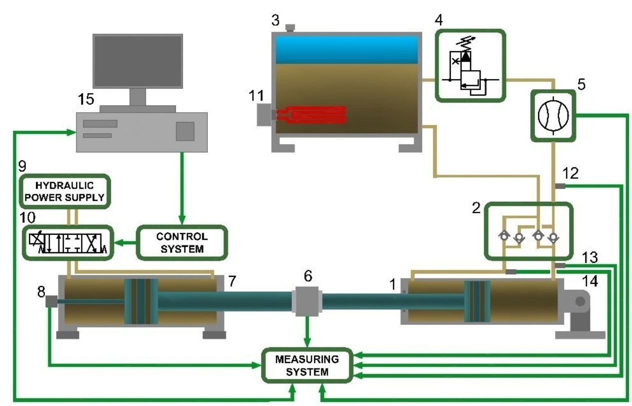

Figure 6: Flowchart of the test stand: 1 – suction-pressure unit, 2 – valve timing, 3 – hydraulic oil tank, 4 – pressure relief valve, 5 – flowmeter, 6 – force sensor, 7 – drive actuator, 8 – position sensor, 9 – hydraulic power supply, 10 – servo valve, 11 – heater, 12–14 – pressure sensors, 15 – computer.

7 Summary

The actual suction-pressure unit presented in this article is significantly different from the ideal one. Thermodynamic transformations of the working fluid as well as mathematical descriptions of factors affecting its work are included in the paper. The presented models of occurring phenomena will be used for further work on low-speed hydraulic pumps, which consist of radially distributed suction-pressure units. Based on the equations developed and the presented factors affecting work of the unit, such as friction, pressure drops, flow resistance, deformation of chambers, and mechanical elements, a multi-piston pump model and simulation tests will be created. The presented considerations will, therefore, enable theoretical studies of pumps and multiplying hydrostatic transmissions. Energy analysis of the real suction-pressure unit is important due to the possibility of conducting theoretical studies on the efficiency of various variants and design solutions for low-speed piston pumps and entire transmissions. The presented test procedure, together with the flowchart of the test stand, is to be used to verify the described model. It will allow determining the compatibility of the model with a real suction-pressure unit. Thus, this study is a significant step in the research of low-speed reciprocating pumps and, in the future, their application into multiplying hydrostatic transmissions.

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